The Localized Subtraction Approach For EEG and MEG Forward Modeling

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Introduction

- EEG and MEG forward problems: simulate electric potentials and magnetic fluxes generated by given neural activity
- Neural activity is typically modeled as a linear combination of point dipoles
- **Problem**: How to incorporate singular point dipole into a FEM formulation?
- Subtraction approach: Subtract singularity out of the problem formulation. This was shown to produce highly accurate results but is not used in practice due to high computational demand [1], [2].
- **Approach: Restrict subtraction to spatially confined region to arrive at a computationally efficient FEM formulation. We call this the *localized subtraction approach*.**

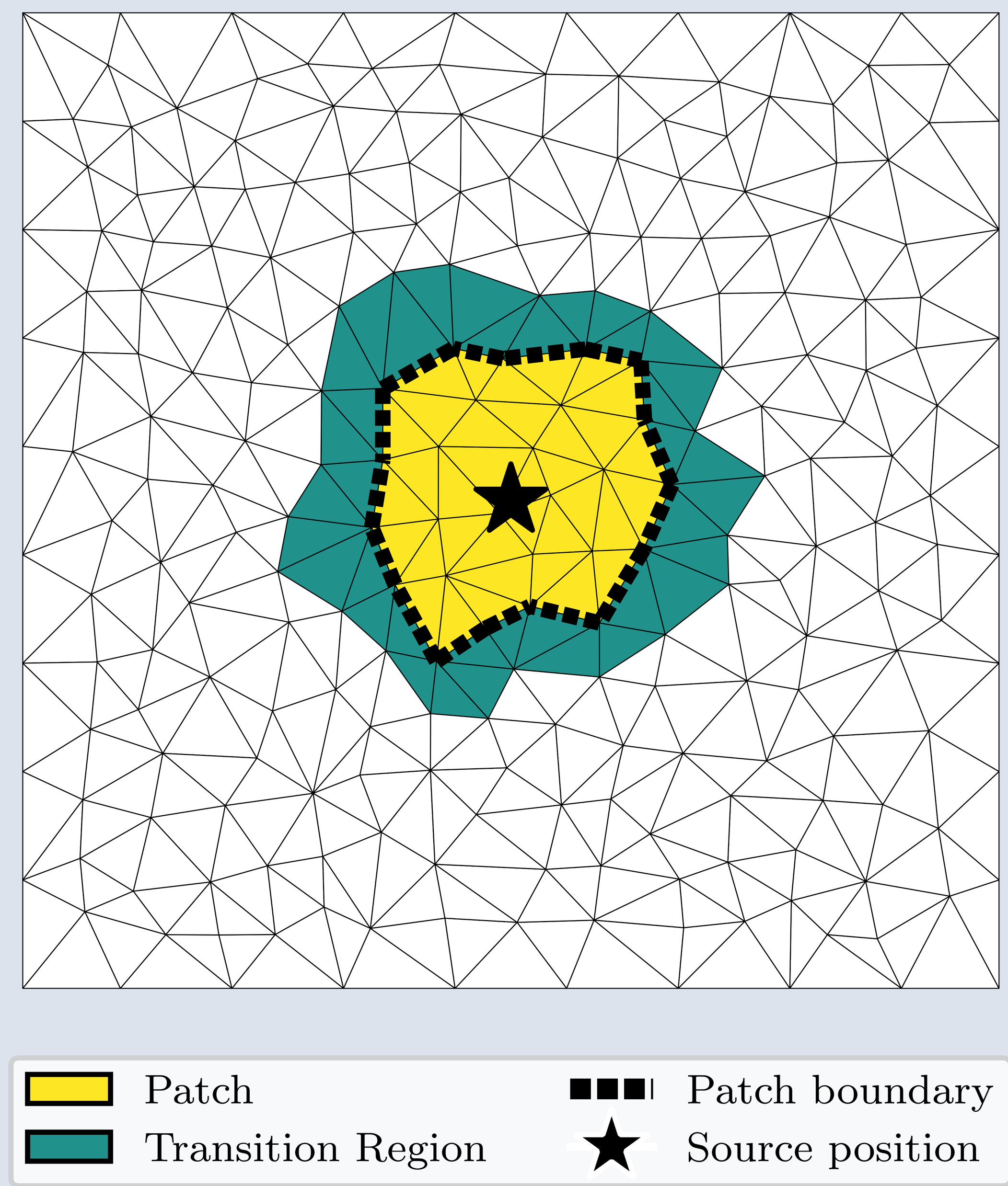
I. Localizing the subtraction

- We cannot (in general) analytically compute the electric potential u generated by dipolar activity inside the head. But in the special case of an unbounded homogeneous conductor, there are analytic formulas for the resulting potential u^∞.
- Let χ be a **cutoff function**, meaning χ is 1 on an environment of the dipole (the **patch**) and then quickly transitions to zero.
- Instead of directly solving for u, we instead solve for the **correction potential**

$$u^c := u - \chi \cdot u^\infty .$$

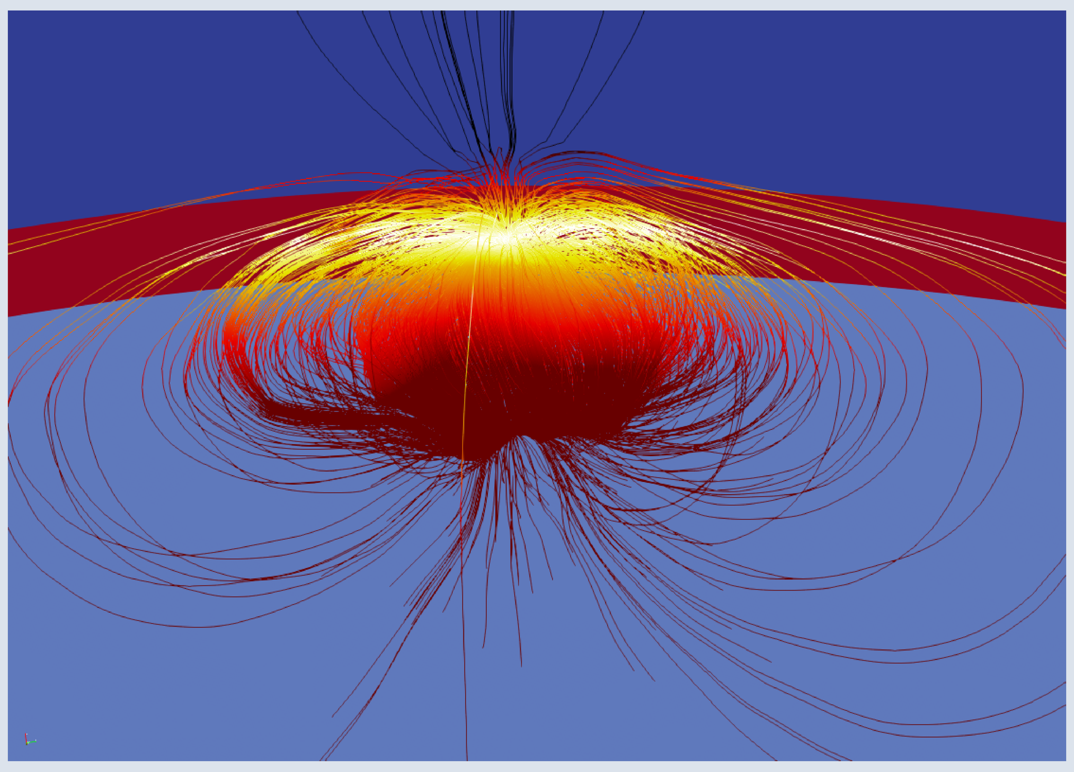
- The equation for u^c does not contain a singularity. Furthermore, the FEM RHS only depends on DOFs corresponding to the patch and the transition region, yielding a fast assembly.

