CutFEM for EEG forward modeling

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***Abstract*—Using the finite element method (FEM) to solve the EEG forward problem requires a volumetric discretization, a mesh of the head domain. When creating the mesh one usually chooses between either hexahedral or tetrahedral elements. The drawback of these options is that they either cannot accurately represent highly folded brain structures or require assumptions such as nestedness of tissue compartments. Methods - The above mentioned issues can be avoided by introducing a category of unfitted finite element approaches where the mesh is disentangled from the geometry which is represented through level set functions. The purpose of this work is to present one such approach, CutFEM, to EEG source analysis. Following a description of the method, we will employ it in a controlled multi-layer sphere scenario. Results - CutFEM outperforms an existing unfitted FEM with regard to memory consumption and speed and a geometry adapted hexahedral model with regard to accuracy while also being able to mesh arbitrarily touching compartments. Conclusion - CutFEM strikes a balance of numerical accuracy, computational efficiency and ability to model complex geometries that was previously not available in FEM-based EEG forward modeling.**

***Index Terms*—EEG forward modeling, unfitted FEM**

I. Introduction

Electroencephalography(EEG) is a widely used tool for the measurement of neural activity in the human brain. Before a source reconstruction of measured data can be undertaken, one has to simulate the electric potential as induced by a current source in the brain, i.e. the EEG forward problem has to be solved. While quasi-analytical solutions to the differential equation underlying the forward problem exist, these are only available in spherical scenarios [4]. To incorporate realistic head tissues and conduction effects such as white matter anisotropy one can employ the finite element method (FEM).

Recently, an unfitted discontinuous Galerkin method (UDG) [6] was introduced to solve the EEG forward problem. Rather than working with mesh elements that are tailored to the geometry, it uses a background mesh which is cut by level set functions, each representing a tissue surface. It was shown to outperform a discontinuous Galerkin approach on a hexahedral mesh while not being limited to the assumptions necessary to create tetrahedral meshes.

Extending on UDG, this paper introduces CutFEM. It reduces the number of degrees of freedom required and adds a ghost penalty stabilization term based on [1].

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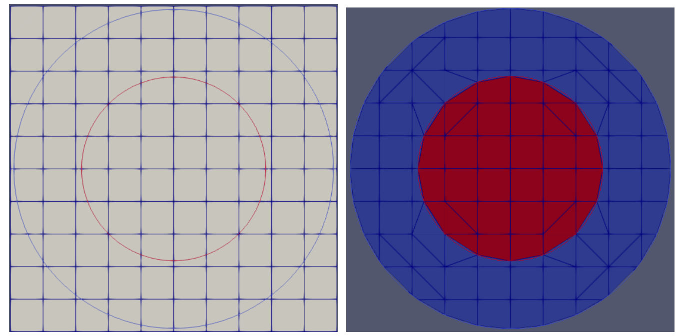

Fig. 1: Left: Fundamental mesh with two spherical level sets. Right: Through TPMC reconstructed compartment snippets

II. Methods

A. CutFEM

As mentioned, CutFEM [2] uses a level set based representation of domain surfaces. Let $\Omega = \cup_i \Omega_i$ be the head domain split into m disjunct open subdomains, i.e. the different tissues. The level set function for compartment i is then defined as $\Phi_i(x) \begin{cases} < 0, & \text{if } x \in \Omega_i \\ = 0, & \text{if } x \in \partial\Omega_i \\ > 0, & \text{else.} \end{cases}$ We proceed by defining a background domain $\hat{\Omega} \subset \mathbb{R}^3$ covering the head domain Ω. This background is then tesselated, yielding a regular hexahedral mesh $\mathcal{T}(\hat{\Omega})$, the fundamental mesh.For each compartment i a submesh $\mathcal{T}_h^i \subset \mathcal{T}_h(\hat{\Omega})$ with a standard continuous Galerkin (CG-) FEM Ansatzfunctionspace V_h^i is created from all fundamental elements that are (partly) contained within the respective domain. These submeshes can and will overlap at compartment boundaries. Each Ansatzfunctions' support is then restricted to its compartment, cutting it off at the boundary and giving rise to the name CutFEM. Each fundamental mesh cell has thus one set of Ansatzfunctions per level set that cuts it.

