The Localized Subtraction Approach For EEG and MEG Forward Modeling

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***Abstract*—One of the basic problems in EEG and MEG source analysis is simulating the sensor measurements that a given neural activity would generate, i.e., the so-called EEG and MEG forward problem. The neural activity is typically modeled as a linear combination of mathematical point dipoles. When using a finite element method (FEM) for the forward problem this leads to difficulties, as it is not clear how the singularity of a point dipole can be properly incorporated. Various FEM approaches have been proposed, and among these are the so-called subtraction approaches. Subtraction approaches are not only well-founded in theory, but also produce accurate results in practice. Their major downside is that they are computationally prohibitively expensive in practical applications. To overcome this we developed a new approach, called the localized subtraction approach. This approach is designed to preserve the mathematical foundation of the subtraction approach, while also leading to sparse right hand sides in the FEM formulation, making it efficiently computable. In this work, this approach will be presented and compared to other state-of-the-art FEM right hand side approaches with regard to accuracy and computational effort. In multi-layer sphere models, the localized subtraction approach will be shown to be as accurate, and in many cases even more accurate, than the other investigated approaches while being largely more efficient than the subtraction approach.**

***Index Terms*—Source analysis, EEG, MEG, FEM, Source modeling**

I. Introduction

When simulating sensor measurements using a FEM approach, the question of how to treat the singularity at the source position arises. Answers to this question are called *potential approaches* and can be roughly divided into so-called *direct approaches*, which directly incorporate the singularity into the FEM right hand side, and *subtraction approaches*, which "subtract" the singularity out of the problem formulation and then post-process the resulting FEM solution to add the singularity back in. State-of-the-art direct approaches, such as the Venant approach [1] and the H(div) approach [2], have the undesirable property that they do not compute the potential induced by a dipole. They instead substitute the singular object with a more well-behaved function, which can be easily incorporated into a FEM model, and looks like a dipole from afar. Subtraction methods do not suffer from this shortcoming, and were shown to produce highly accurate results [3]. Despite this, they are typically not used in practical applications. The main reason for this is their exceedingly high computational demand, leading researchers and practitioners to instead use the still quite accurate direct approaches, whose corresponding FEM right hand sides can be computed rapidly [4]. Different approaches have been suggested to remedy this drawback of subtraction type methods, such as reducing the computational burden by approximating complicated integrands by simpler ones [5], [6], or by deriving analytical formulas and thus removing the need for an expensive numerical computation [4]. While these approaches all lead to faster computation times, none of them have achieved the goal of making subtraction methods viable in practice.

The fundamental reason for this is that all previous subtraction type methods lead to FEM formulations with dense right hand sides, whose computation requires an iteration over the whole mesh for every dipole position under consideration. Especially for realistic head models, consisting of millions of elements, this leads to the aforementioned severe computational demand. The direct approaches in contrast produce sparse right hands sides, yielding a fast assembly. Based on this observation, we developed a new approach, called the *localized subtraction approach*, possessing the accuracy and mathematical rigor of the subtraction approach, while also leading to sparse right hand sides, whose computation is only based on the local mesh structure around the source position and can be performed in a fast manner.

Another difficulty for subtraction methods arises in the context of MEG forward simulations. Here, one typically first computes the EEG forward solution and then applies Biot-Savart's law [7]. For subtraction methods, this leads to integrals over singular functions. This was addressed in [8], whose work we expanded on and transferred to our new approach.

All methods described in this paper are implemented in the open source DUNEuro toolbox [9] and are available through https://gitlab.dune-project.org/duneuro/duneuro.

