The Unfitted Discontinuous Galerkin Method in Brain Research

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Introduction

Patient specific simulation is a useful tool in different areas of brain research. A main method in use for such simulations is the *finite element method* (FEM) [1]. It uses a volume tessellation and can treat models with anisotropic conductivities or complex domains with fine structures or holes. While hexahedral meshes reduce the complexity of a simulation pipeline, tetrahedral meshes offer a better geometric approximation. However, the automatic construction of a high quality triangulation from quasi-non-invasive magnetic resonance imaging (MRI) is a difficult task.
The *unfitted discontinuous Galerkin* method (UDG) [2] avoids these problems by using a structured mesh that does not resolve the geometry. It works directly on a level set segmentation and includes the geometry in its mathematical formulation. In addition it maintains conservation laws on a discrete level.

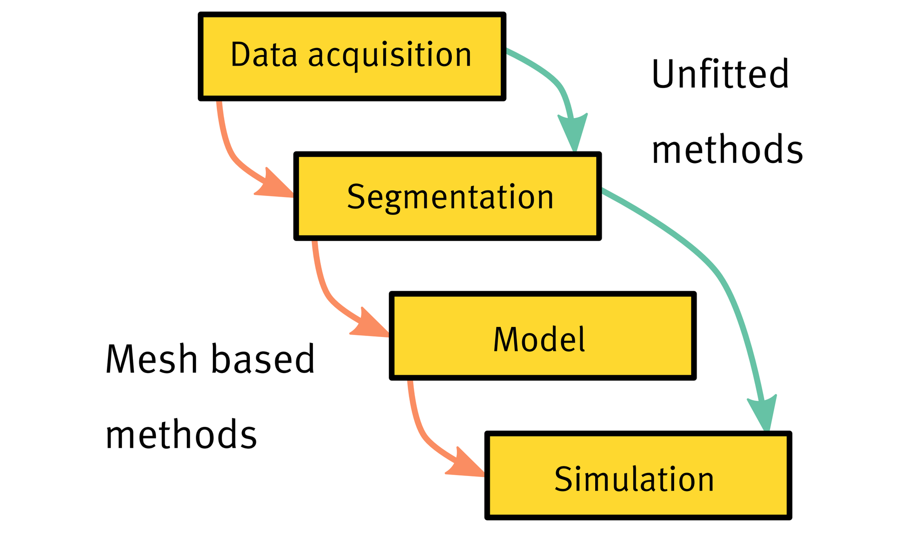

Figure 1: General pipeline of a patient specific simulation

Unfitted Discontinuous Galerkin

The UDG method is a method for discretizing partial differential equations, in this case the Poisson equation $\nabla \cdot \sigma \nabla U = f$

Unfitted

The computational mesh does not resolve the geometry. The latter is given as level sets. The elements of the mesh are restricted to the different domains.

Discontinuous Galerkin

Galerkin method similar to the finite element method. Allow discontinuities of the potential between elements. Consider continuity in the weak formulation.

$$a(u,v) = \int_\Omega \nabla u \nabla v dx - \int_\Gamma ([\![u]\!] \cdot \{\nabla v\} + \{\nabla u\} \cdot [\![v]\!]) dx + \frac{\eta}{h} \int_\Gamma [\![u]\!] \cdot [\![v]\!] dx$$

