The Discontinuous Galerkin Finite Element Method for Solving the MEG Forward Problem

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ChildBrain
ADVANCING BRAIN RESEARCH IN CHILDREN'S DEVELOPMENTAL NEUROCOGNITIVE DISORDERS

In source reconstruction, solutions for both the forward and inverse problem are required, and the accuracy of the inverse solution depends also on the one of the forward solution. When dealing with realistic head models, numerical methods have to be adopted for solving the forward problem [1]. Among others, the Discontinuous Galerkin – Finite Element Method (DG - FEM) allows for the fulfilling of conservation laws, even on a discrete level [2]. In EEG studies it has already been remarked how this property prevents the occurrence of unwanted effects, e.g. unphysical leakages of volume currents in regular hexahedral meshes with insufficient resolution [3]. Moreover it puts the basis for the application of the so called Unfitted Discontinuous Galerkin FEM (UDG-FEM), an extended version of the DG-FEM that allows smooth tissue surface representations, whose advantages have been already shown in EEG studies [4]. Our goal in this work is to investigate accuracy and leakage aspects of CG-FEM and DG-FEM for the MEG forward problem as the first important step towards an UDG-FEM implementation for MEG.

EEG Forward Problem

$$\nabla \cdot (\sigma \nabla u) = f \quad (on\,\Omega)$$
$$\nabla u \cdot \boldsymbol{n} = 0 \quad (on\,\partial\Omega)$$

MEG Forward Problem

$$\Phi = \Phi^{prim} + \Phi^{sec}$$
$$\Phi_c^{sec} = \left(\int_\Omega (\boldsymbol{j}^{sec} \times \boldsymbol{D})\, d\Omega\right) \cdot \boldsymbol{p_c}, \forall\, c\; coil$$

CG - FEM

$$\boldsymbol{j}_{cg}^{sec} = \sigma \nabla u$$

In the CG framework, the potential *u* is discretized in the space of globally continuous functions and piecewise polynomials. The DOF are nodes. It is not possible to conclude about the property of conservation of charge.

Fig1a-b: Illustration of current flow/leakage effect for CG-FEM (1a) and DG-FEM (1b). While for the CG-FEM an *unphysical* current flow through a single vertex occurs, the DG-FEM only allows current flow over faces, [3].

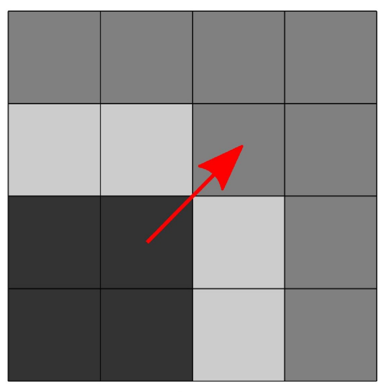

Fig1a

DG - FEM

$$\boldsymbol{j}_{dg}^{sec} = \Pi_{RT0}\left(\{\sigma \nabla u\} - \eta \frac{\sigma_\gamma}{h_\gamma} [\![u]\!]\right)$$

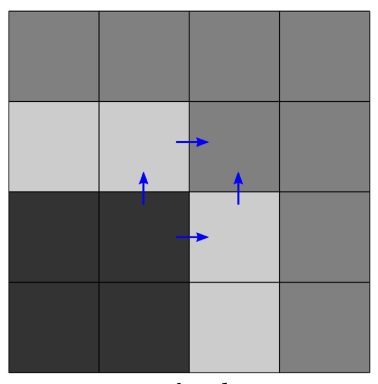

Fig1b

In the DG framework, the potential *u* is discretized in the space of globally integrable functions, not globally continuous and piecewise polynomials. The DOF are elements. The property of conservation of charge is fulfilled:

$$\int_{\partial K} \boldsymbol{j}_{dg}^{sec} \cdot \boldsymbol{n}\, dS = \int_K f\, dK$$

MEG Results (Statistics)