II. Construction of cutoff



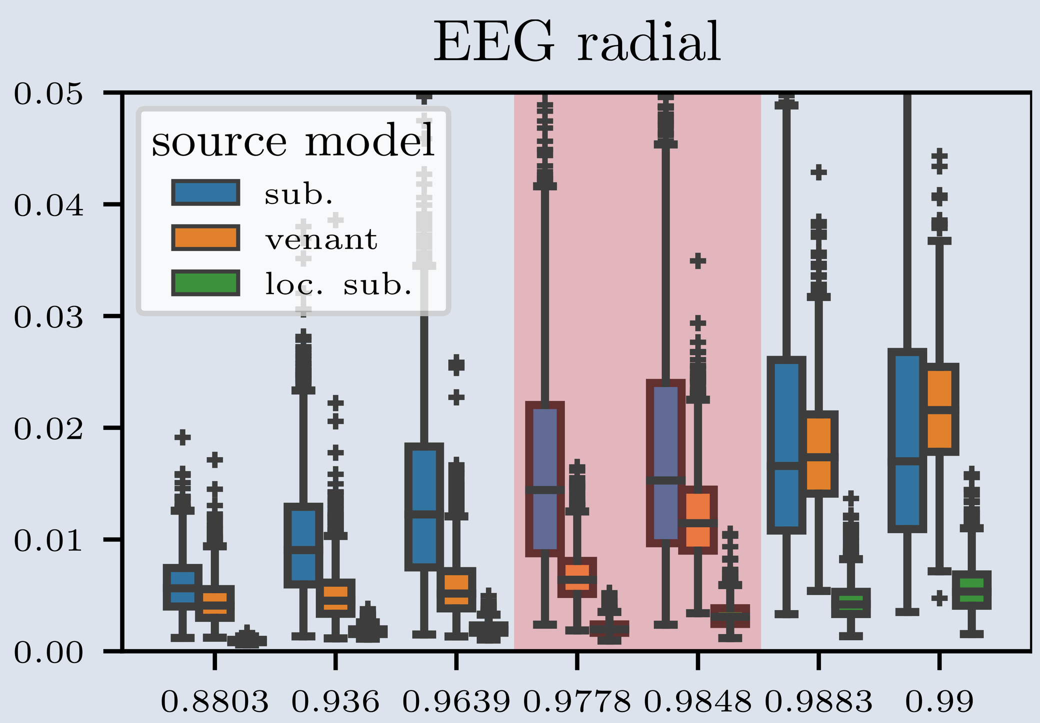
III. Simulated flux

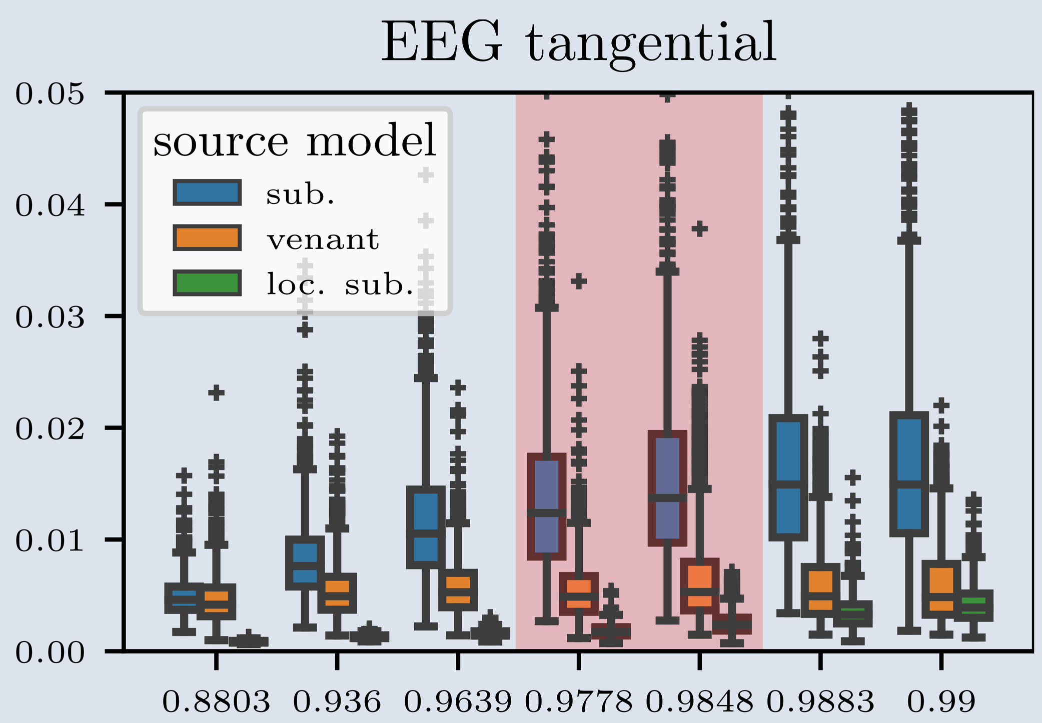
- Radial dipole in multilayer sphere model at 1mm distance to conductivity jump
- Using localized subtraction approach, simulate volume current for 2000 random points in a sphere of radius 2mm around source position