The Ansatzfunctions are only continuous on their respective compartment and cut off at the boundary, yielding discontinuities in potential and electric current. This is resolved by adding internal boundary conditions. These additions to the standard EEG forward problem can be seen in the following.

$$\nabla \cdot \sigma \nabla u = f, \quad \text{in } \bigcup_i \Omega_i \tag{1}$$

$$\langle \sigma \nabla u, n \rangle = 0, \quad \text{on } \partial\bar{\Omega} \tag{2}$$

$$[\![u]\!] = 0, \quad \text{on } \Gamma \tag{3}$$

$$[\![\sigma \nabla u]\!] = 0, \quad \text{on } \Gamma. \tag{4}$$

$[\![v]\!]$ defines the jump of a function $v \in V_h$ on Γ, σ a positive definite conductivity tensor. Equation (2.6) ensures that the

	CutFEM	UDG	Hex
number DoFs	552 985	3 601 824	3 341 280
max. RAM used	6.91 GB	64.77 GB	40.2GB
Driver setup	44s	45s	52s
Matrix assembly	319s	161s	25s
Solver setup	353s	235s	45s
Solving	1111s	2367s	1550s
Lead field	22s	20s	125s
Total time	1849s	2828s	1797s

TABLE I: Computation times, RAM/DoF usage in the shifted sphere model.

electric potential is continuous over internal boundaries while (2.7) ensures conservation of charge over the compartments. These conditions are only enforced weakly, similar to a discontinuous galerkin approach. As Test-/Ansatzfunctionspace we employ V_h as direct sum of all V_h^i.

To integrate over the compartments, a topology preserving marching cubes (TPMC) algorithm [5] is employed. Mesh cells cut by a level set are replaced by a set of snippets, each completely contained within one compartment and of a simple geometry (simplices or cubes). See Fig. 1 for an illustration.

To stabilize distorted or sliver-like snippets, a ghost penalty based on [1] is added, weakly coupling the gradients of the Ansatzfunctions on the snippets to their neighbors.

B. Shifted 4-layer sphere model

We test CutFEM in a 4-layer sphere model with brain, CSF, skull and scalp as compartments. The brain sphere will be shifted to one side such that there is exactly one point where skull and brain touch. Such points occur naturally in the MRI when the subject lies down and the brain sinks to the back of the skull. Conductivities were chosen according to [3], with the exception that CSF and brain use the same conductivity. In terms of volume conduction the model is thus indistinguishable from a 3-layer concentric sphere model.

We compare the previously existing UDG method, a geometry adapted hexahedral CG-approach and CutFEM with respect to accuracy and computation time.

Error measures are the relative difference measure (RDM) and magnitude error (MAG). They respectively state the difference in potential distribution and strength as compared to the analytical solution.

III. Results

Computation times and memory usage can be found in Table 1, error comparisons in Fig 2. CutFEM is as computationally efficient as the hexahedral version while only using a fraction of the memory and degrees of Freedom. Compared to UDG, it is around 35 percent faster and also much more memory-efficient.

CutFEM outperforms hexahedral CG in all eccentricity categories and for both radial and tangential source directions. The head model including the TPMC reconstruction is completely identical for both CutFEM and UDG, indicating that the larger variance in the UDG results can be explained by CutFEM's use of the ghost penalty term. The overall largest absolute error values for CutFEM are 3.08 % RDM and 8.21 % MAG, underlining its stability.

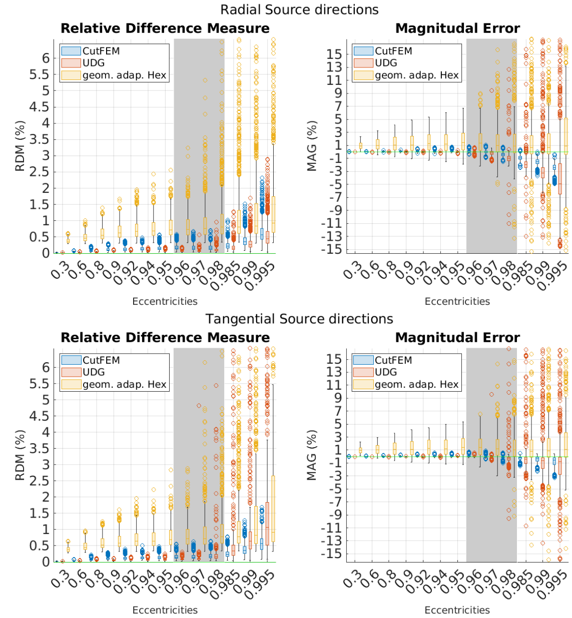

Fig. 2: EEG-Errors for hexahedral CG-FEM and unfitted FEM approaches in a shifted sphere scenario Top: Errors for tangential source directions. Bottom: Errors for radial source directions. Errors are in percent and grouped by eccentricities. The green line marks optimal error values.

IV. Conclusion

In this work CutFEM, an unfitted finite element approach, was presented for EEG. It combines the numerical accuracy of tetrahedral meshing approaches while requring no assumptions about the structure and nestedness of the head compartments. It was shown to be more accurate than a geometry adapted hexahedral mesh approach and more economic than a previously existing unfitted FE approach.

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