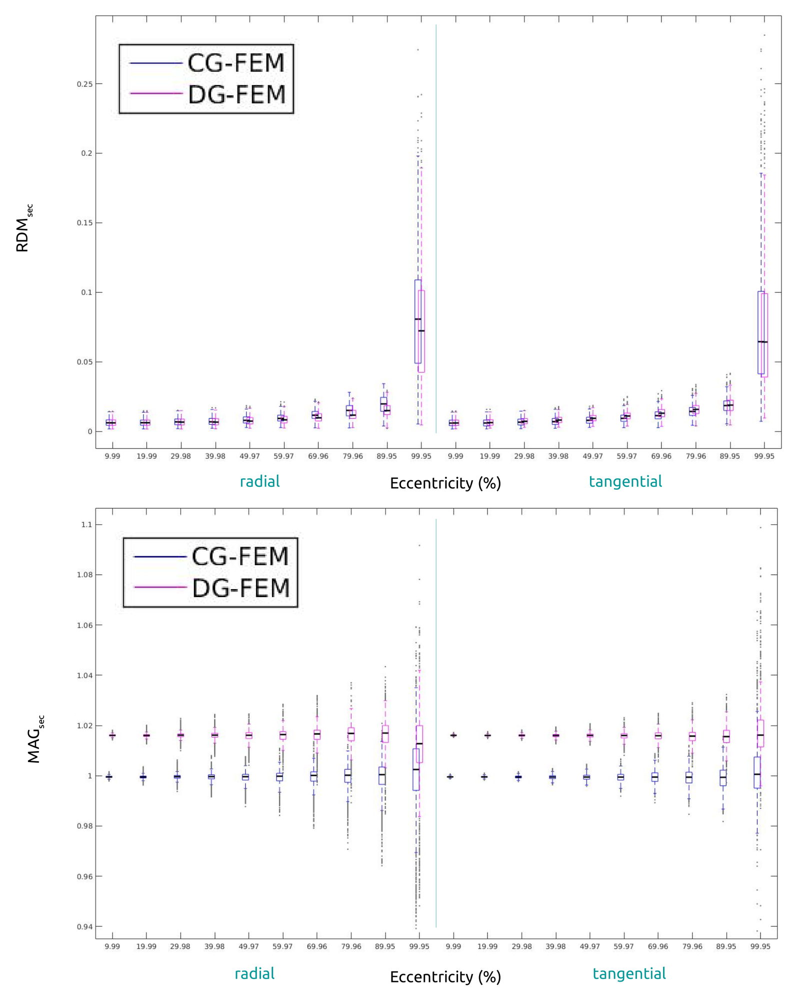

Fig 2: Boxplots of the RDM_{sec} (top) and MAG_{sec} (bottom) error indices referred to the secondary magnetic field. In the x axis there are the 10 eccentricities both for 10K radial dipoles and 10K tangential dipoles.

Visualizations of MEEG Forward Solutions (Leaky Scenario)

