The Discontinuous Galerkin Finite Element Method for Solving the MEG and the Combined MEG/EEG Forward Problem

M.C. Piastra[1,2], A. Nüßing[1,2], J. Vorwerk[3], H. Bornfleth[4], R. Oostenveld[5,6], C. Engwer[2,7], C.H. Wolters[1]

mcpiastra@uni-muenster.de

[1]Institute for Biomagnetism and Biosignalanalysis, University of Münster, Münster, Germany
[2]Institute for Computational and Applied Mathematics, University of Münster, Münster, Germany
[3]Computing and Imaging Institute, University of Utah, Salt Lake City, UT, United States
[4]BESA GmbH, Graefelfing, Germany
[5]Donders Institure, Radboud University, Netherlands
[6]NatMEG, Department of Clinical Neuroscience, Karolinska Institutet, Stockholm, Sweden
[7]Cluster of Excellence EXC 1003, Cells in Motion, CiM, University of Münster, Münster, Germany

Westfälische Wilhelms-Universität Münster

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In source reconstruction, solutions for both the forward and inverse problem are required, and the accuracy of the inverse solution depends also on the one of the forward solution. When dealing with realistic head models, numerical methods have to be adopted for solving the forward problem [1]. Among others, the Discontinuous Galerkin – Finite Element Method (DG - FEM) allows for the fulfilling of conservation laws, even on a discrete level [2]. In EEG studies it has already been remarked how this property prevents the occurrence of unwanted effects, e.g. unphysical leakages of volume currents in regular hexahedral meshes with insufficient resolution [3]. Moreover it puts the basis for the application of the so called Unfitted Discontinuous Galerkin FEM (UDG-FEM), an extended version of the DG-FEM that allows smooth tissue surface representations, whose advantages have been already shown in EEG studies [4]. Our goal in this work is to investigate accuracy and leakage aspects of CG-FEM and DG-FEM for the MEG forward problem as the first important step towards an UDG-FEM implementation for MEG.

MEG Forward Problem

$$\mathbf{B}(\mathbf{r}) = \mathbf{B}^p(\mathbf{r}) - \underbrace{\frac{\mu_0}{4\pi}\int_\Omega \mathbf{j}^s(\mathbf{r}') \times \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\, d^3\mathbf{r}'}_{\mathbf{B}^s(\mathbf{r})}$$

where

- $\mathbf{r}$ is the coil position
- $\mathbf{j}^s$ is the secondary electric current density

Materials and Methods

We developed, implemented and evaluated one CG-FEM and two new DG-FEM approaches, a conservative and a non-conservative one, to solve the MEG forward problem.

- The subtraction approach (see [5]) was adopted to discretize the source term
- For the Statistics, we used the head model described in Table1, discretized with three hexahedral mesh with 4 mm, 2mm and 1mm resolution. We generated 8,000 dipoles with purely tangential orientations and unit strengths, uniformly distributed inside the brain compartment on spherical surfaces with 8 different logarithmically scaled eccentricities. We used 256 point-magnetometers outside the sphere model at a fixed radius of 110 mm. We measured the topographical (RDM%) and magnitude errors (MAG%) of the secondary and full B-field.
- For the Leaky Scenario, we used the same head model as for the Statistics, but we thinned the skull compartment.
- The transfer matrix approach was used to speed up computations, see [6].
- All methods were implemented in **duneuro**.

material	out. radius (mm)	conductivity (S/m)
brain	78	0.33
csf	80	1.79
skull	86	0.01
skin	92	0.43

Table1: head model features [3]

$$RDM\%(f^{ana}, f^{num}) := \left\|\frac{f^{num}}{\|f^{num}\|_2} - \frac{f^{ana}}{\|f^{ana}\|_2}\right\|_2$$

$$MAG\%(f^{ana}, f^{num}) := \frac{\|f^{num}\|_2}{\|f^{ana}\|_2}$$

- **DUNE** (Distributed and Unified Numerics Environment) is a C++ open source library for the discretization and solution of partial differential equations (PDEs).
- **duneuro** is a module of DUNE specialized in solving PDEs in Neuroscience (www.duneuro.org)

http://dune-project.org/

I. conservation property

In the DG framework, the property of conservation of charge is fulfilled, therefore there are two formulations of the secondary B-field:

$$\mathbf{B}^{s,NONcons}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_\Omega \mathbf{j}^{s,NONcons}(\mathbf{r}') \times \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\, d^3\mathbf{r}'$$

$$\mathbf{B}^{s,cons}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_\Omega \mathbf{j}^{s,cons}(\mathbf{r}') \times \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\, d^3\mathbf{r}'$$