IV. EEG Accuracy comparison

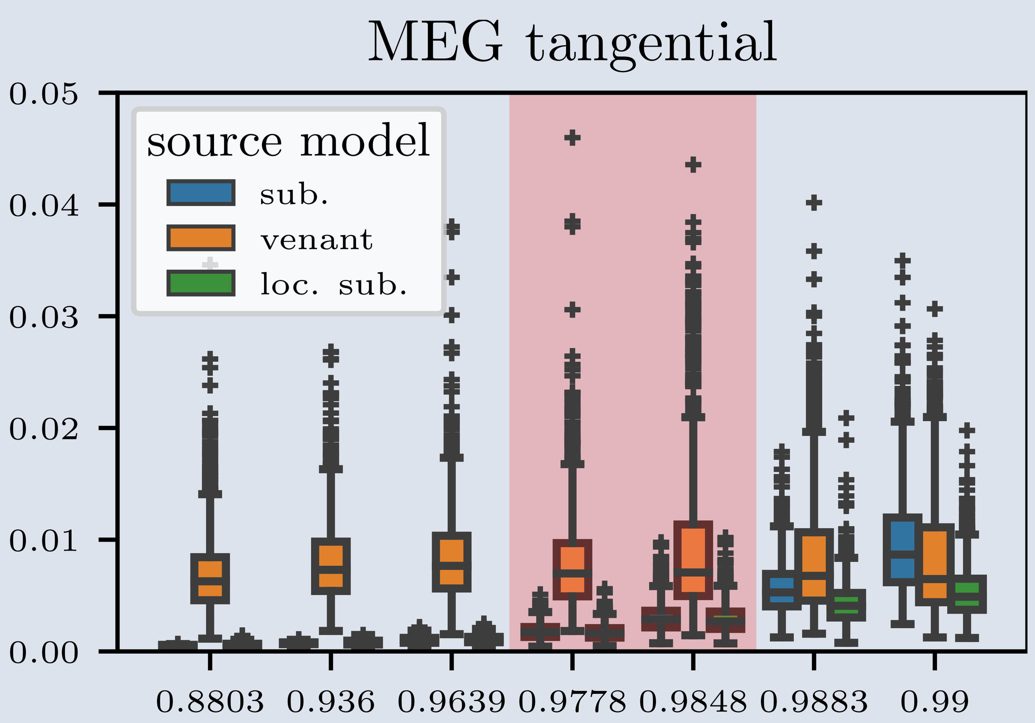
- In 4-layer sphere models, where quasi-analytical solutions for mathematical point dipoles exist, we validate the accuracy of the localized subtraction approach in comparison to the analytical subtraction approach [2] and the venant approach [3].
- In a tetrahedral mesh consisting of 787k nodes and using 200 uniformly distributed electrodes at the sphere surface, we computed the relative errors of the different FEM approaches at different eccentricities for 1000 radial and tangential dipoles each. The physiologically most relevant sources at 1-2mm distance from the CSF are highlighted [4].





V. MEG Accuracy comparison

- Using the model from IV, we computed the magnetic field at 256 coil positions distributed uniformly around the sphere
- Low errors of subtraction approaches show the strong influence of the currents close to the source on the magnetic field



VI. Runtime comparison

We measured the time needed to solve the EEG/MEG forward problem for 1000 dipoles using the localized subtraction and the subtraction approach. The times were measured on an AMD Ryzen Threadripper 3960X CPU using the mesh from IV. and a transfer matrix approach [5].

Approach	EEG time (s)	MEG time (s)
loc. subtr.	0.93	13
subtr.	655	18925

Conclusion

The localized subtraction approach was shown to outperform existing FEM right hand side approaches in terms of both computational efficiency and accuracy in spherical head modeling studies.

Acknowledgements

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