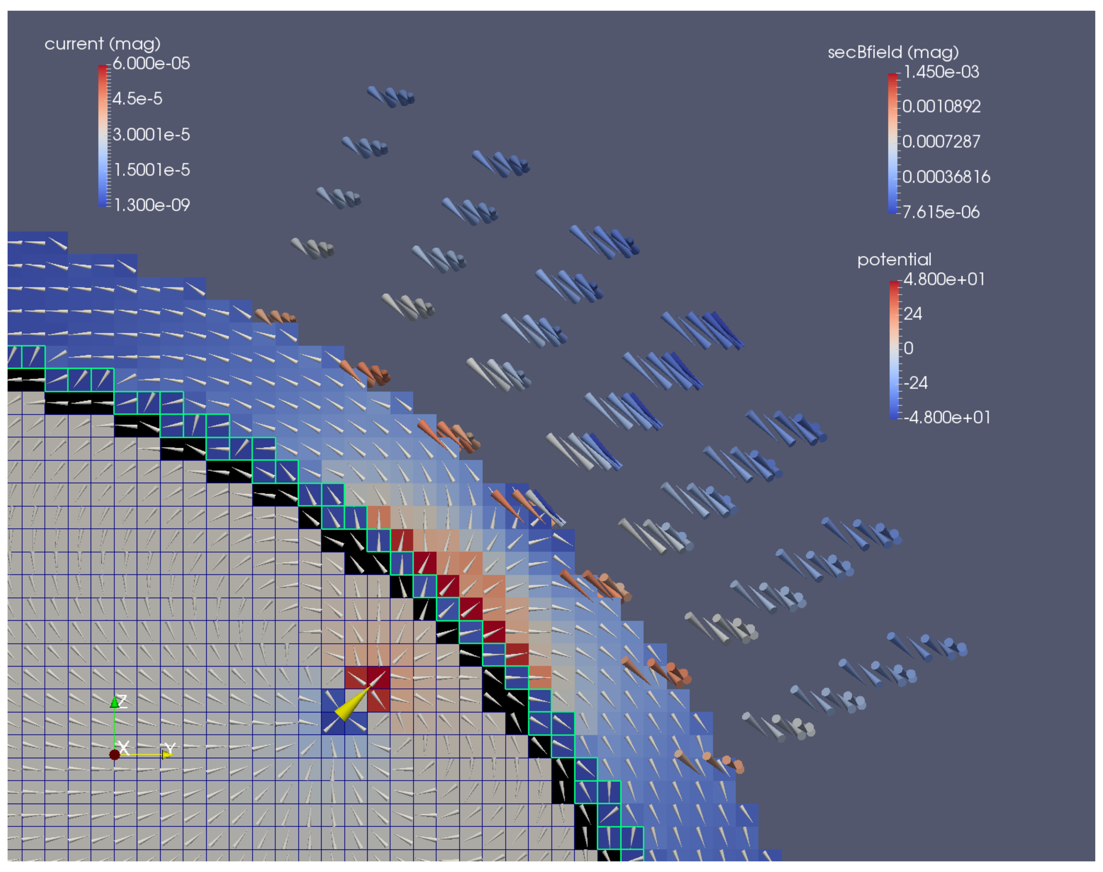

Fig3a

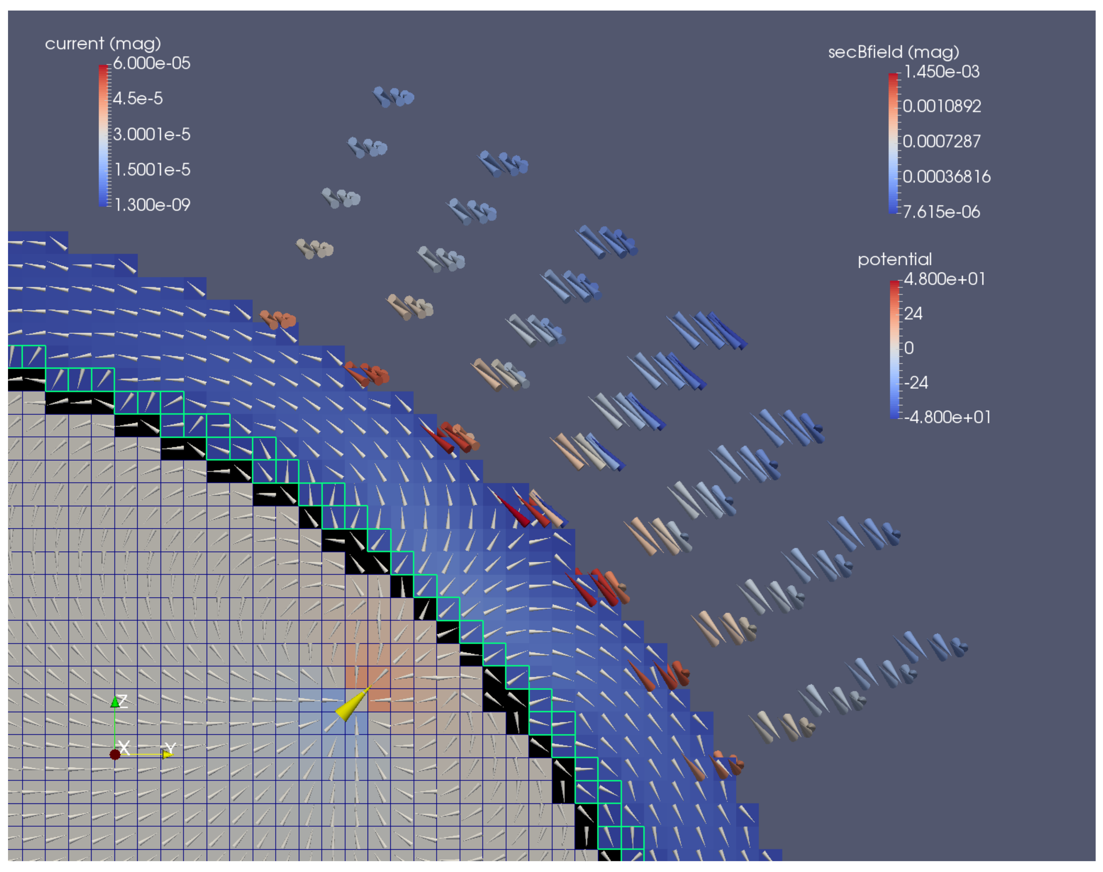

Fig3b

Fig 3a-b: MEEG solutions visualized in a leaky scenario. In the brain compartment, the coloring refers to the potential, in the skull and skin compartment (separated by green lines) it refers to the current strength, the CSF is black. In the exterior of the spheres, the magnetic flux is visualized on each coil position. Fig3a refers to the CG solution, Fig3b to the DG solution. The dipole is the yellow cone.

Simulation Features

We evaluated the DG method for the EEG solution using a partial integration approach for the source model. We then computed the MEG solution.

- For the Statistics, we used the head model described in Table1, discretized with a hexahedral mesh of resolution equal to 2mm. We generated 10K random dipoles on each of the 10 eccentricities in the inner compartment and measured the secondary magnetic field on 256 external coils. The secondary magnetic field is then compared to the analytical solution and the error is measured through RDM_{sec} and MAG_{sec}.
- For the Leaky Scenario, we used the same head model as for the Statistics, but we thinned the skull compartment (from 6 to 2 mm). We then visualized the EEG and MEG solution as described in the caption of Fig3a-b.

material	out. radius (mm)	conductivity (S/m)
