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A mixed finite element approach to solve the EEG forward problem

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Introduction

It was shown in several studies that approaches based on the finite element method (FEM) can achieve a high accuracy in solving the EEG forward problem [1,2]. The FEM allows the simulation of important features of the human head, e.g. anisotropic conductivities [3] as well as the modeling of complicated geometrical structures in the human head, allowing the inclusion of the CSF [4], the distinction between skull compacta and spongiosa [5] or the modeling of skull holes [6].
A crucial point in the implementation is the treatment of the strong singularity at the source position due to the model of the current dipole. Different approaches to solve this problem have been developed, e.g. the Venant approach, the partial integration approach, the subtraction approach (see recent comparison in [2]) or an approach based on Raviart-Thomas-type sources [7]. All of these approaches use a discretization based on classical Lagrange elements (hat-functions) to solve the EEG forward problem/Poisson equation.
Here, we present a novel approach to solve the EEG forward problem based on mixed finite elements [8]. Mixed finite elements have achieved a high accuracy and shown great robustness in a variety of applications. Furthermore, this approach enables us to directly introduce an atomic current source instead of approximating this by a distribution of electrical monopoles like it is done in the Venant or partial integration approach.

Methods

Instead of discretizing the 2nd order partial differential equation
$\nabla(\sigma\nabla u) = \nabla \mathbf{j}^{\mathbf{P}}$
directly, the problem is split into a system of two coupled 1st order differential equations, leading to the weak formulation:

$$\int_\Omega \sigma^{-1}\mathbf{j}\cdot\mathbf{q}\,\mathrm{d}x - \int_\Omega u\nabla\cdot\mathbf{q}\,\mathrm{d}x = \int_\Omega \sigma^{-1}\mathbf{j}^{\mathbf{P}}\cdot\mathbf{q}\,\mathrm{d}x \quad \text{for all } \mathbf{q}\in H(div,\Omega),$$
$$\int_\Omega (\nabla\cdot\mathbf{j})v\,\mathrm{d}x = 0 \quad \text{for all } v\in L^2(\Omega)$$

Based on a regular discretization $\mathcal{T}$ of Ω, we approximate the space $H(div,\Omega)$ by the space of lowest order Raviart-Thomas elements $RT_0(\mathcal{T})$ and $L^2(\Omega)$ by the space $P_0(\mathcal{T})$ of piecewise constant functions.
After approximating the scalar unknown u by its projection into the space $L^2(\Omega)$ and the vector-valued unknown $\mathbf{j}$ by its projection into the space $RT_0(\mathcal{T})$, this leads to the equation system

$$\begin{pmatrix} A & B^T \\ B & 0 \end{pmatrix}\begin{pmatrix} j \\ u \end{pmatrix} = \begin{pmatrix} b \\ 0 \end{pmatrix} \qquad A_{i,j} = \int_\Omega \sigma^{-1}\mathbf{q}_i\cdot\mathbf{q}_j\,\mathrm{d}x \qquad B_{i,j} = \int_\Omega v_i(\nabla\cdot\mathbf{q}_j)\,\mathrm{d}x$$

Source Model

Until now, we did not make any assumptions about the shape of the source term. In general, there is a variety of possibilities to model the source term, especially shape functions with higher regularity than the classical current dipole are desirable. For now, we will stick with the classical model of a current dipole and thus gain

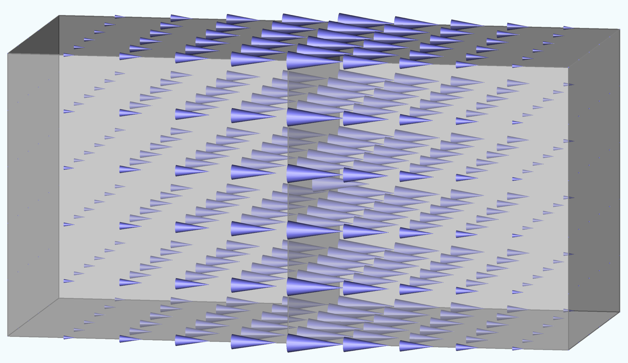
$$b_i = \int_\Omega \sigma^{-1}(\mathbf{M}\delta_{\mathbf{x_0}})\cdot\mathbf{q_i}\,\mathrm{d}x = \begin{cases} \sigma^{-1}\mathbf{M}\cdot\mathbf{q_i}(\mathbf{x_0}) \text{ if } \mathbf{x_0}\in supp(\mathbf{q_i}), \\ 0 \text{ otherwise.} \end{cases}$$

