A Discontinuous Galerkin Finite Element approach for the EEG forward problem

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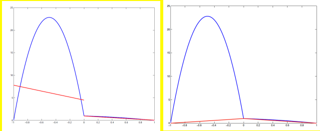
Introduction and Motivation

In EEG source analysis, numerical approaches are needed to compute head surface field distributions from dipolar current sources using realistic head volume conductor models (Wolters et al., 2007). Here, we have implemented and evaluated a Discontinuous Galerkin Finite Element Method (DG-FEM) (Ludewig, 2013; Arnold et al., 2002; Bastian and Engwer, 2009). In contrast to standard FEM (i.e., conforming, Lagrange), DG-FEM uses basis functions with only local support on each element, which might lead to discontinuities over element borders (Fig.1). Continuity and boundary conditions are only enforced weakly through penalty terms on element interfaces and on the boundary.

Fig.1: 1D model problem: Equation like in source analysis, but with given and smooth right-hand side and homogeneous Dirichlet boundary conditions. Conductivity is jumping from 0.0056 S/m in interval (-1,0) to 1 S/m in (0,1). Analytical solution (blue) and numerical solutions using linear basis functions (red) obtained by DG-FEM (left) and by standard FEM (right).



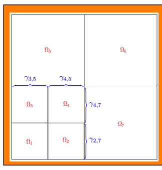


Fig. 2: A non-conforming hexahedral mesh that can easily be handled with DG-FEM, but which leads to hanging nodes in standard FEM.

Motivations for a DG-FEM approach are
- possibly more accurate solutions (Fig.1)
- handling of non-conforming meshes (Fig.2),
- straight-forward implementation of higher order basis functions and p-adaptivity
- easier matrix structure and easier parallelization
- mass conservation properties

Theory

Here we present the basic mathematical framework for a DG-FEM approach for the EEG forward problem when using the FE subtraction method for modeling the singularity arising from the choice of a mathematical current dipole as a source term (Wolters et al., 2007); for more informations, see (Ludewig, 2013):

Definition (Jump)
The *Jump* of a piecewise continuous function x on the interface between two adjacent elements E_e and E_f is denoted by the linear operator

$$[\![x]\!]_{e,f} = x|_{\partial E_e}\vec{n}_{E_e} + x|_{\partial E_f}\vec{n}_{E_f}$$

Definition (Average)
The (weighted) average of a piecewise continuous function x on the interface is given by

$$\langle x\rangle_{e,f} = \omega_e x|_{\partial E_e} + \omega_f x|_{\partial E_f}$$

where $\omega_e := \frac{\sigma_f}{\sigma_e+\sigma_f}$ and $\omega_f := \frac{\sigma_e}{\sigma_e+\sigma_f}$.

Weak DG formulation of the subtraction approach:

$$\begin{aligned}
&-\int_\Omega \sigma(x)\nabla u^{corr,y}(x)\cdot\nabla v(x)dx \\
&\quad+\sum_{\gamma_{e,f}\in\Gamma_{int}}\int_{\gamma_{e,f}}\langle\sigma\nabla u^{corr,y}\rangle(s)[\![v]\!](s)+\langle\sigma\nabla v\rangle(s)[\![u^{corr,y}]\!](s)ds \\
&-\eta\frac{\kappa_{e,f}}{h_{e,f}}\sum_{\gamma_{e,f}\in\Gamma_{int}}\int_{\gamma_{e,f}}[\![u^{corr,y}]\!](s)[\![v]\!](s)ds \\
&=\int_\Omega \sigma^{corr,y}(x)\nabla u^{\infty,y}(x)\cdot\nabla v(x)dx+\int_{\partial\Omega}\sigma^{\infty,y}(s)\nabla u^{\infty,y}(s)\cdot\vec{n}v(s)ds \\
&-\sum_{\gamma_{e,f}\in\Gamma_{int}}\int_{\gamma_{e,f}}\langle\sigma^{corr,y}\nabla u^{\infty,y}\rangle(s)[\![v]\!](s)ds
\end{aligned}$$

Methods

The new approach was implemented in the DUNE[1] PDELab framework. For validation purposes, we used the following setup:

- 4 layer sphere model (78, 80, 86, 92 mm; 0.33,1.79,0.01,0.43 S/m)
- Relative Error (RE) and Relative Difference Measure (RDM) in coarse (190,060 elements, 31,627 nodes) and fine (3,159,575 elements, 518,713 nodes) tetrahedral FE models measured at 222 uniformly distributed surface electrodes
- Radial and tangential dipoles distributed on excentricities up to 0.9925 (coarse model) or 0.995 (fine model), i.e., only 0.585 mm or 0.39 mm to next conductivity jump (0.987 corresponds to 1mm), resp. (**thus: very high excentricity**). Boxplots consist of 10 randomly chosen dipoles per excentricity.