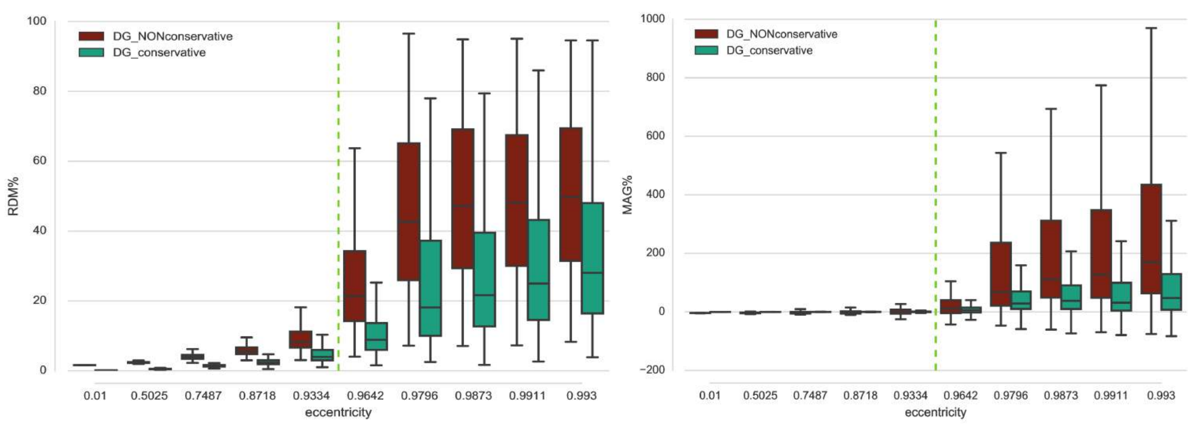

Figure1: Accuracy comparison for secondary B-field computation between DG-FEM with non-conservative flux (in red) and DG-FEM with the conservative flux (in green) in a 4 mm hexahedral sphere model: visualized are the boxplots of the RDM% (left) and MAG% (right), for tangentially oriented sources at logarithmically-scaled eccentricities. The dashed green line represents the eccentricity of 4 mm distance to the brain-CSF boundary.

II. convergence of DG-FEM

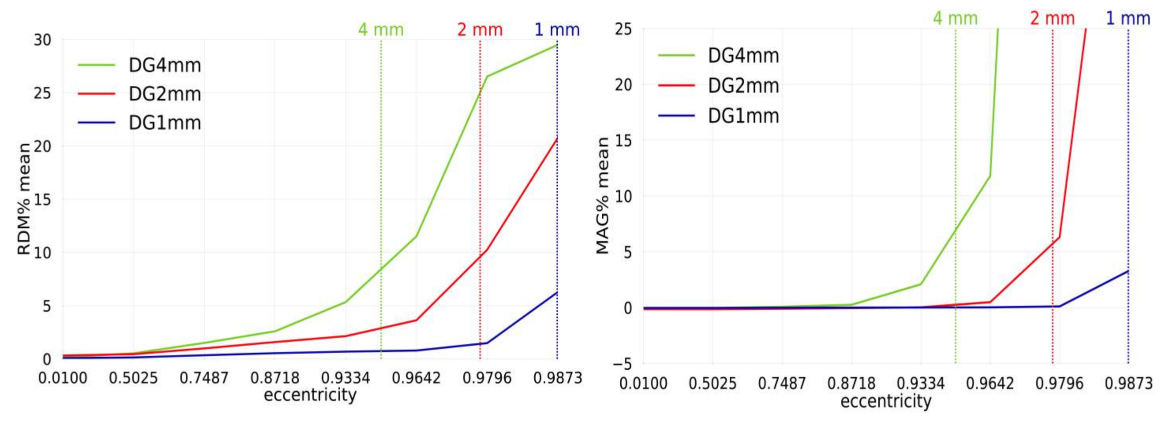

Figure2: Validation and convergence analysis for secondary B-field computation of DG-FEM with conservative flux in a 4 mm (green), 2 mm (red) and 1 mm (blue) hexahedral sphere model: visualized are the means RDM% (left) and MAG% (right), for tangentially oriented sources at logarithmically-scaled eccentricities.

III. general behavior of DG-FEM

The full B-field is computed in three meshes with increasing resolution, i.e., 4 mm, 2mm and 1 mm.
The CG-FEM results are compared with DG-FEM results.
RDM% and MAG% are analyzed and visualized in Figure 3.