where $\mathbf{x_0}$ is the source position, $\mathbf{M}$ the dipole moment of the source and $supp(\mathbf{q_i})$ the support of $\mathbf{q_i}$.

Implementation & Evaluation

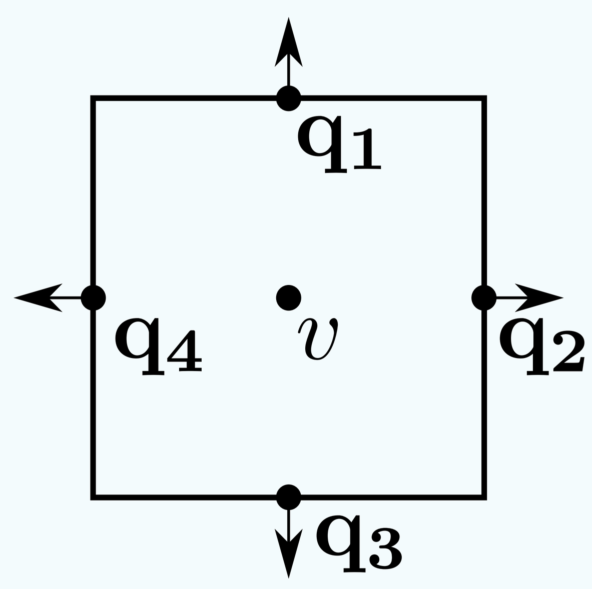
We implemented the presented approach for hexahedral meshes using DUNE-PDELab, part of the open-source toolbox DUNE [9].
As basis for the space $P_0(\mathcal{T})$ we use the indicator function on each element, for the space $RT_0(\mathcal{T})$ we choose face-based basis functions, as exemplarily depicted in Figure 1. Figure 2 depicts the resulting degrees of freedom in a 2d-example.
We constructed a three layer sphere model (center at (127 mm, 127 mm, 127 mm)) with an isotropic resolution of 2 mm, compartment boundaries at radii of 80 mm, 86 mm and 92 mm and conductivities of 0.43 S/m, 0.01 S/m and 0.33 S/m. Dipoles were placed on the line (0,0,1)*x + (127,127,127) in 2 mm steps so that they are located in element centers.
As error measure we calculated the relative difference measure (RDM) (normalized L_2-difference) over all surface points with an analytical solution as reference.

