Efficient Computation of Lead Field Bases and Influence Matrix for the FEM-based EEG and MEG Inverse Problem

C.H.Wolters[1,2,3], L.Grasedyck[2] , A.Anwander[3] and W.Hackbusch[2]

(1) Scientific Computing and Imaging Institute, University of Utah, Salt Lake City, USA
(2) Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany
(3) Max Planck Institute for Human Cognitive and Brain Sciences, Leipzig, Germany

ABSTRACT

The inverse problem in EEG and MEG aims at reconstructing the underlying current distribution in the human brain. The finite element method, used for the forward problem, is able to realistically model tissue conductivity inhomogeneities and anisotropies. So far, the computational complexity is quite large when using the necessary high resolution finite element models. It is already known that the so-called reciprocity can strongly reduce this complexity with regard to the EEG modality. We will derive algorithms for the efficient computation of EEG and MEG lead field bases which exploit the fact that the number of sensors is generally much smaller than the number of reasonable dipolar sources. Each finite element forward solution is then reduced to a simple matrix-vector multiplication instead of an expensive iterative finite element solution process. Our approaches can be applied to inverse reconstruction algorithms in both continuous and discrete source parameter space for EEG and MEG. In combination with modern solver methods, the presented approach leads to a highly efficient solution of FE-based source reconstruction problems.

KEY WORDS

EEG/MEG Source Reconstruction, Conductivity Anisotropy and Inhomogeneity, Finite Element Method, Inverse Algorithms, Reciprocity, Lead Field Bases, Algebraic Multigrid Solver, Preconditioned Conjugate Gradient Methods

INTRODUCTION

For the forward problem in EEG/MEG source reconstruction, the volume conductor head has to be modeled. It is known that the head tissue compartments scalp, skull, cerebro-spinal fluid, brain gray matter and white matter have different conductivities and that the layers skull and white matter are anisotropic conductors [Wolters, 2003]. Different numerical approaches for the forward problem have been used such as Multi-Layer Sphere, Boundary Element (BE) and Finite Element (FE) head modeling, where only the FE method is able to treat both realistic geometries as well as inhomogeneous and anisotropic material parameters. The influence of skull and white matter conductivity anisotropy on the EEG/MEG forward and inverse problem was studied in realistic FE models in [Marin, 1998], [Haueisen, 2002], [Wolters, 2003]. In those studies it has been shown that an exact modeling of tissue conductivity inhomogeneity and anisotropy is crucial for an accurate reconstruction of the sources.
An important question is how to handle the computational complexity of FE-modeling with regard to the inverse problem. It is the state-of-the-art approach for FE-based forward modeling in EEG/MEG inverse methods to solve an FE equation system for each possible dipolar source [Buchner, 1997]. For the FE method, we recently developed fast solver approaches based on multigrid ideas, speeding the computations by factors of up to 100 compared to standard approaches [Wolters, 2002]. In that study we used parallelization to speed the computations and to distribute the memory between the computational nodes. Still, the repeated solution of such a system with a constant geometry matrix for thousands of right-hand sides (the sources) is the major time consuming part within the inverse localization process and limits the resolution of the models.
A further very efficient concept for the reduction of the computational complexity has been described for the EEG, the concept of reciprocity [Weinstein, 2000], [Vanrumste, 2001]. The reciprocity theorem for the electric case states that the field of the so-called lead vectors is the same as the current field raised by feeding a reciprocal current to the lead. The concept allows to switch the role of the sensors with the dipole locations. For FE based EEG source reconstruction, it was shown in [Weinstein, 2000] how to use this principle for the efficient computation of a so-called *node-oriented lead field basis*, a matrix with ``number sensors'' rows and ``number FE nodes'' columns. This matrix can then be exploited within the EEG inverse problem. In [Vanrumste, 2001], reciprocity was used for the efficient solution of the EEG inverse problem when using the Finite Difference method for the forward problem. The application of reciprocity to MEG is non-trivial and has been studied in [Nolte, 2003], where the *magnetic lead field theorem* was proven. Nevertheless, as far as we know, it is not yet clear how to efficiently compute the lead field basis for the MEG in combination with the FE method for the forward problem.
In this paper, we will simply apply the mathematical law of associativity with respect to the matrix multiplication. Then, for each head model, we only have to solve ``number of EEG/MEG sensors'' times a large sparse FE system of equations in order to compute the lead field basis for both EEG and MEG. This setup can be computed efficiently using the MultiRHS-AMG-CG solver [Wolters, 2004/2]. Each forward solution is then reduced to the multiplication of the lead field basis to an FE right-hand side vector. Simulation studies with a high resolution anisotropic FE head model and a blurred dipole model will then show that an influence matrix with a resolution of 2mm can be computed in only a few seconds on a simple single processor PC.