brain	78	0.33
csf	80	1.79
skull	86	0.01
skin	92	0.43

Table1: head model features [3]

$$RDM_{sec}(f^{ana}, f^{num}) := \left\| \frac{f^{num}}{\|f^{num}\|_2} - \frac{f^{ana}}{\|f^{ana}\|_2} \right\|_2$$

$$MAG_{sec}(f^{ana}, f^{num}) := \frac{\|f^{num}\|_2}{\|f^{ana}\|_2}$$

Simulation Tools

- DUNE (Distributed and Unified Numerics Environment) is a C++ open source library for the discretization and solution of partial differential equations (PDEs).
- DUNEURO is a module of DUNE specialized in solving PDEs in Neuroscience.

Dune
http://dune-project.org/

duneuro

Conclusions and Outlook

We present first applications of the DG-FEM to the MEG forward problem, the first important step before proceeding with the implementation of UDG-FEM for the MEG forward problem, method that has clear advantages already pointed out in the EEG case [4]. With regards to the Statistics, we can notice that the accuracy of the DG-FEM solution is in the same range of the one reached by the CG-FEM one. In the Leaky Scenario, the MEG solutions result to be less influenced from Leakages than for the EEG case, possibly due to the fact that MEG solutions are blind for radial sources, and the leaky current is mainly radially oriented. Further investigations are needed.

References:

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