The Unfitted Mesh

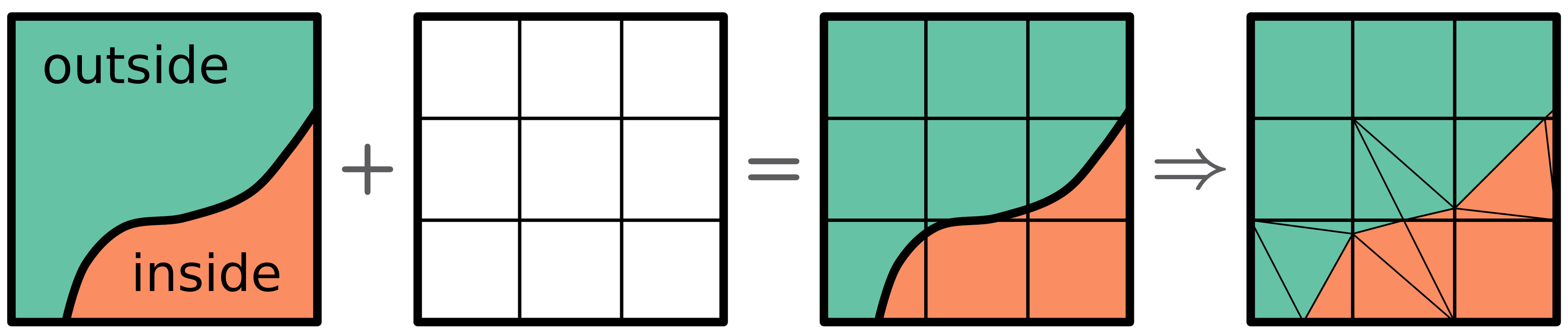

Figure 2: Construction of the unfitted mesh for a level set on a 2D grid

DUNE Framework

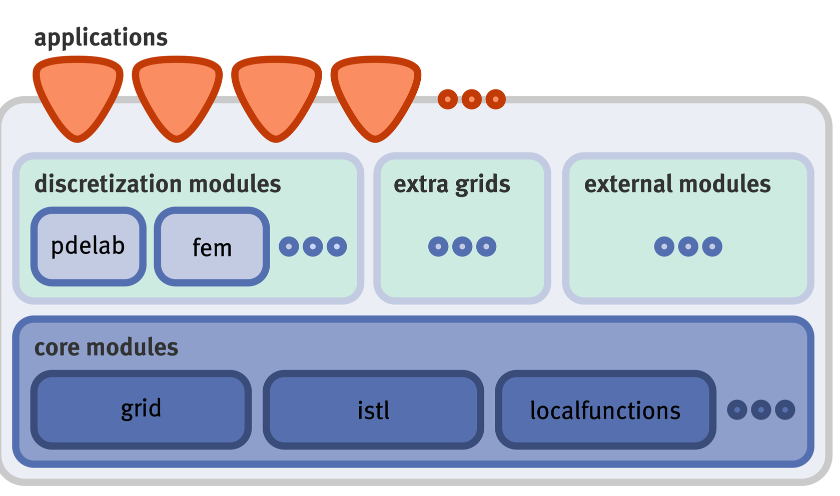

Figure 3: The modular structure of the DUNE library

DUNE = Distributed and Unified Numerics Environment

http://www.dune-project.org

- C++ open source library for the discretization and solution of partial differential equations
- modular structure, general interfaces

Conclusion and Outlook

We presented first promising results of the application of the UDG method in EEG and tES modeling. It shows proper convergence behavior when reducing the element size. We can observe higher (RDM) or at least comparable (MAG) accuracy to a DG method on a conforming mesh with a similar number of unknowns.
Besides the PI approach, we are deriving more accurate source models for the UDG method. For the application to realistic head models, we are currently investigating a smoothing procedure based on constrained mean curvature flow. In addition, we are evaluating the effect of the method on leakage behavior and are considering its application in a tDCS optimization scheme.

References

contact

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EEG Forward Problem

We evaluate the UDG method using a *partial integration* (PI) approach for the EEG forward problem on a *multilayer sphere model*. We use 4 layers with conductivities from outer to inner compartment: 0.33, 0.0042, 1.79 and 0.33 S/m. We generate 500 random dipoles on each of 15 eccentricities in the inner compartment and measure the potential at 200 surface electrodes. The potential is compared to the analytic solution and the error is measured as:

$$\mathrm{RDM\%}(U_{num}, U_{ana}) = 50 \cdot \left\| \frac{U_{num}}{\|U_{num}\|} - \frac{U_{ana}}{\|U_{ana}\|} \right\| \in [0, 100] \qquad \mathrm{MAG\%}(U_{num}, U_{ana}) = 100 \cdot \left(\frac{\|U_{num}\|}{\|U_{ana}\|} - 1\right) \in [-100, \infty)$$

Both measures have an optimal value of 0. The method is evaluated on grids with different element diameter and compared to the DG method on a conforming mesh of similar size.