METHODS

The electric forward problem:

From the quasistatic Maxwell equations, we can derive the equation

$$-\nabla \cdot (\sigma \nabla \Phi) = -\nabla \vec{j}^{\,p}, \tag{1}$$

which describes the potential distribution Φ in the head domain due to a primary source $\vec{j}^{\,p}$ in the brain. We assume that the 3x3 conductivity tensors σ are given for the head domain. For the forward problem, the primary current and the conductivity distribution in the volume conductor are known and the equation has to be solved for the unknown potential distribution. The boundary condition $(\sigma_1 \nabla \Phi_1, \vec{n})\big|_{\text{surface}} = (\sigma_2 \nabla \Phi_2, \vec{n})\big|_{\text{surface}}$ with $\vec{n}$ the unit surface normal expresses the continuity of the current density across any surface between regions of different conductivity. We find homogeneous Neumann conditions on the head surface Γ, $(\sigma \nabla \Phi, \vec{n})\big|_{\Gamma} = 0$, and, additionally, a reference electrode with given potential, i.e., $\Phi_{\text{ref}} = 0$.

The magnetic forward problem:

Since the divergence of the magnetic induction $\vec{B}$ is zero, a magnetic vector potential $\vec{A}$ with $\vec{B} = \nabla \times \vec{A}$ can be introduced. From the Maxwell equations, using Coulomb's gauge $\nabla \cdot \vec{A} = 0$, we can therefore derive a Laplace equation for $\vec{A}$ and, since the magnetic permeability μ is constant over the whole space, solve it as shown in (2) and described in more detail in [Wolters, 2004/1]. Let Ω be the head domain and let F be the surface enclosed by the MEG magnetometer flux transformer $\Upsilon = \partial F$. The magnetic flux Ψ through Υ is determined as a surface integral over the magnetic induction for the coil area F, or, using Stokes theorem, as

$$\Psi = \int_F \vec{B} \cdot d\vec{f} = \oint_{\Upsilon} \vec{A}(\vec{x}) \cdot d\vec{x} = \oint_{\Upsilon} \frac{\mu}{4\pi} \int_{\Omega} \frac{\vec{j}^{\,p}(\vec{y})}{|\vec{x} - \vec{y}|} d\vec{y} \cdot d\vec{x} - \oint_{\Upsilon} \frac{\mu}{4\pi} \int_{\Omega} \frac{\sigma(\vec{y}) \nabla \Phi(\vec{y})}{|\vec{x} - \vec{y}|} d\vec{y} \cdot d\vec{x}. \tag{2}$$

The first part of this magnetic flux is called the *primary magnetic flux* and in the following denoted with Ψ_p and the second is the so-called *secondary magnetic flux* Ψ_{sec}. Ψ_p is only dependent on the source model and can in general be computed by simply evaluating an analytical formula [Pohlmeier,1996]. If we define

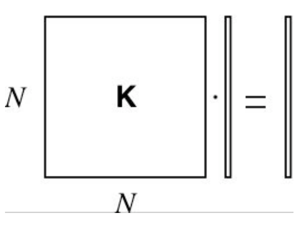
$$\vec{C}(\vec{y}) = \oint_{\Upsilon} \frac{1}{|\vec{x} - \vec{y}|} d\vec{x} \tag{3}$$

and if the potential distribution Φ is given, the final equation for Ψ_{sec} emerges from the secondary (return) currents and can be given by

$$\Psi_{\text{sec}} = -\frac{\mu}{4\pi} \int_{\Omega} (\sigma(\vec{y}) \nabla \Phi(\vec{y}), \vec{C}(\vec{y})) d\vec{y} \tag{4}$$

Finite element discretization aspects for the EEG forward problem:

For the numerical solution, we choose a finite dimensional subspace with dimension N and a standard nodal finite element basis $\psi_1, \ldots, \psi_N$. The numerical solution process depends on the chosen model for the primary source. Here, we will refer to the literature for a deeper discussion and restrict ourselves to the following remarks. The mathematical dipole model together with the subtraction approach leads to a right-hand side vector $\underline{j}^{\text{math}} \in R^N$ with N non-zero entries [Wolters, 2003]. The blurred dipole model [Buchner,1997] [Wolters, 2003] follows the law of St.Venant and is made up from monopolar loads on all neighboring FE nodes so that the dipolar moment is fulfilled and the source load is as regular as possible. The dipole moment is then only a means for visualization. In this case, the right-hand side vector $\underline{j}^{\text{blur}} \in R^N$ has only c_{nz} nonzero entries with c_{nz} the number of neighboring FE nodes. In the following derivation of the theory, we will only consider the case $\underline{j} = \underline{j}^{\text{blur}}$. Nevertheless, as shown in [Wolters, 2004/1], for $\underline{j} = \underline{j}^{\text{math}}$, the theory is very similar. The application of variational and FE techniques yields a system of linear equations

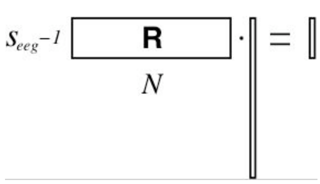


$$\mathbf{K}\underline{\Phi} = \underline{j}, \tag{5}$$

where the stiffness or geometry matrix has the entries

$$\mathbf{K}_{ij} = \int_{\Omega} (\nabla \psi_j(\vec{y}), \sigma(\vec{y}) \nabla \psi_i(\vec{y})) d\vec{y}, \forall 1 \le i, j \le N$$

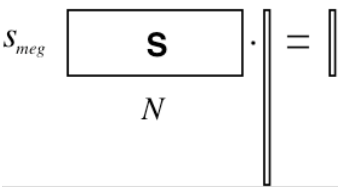
and is symmetric positive definite. Let us further assume that the $(s_{\text{EEG}} - 1)$ non-reference EEG electrodes directly correspond to FE nodes at the surface of the head model. It is then easy to determine a restriction matrix $\mathbf{R} \in R^{(s_{EEG}-1) \times N}$, which has only one non-zero entry with the value 1 in each row and which maps the potential vector onto the non-reference EEG electrodes:



$$\mathbf{R}\underline{\Phi} =: \underline{\Phi}_{\text{EEG}} \tag{6}$$

Finite element discretization aspects for the MEG forward problem:

For the magnetic forward problem, the flux transformers of the MEG device have to be modeled. Following [Pohlmeier, 1996], we model such a coil by means of a thin, closed conductor loop, using isoparametric quadratic row elements. When approximating the potential Φ by means of its Galerkin projection, Equation (4) can be written in matrix form



$$\mathbf{S}\underline{\Phi} =: \underline{\Psi}_{\text{sec}} \tag{7}$$

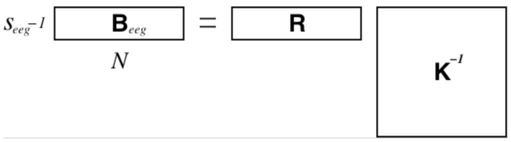
with $\mathbf{S} \in R^{s_{\text{MEG}} \times N}$ the so-called *secondary flux matrix*. $\mathbf{S}$ maps the potential onto the secondary flux vector $\underline{\Psi}_{\text{sec}} \in R^{s_{\text{MEG}}}$. The secondary flux matrix has the entries

$$\mathbf{S}_{ij} = -\frac{\mu}{4\pi}\int_{\Omega}(\sigma(\vec{y})\nabla\psi_j(\vec{y}),\vec{C}_i(\vec{y}))d\vec{y}, \forall 1 \le j \le N$$

where $\vec{C}_i(\vec{y})$ denotes the function (3) for the i[th] MEG magnetometer $\Upsilon_i, \forall 1 \le i \le s_{\text{MEG}}$. For the computation of the matrix entries of $\mathbf{S}$, a FE ansatz for the integrand and Gauss integration is used [Pohlmeier, 1996].