For the most relevant eccentricity of 0.9796 (i.e., 1.59 mm from the CSF compartment), the highest resolved model (1 mm resolution) reaches mean RDM% and MAG% errors of 1.5% and 0.1%, respectively.

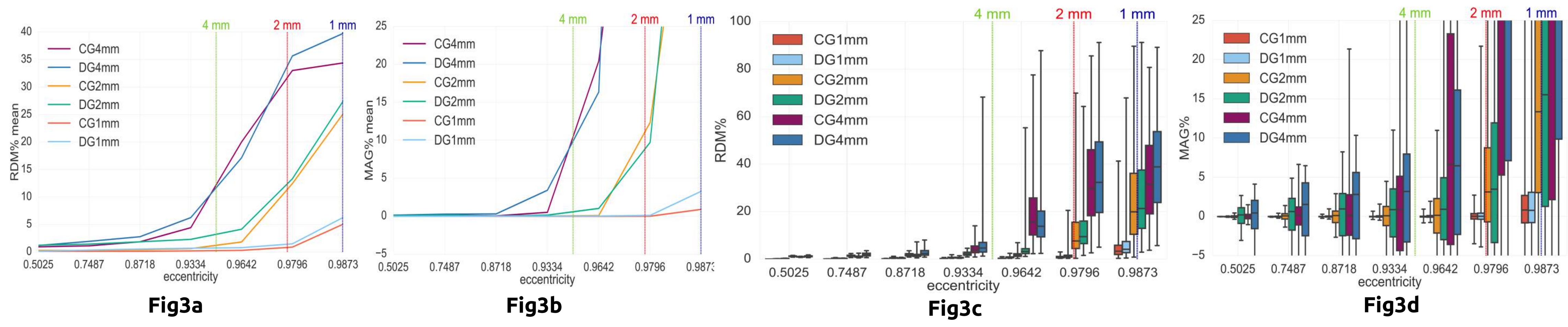

Figure3a-d: Accuracy comparison between CG- and DG-FEM for solving the MEG forward problem, i.e., the full B-field, for different mesh resolutions. Visualized are the means (3a-3b) and the boxplots (3c-3d) of the RDM% (3a-3c) and MAG% (3b-3d), for tangentially oriented sources at logarithmically-scaled eccentricities. Dipoles not belonging to the brain compartment are excluded from the statistics. Dashed lines represent the eccentricities of 4 mm (green), 2 mm (red) and 1 mm (blue) distances to the brain-CSF boundary. Note the different scaling of the y-axes.

IV. leaky scenario

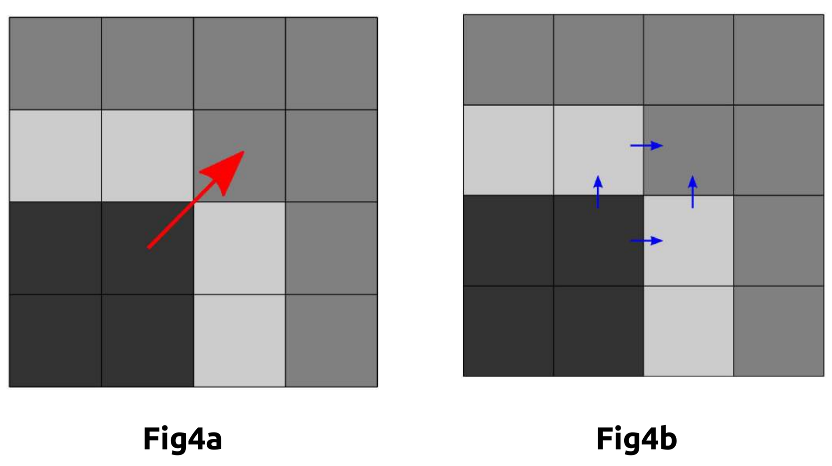

Fig4a-b: Illustration of current flow/leakage effect for CG-FEM (4a) and DG-FEM (4b). While for the CG-FEM an *unphysical* current flow through a single vertex occurs, the DG-FEM only allows current flow over faces, see [3].