II. Methods

Let Ω be the head domain and $\partial\Omega$ its boundary. Let $\sigma : \Omega \to \mathbb{R}^{3\times 3}$ be the conductivity tensor. Using the quasistatic Maxwell equations, as is typically done, and assuming a

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dipolar source, the electric potential u can be described by

$$\operatorname{div}(\sigma \nabla u) = \operatorname{div}(j^p) \quad \text{on } \Omega, \tag{1}$$

$$\langle \sigma \nabla u, \eta \rangle = 0 \quad \text{on } \partial\Omega, \tag{2}$$

where η is the unit outer normal and $j^p = M \cdot \delta_{x_0}$ is a mathematical point dipole with moment M and position x_0 (see e.g. [10]). We assume that σ is constant on a neighborhood of the source position x_0. Let σ^∞ be the value of σ on this neighborhood. Then the equation

$$\operatorname{div}(\sigma^\infty \nabla u^\infty) = \operatorname{div}(j^p) \quad \text{on } \mathbb{R}^3 \tag{3}$$

has a solution u^∞ with an analytic representation, which is hence easily computable (see e.g. [5]). Now let $\chi : \Omega \to \mathbb{R}$ be a function such that

- on a neighborhood of the source position we have $\chi = 1$.
- χ is only nonzero on a small set $\widetilde{\Omega} \subset \Omega$.

We then define the *correction potential* u^c by the equation

$$u = u^c + \chi \cdot u^\infty. \tag{4}$$

The approach now is to use (1), (2), (3) and (4) to derive an equation for u^c. By construction, u^∞ cancels the point dipole, leading to an equation without a singularity. This enables a straightforward computation of u^c. Furthermore, by construction the corresponding FEM right hand side is only non-zero on degrees of freedom corresponding to $\widetilde{\Omega}$, leading to sparse right hand sides, and hence fast computation times.

III. Results

We validate the localized subtraction approach in multi-layer sphere models, where analytical formulas for electric potentials and magnetic fields caused by a dipolar source exist [11], [12]. Comparisons in this setting show that the localized subtraction approach is as accurate as other state-of-the-art potential approaches and in many cases even more accurate. More concretely, in high-quality meshes with sufficiently small elements the localized subtraction approach is about as accurate as the Venant approach [1]. If on the other hand there are badly shaped or sized mesh elements, employing the Venant approach can lead to inaccuracies, while the localized subtraction approach stays accurate. This is illustrated in Figure 1, where a computation in a suboptimal mesh produced many outliers for the Venant approach. Furthermore, the localized subtraction approach is largely more efficient than previous versions of the subtraction approach. On an AMD Ryzen Threadripper 3960X CPU, where the experiments of this work were run, simulating the electric potential for 1000 dipolar sources took less than 1 second for the localized subtraction approach, while the corresponding computation for the ordinary subtraction approach, as described in [4], took about 10 minutes.

IV. Conclusion

The localized subtraction approach is a theoretically sound method that rivals, and in many cases even surpasses, other state-of-the-art potential approaches in terms of efficiency and accuracy.

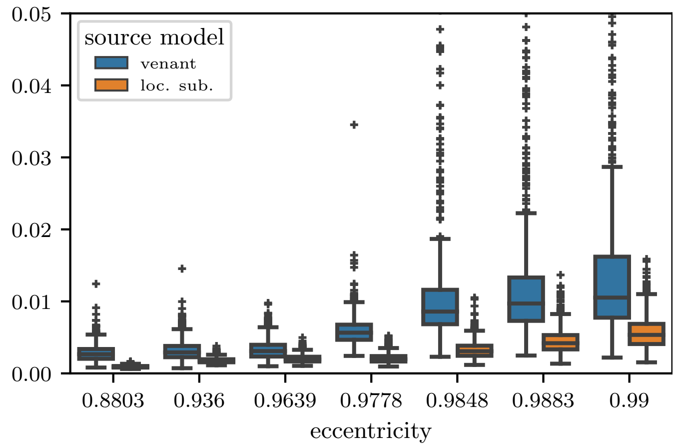

Fig. 1: Relative error between analytical and numerical solutions for the Venant [1] and the localized subtraction approach. For each eccentricity, 1000 radial dipoles were simulated. We used 200 electrode positions distributed approximately uniformly on the sphere surface. Relative errors were computed by first shifting the potential vectors to have zero mean and then using the euclidean distance. The eccentricity is computed as $\frac{\|x_0 - c\|}{r}$, where x_0 is the dipole position, c is the sphere center and r is the radius of the innermost sphere. A dipole is called *radial* if its moment is a multiple of $x_0 - c$. The whiskers extend to 1.5 times the IQR.

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