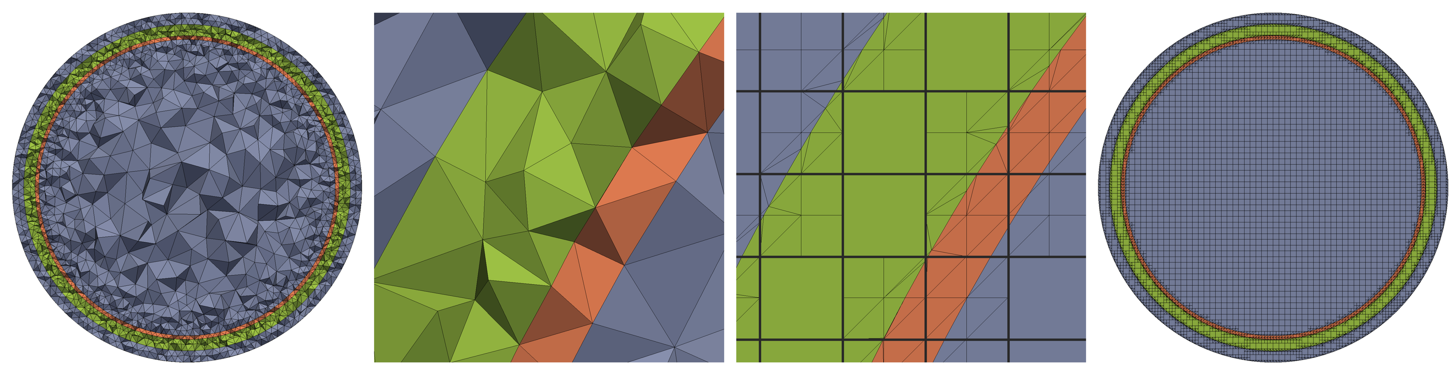

Figure 4: Multilayer sphere model used for DG (left) and UDG (right) simulation

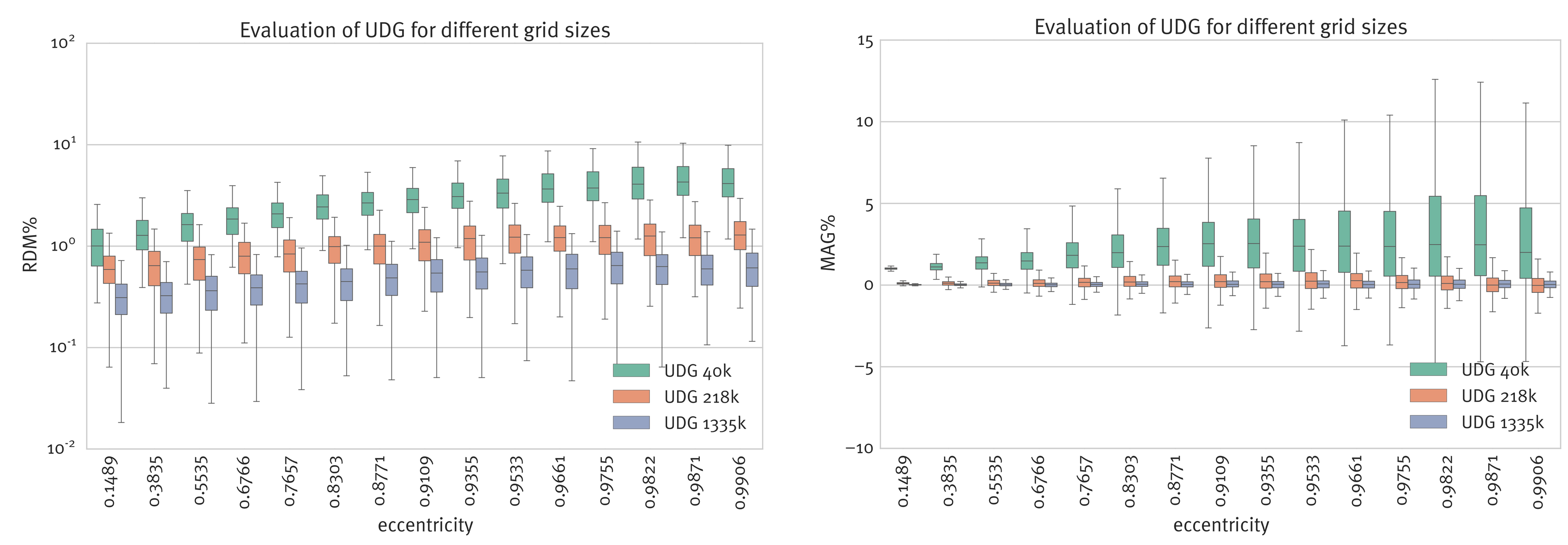

Figure 5: RDM% (left) and MAG% (right) errors for the UDG method with different grid sizes