Figure 1



Visualization of an RT_0-element

Figure 2



Exemplary depiction of the degrees of freedom in 2d

Results

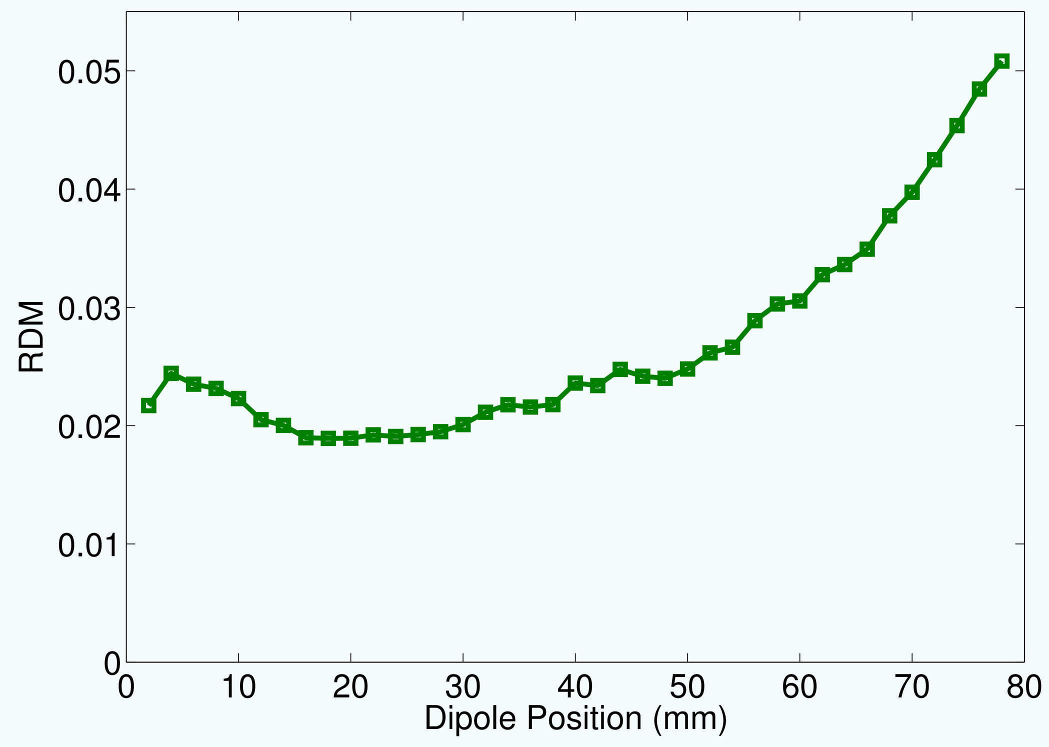


Figure 3: RDM-error of the mixed approach in a three layer sphere model for tangential sources

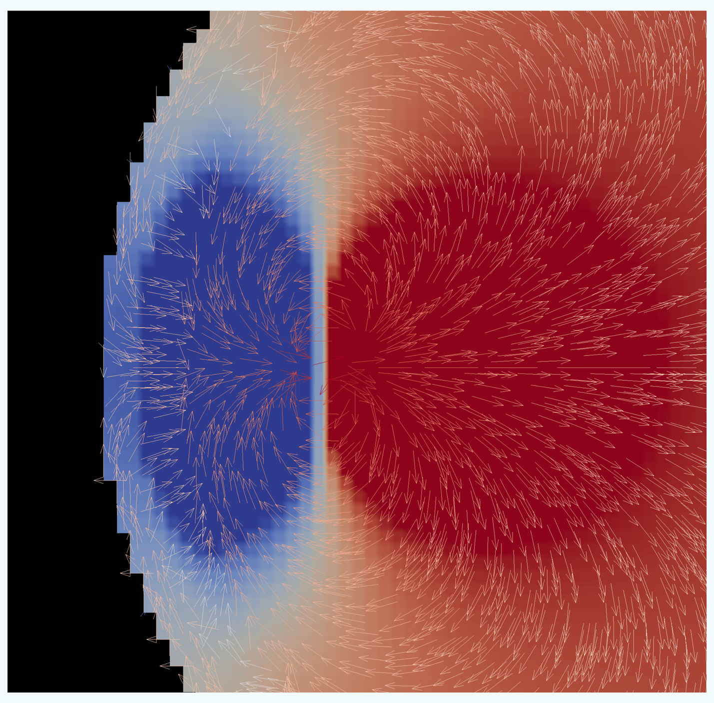


Figure 4: Visualization of potential and current in the z = 127 plane

Conclusion & Outlook

We have presented a novel approach to solve the EEG forward problem based on mixed finite elements. Thereby, it was possible to circumvent the problems due to the model of the current dipole occuring in other approaches. The first numerical results show promising accuracies.
The current implementation still has to be improved, especially with regard to the choice of solvers in order to increase speed and stability.
In further studies, the solution accuracy has to be investigated more systematically and in more realistic scenarios. The convergence towards the numerical solution in meshes with higher resolutions has to be tested. The dependency of the numerical accuracy on the position in the mesh element has to be investigated to be able to optimally place the sources for leadfield computations, as it was done for the Venant and partial integration approach.
Further improvements of the results could be achieved by the use of geometry-adapted meshes that enable a better fit to the underlying geometry. This could also be achieved by an implementation enabling the use of tetrahedral meshes.

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