Finally, we show a DG-FEM result in a realistic head model with compartments skin (0.33 S/m), skull compacta (0.008 S/m), skull spongiosa (0.025 S/m), cerebrospinal fluid (1.79 S/m), brain grey (0.33 S/m) and white matter (0.14 S/m).

Results

Figures 3 (coarse tetrahedral model) and 4 (fine tetrahedral model) show the numerical validation results in the 4 layer sphere model for standard FEM with linear basis functions (red), DG-FEM with linear (blue), quadratic (green) and cubic (pink) basis functions and Figure 5 a DG-FEM computed current flow field in a realistic head model.

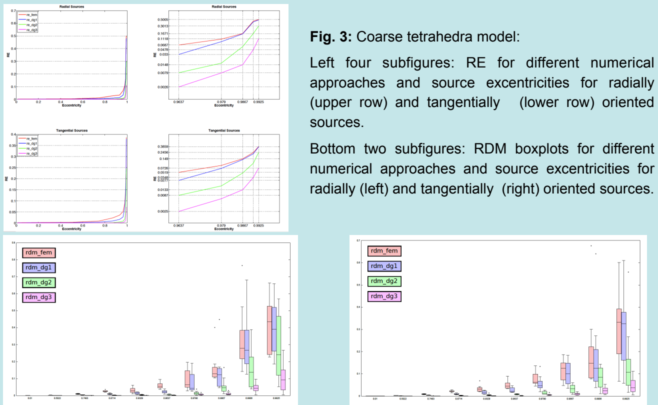


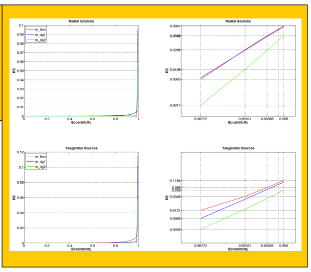
Fig. 3: Coarse tetrahedra model:

Left four subfigures: RE for different numerical approaches and source excentricities for radially (upper row) and tangentially (lower row) oriented sources.

Bottom two subfigures: RDM boxplots for different numerical approaches and source excentricities for radially (left) and tangentially (right) oriented sources.

Fig. 4: Fine tetrahedra model:

Right four subfigures: RE for different numerical approaches and source excentricities for radially (upper row) and tangentially oriented sources (lower row).



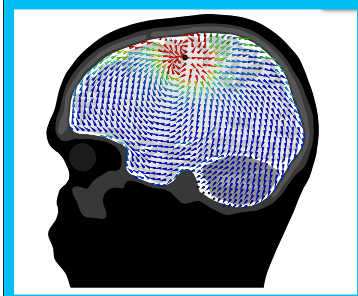


Fig. 5: DG-FEM-computed visualization of current flow caused by a source in somatosensory cortex in a realistic (six tissue) head model.

Discussion and outlook

- With RE's below 0.015 (coarse model, cubic basis functions) and below 0.0028 (fine model, quadratic basis functions) at 1 mm from the next conductivity jump (excentricity of 0.987), our DG-FEM implementation shows very high numerical accuracies.
- For the subtraction approach and for excentric sources, higher order basis functions are important for both standard FEM and DG-FEM
- For the linear basis functions, DG-FEM showed slightly higher accuracies than standard FEM, but on the cost of a currently much higher memory and computational load in our current implementation (Ludewig, 2013).
- We continue evaluating the various motivational aspects for DG-FEM in EEG/MEG source analysis and in the field of simulation of tCS/TMS brain stimulation

References:
- Arnold et al. (2002), Unified Analysis of Discontinuous Galerkin Methods for Elliptic Problems. SIAM J. Num. Anal..
- Bastian and Engwer (2009), An unfitted finite element method using discontinuous Galerkin, Int. J. Num. Meth. Eng..
- Ludewig (2013), Discontinuous Galerkin Methods for the EEG forward problem. Master Thesis in Mathematics, WWU Münster.
- Wolters et al. (2007) Numerical mathematics of the subtraction approach for the modeling of a current dipole in EEG source reconstruction using finite element head models. SIAM J. Sci. Comp.

Software: [1] http://www.dune-project.org, DUNE, a free and open-source modular C++ toolbox for solving partial differential equations using grid-based methods.

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