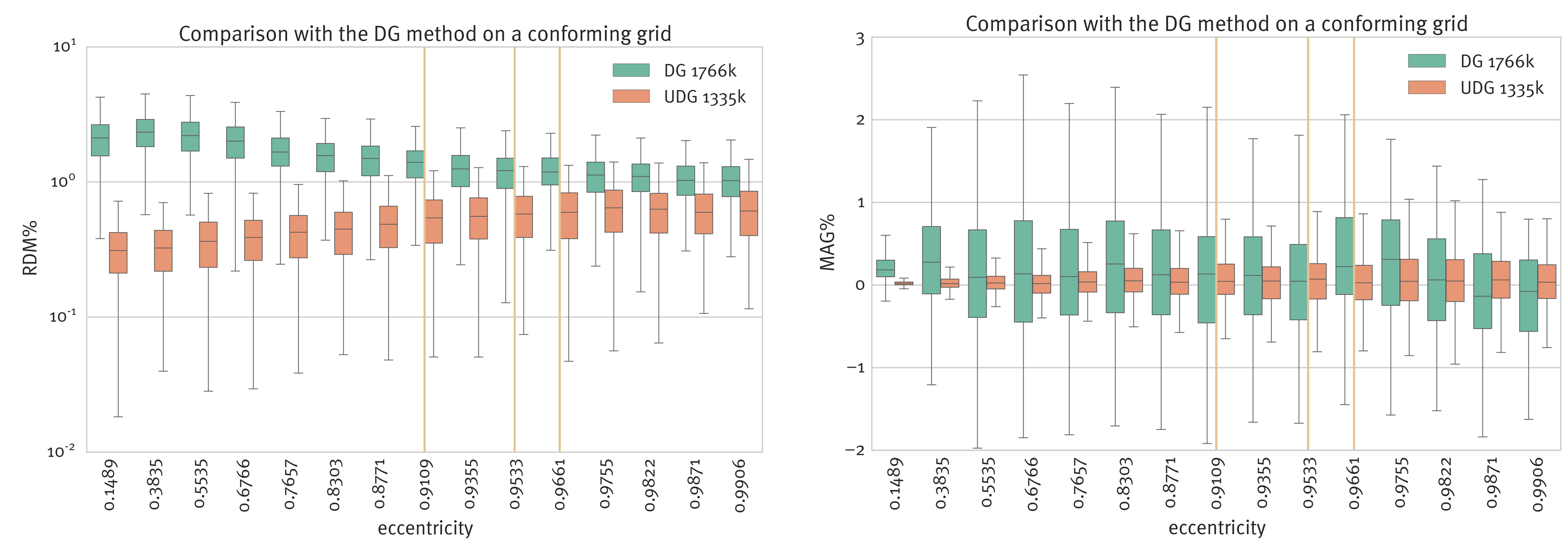

Figure 6: RDM% (left) and MAG% (right) errors for the DG and UDG method

Brain Stimulation

We test the UDG method for a tDCS simulation [3] on a 4 compartment isotropic head model. We use the same conductivities as for the EEG forward problem. The level sets are generated artificially from a voxel segmentation.

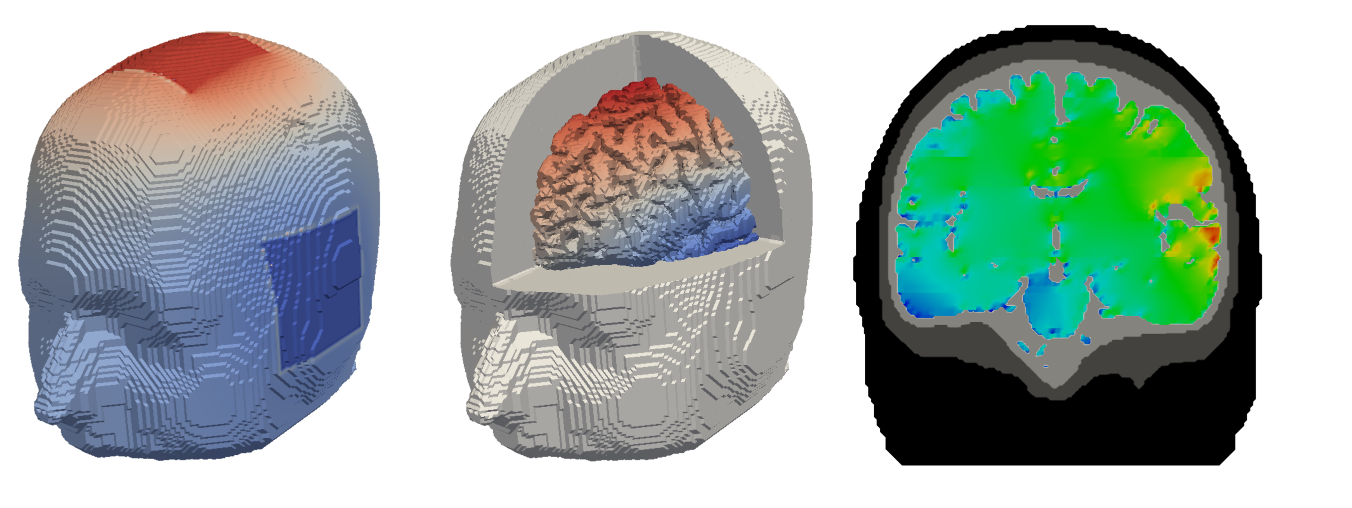

Figure 7: The potential u at the scalp and brain surface and the current density $|\sigma \nabla u|$