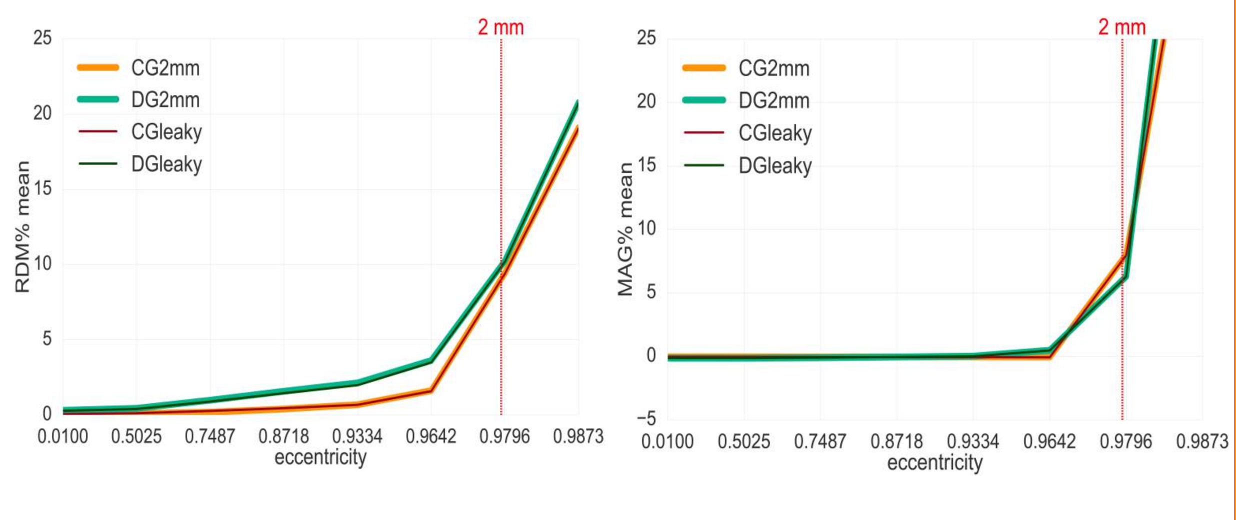

Figure5: Accuracy comparison for secondary B-field computation between CG-FEM (in warm colors) and DG-FEM with the conservative flux (in cold colors), in two different 2 mm hexahedral sphere models, i.e., a spherical head model with and without skull leaky points. Visualized are the means of the RDM% (left) and MAG% (right), for tangentially oriented sources at logarithmically-scaled eccentricities.

V. realistic head model

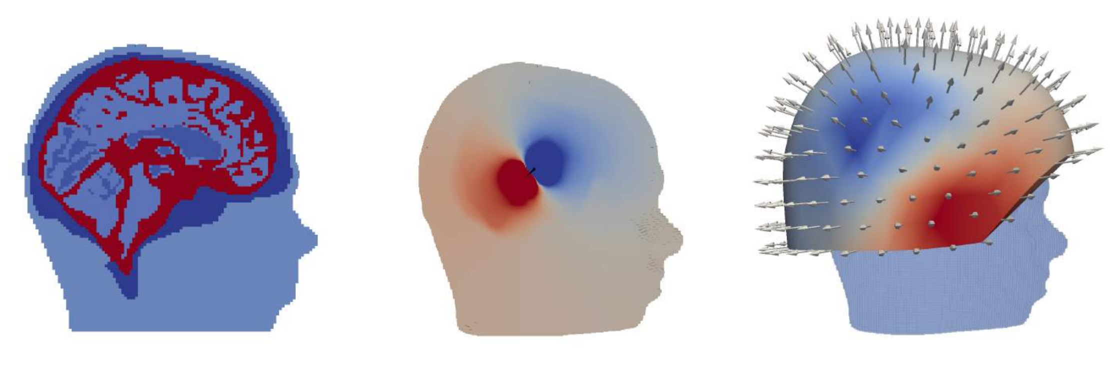

Figure6: Exemplary EEG and MEG forward computation for an auditory source computed using DG-FEM in a realistically shaped head model. Hexahedral mesh with 2 mm resolution, 6 compartments, sagittal slice (left); electric potential distribution visualized on the clipped volume conductor model in the sagittal plane where the auditory dipole (black cone) lies (middle); MEG solution interpolated on the radial magnetometers including a volume rendering of the head model (right).

Conclusions and Outlook

- a **conservative** representation of the **flux** increases the accuracy of DG-FEM results in MEG
- for the finest mesh resolution of 1 mm, sources with a distance of 1.59 mm from the brain-CSF surface, DG-FEM yielded mean **RDM%** of **1.5%** and mean **MAG%** of **0.1%** for the magnetic field
- skull leakages do **not** play a role for the MEG modality
- in a combined EEG and MEG (EMEG) source reconstruction analysis is desirable to employ the **same forward model** for both EEG and MEG data
- DG-FEM complements, and in some cases as the skull leakage scenarios, outperforms CG-FEM in EEG or combined EMEG

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