Computation of the lead field bases:

The inverse of the geometry/stiffness matrix, $\mathbf{K}^{-1}$, exists, but its computation is a difficult task, since the sparseness of $\mathbf{K}$ will be lost while inverting. But with regard to the EEG inverse problem, we are only interested in computing

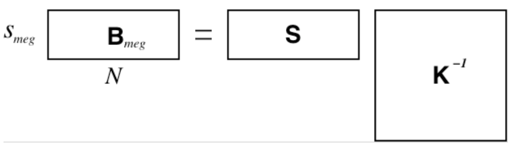


$$\mathbf{B}_{\text{EEG}} := \mathbf{R}\mathbf{K}^{-1} \in R^{(s_{\text{EEG}}-1)\times N} \tag{8}$$

which describes the direct mapping of a FE right-hand side vector to the non-reference electrodes:

$$\mathbf{B}_{\text{EEG}}\underline{j} = \mathbf{R}\mathbf{K}^{-1}\underline{j} = \mathbf{R}\underline{\Phi} = \Phi_{\text{EEG}} \tag{9}$$

[Weinstein, 2000] introduced the notation *EEG lead field basis* for $\mathbf{B}_{\text{EEG}}$. We will now see that we face a comparable situation with regard to the MEG inverse problem. In fact, let us define the *MEG lead field basis*:



$$\mathbf{B}_{\text{MEG}} := \mathbf{S}\mathbf{K}^{-1} \in R^{s_{\text{MEG}} \times N} \tag{10}$$

One should note that the rows of $\mathbf{B}_{\text{EEG}}$ do indeed form a basis in the mathematical sense, while this is not necessarily true for $\mathbf{B}_{\text{MEG}}$. $\mathbf{B}_{\text{MEG}}$ describes the direct mapping of the FE right-hand side vector to the secondary magnetic flux vector:

$$\mathbf{B}_{\text{MEG}}\underline{j} = \mathbf{S}\mathbf{K}^{-1}\underline{j} = \mathbf{S}\underline{\Phi} = \Psi_{\text{sek}} \tag{11}$$

The lead field basis can be computed as follows: If we multiply the matrix equation

$$\begin{bmatrix}\mathbf{B}_{\text{EEG}} \\ \mathbf{B}_{\text{MEG}}\end{bmatrix} = \begin{bmatrix}\mathbf{R} \\ \mathbf{S}\end{bmatrix}\mathbf{K}^{-1} \tag{12}$$

with $\mathbf{K}$ from the right side, transpose both sides and use symmetry of $\mathbf{K}$, we obtain

$$\mathbf{K}\left[\mathbf{B}_{\text{EEG}}^{\text{tr}} \quad \mathbf{B}_{\text{MEG}}^{\text{tr}}\right] = \left[\mathbf{R}^{tr} \quad \mathbf{S}^{tr}\right] \tag{13}$$

From the last equation, we can compute the lead field bases by solving $s := (s_{\text{EEG}} - 1) + s_{\text{MEG}}$ large sparse FE equation systems using iterative solver methods as described in [Wolters, 2004/2]. For $\underline{j} = \underline{j}^{\text{math}}$, the theory is very similar [Wolters, 2004/1]. With regard to the forward computation complexity, the main difference between both source models is that a vector with only nonzero entries has to be multiplied to the lead field bases in contrast to only c_{nz} nonzeros for $\underline{j} = \underline{j}^{\text{blur}}$.

RESULTS

The new approach was tested by means of an influence matrix computation, which is the basis for all current density reconstruction methods. Nevertheless, the presented approach can be used for all inverse algorithms in discrete and also continuous parameter space such as MUSIC, dipole fitting etc.. For the following computational complexity considerations, we chose an anisotropic tetrahedral FE head model with 892,119 elements and 147,287 nodes. The brain surface was represented by a triangular mesh with 2mm mesh resolution, resulting in an influence space with 19106 triangles and 9555 nodes. We chose a 71 electrode EEG configuration and a 147 channel whole head BTI MEG. The influence matrices were computed without normal constraint but with a tangential constraint for the MEG, so that 9555*3=28665 forward computations were necessary for the EEG influence matrix and 9555*2=19110 for the MEG. We performed the simulations on two platforms, a 3.2GHz Pentium 4 PC with 2GB main and 1024 KB cache memory running Red-Hat Linux and a 1GHz G4 Apple Macintosh PowerBook with 1GB main and 512 KB cache memory running OSX. We compared the new lead field bases approach, i.e., solving (13) in a setup phase and then, for each forward solution, Equation (9) or (11), with the standard approach in FEM source reconstruction [Buchner, 1997], i.e., for each dipole in the influence space first solving Equation (5) and then Equation (6) or (7). The iterative FE solver method is indicated in column 3 of Table 1, we refer to [Wolters, 2004/2] for more informations. The setup time, column 4 in Table 1, only occurs once per head model. For both methods we measure the setup time for the preconditioner. For the lead field bases approach, we furthermore have to add the time for solving (13). Simulations concerning the computational complexity for the influence matrix were performed for both, the blurred [Buchner, 1997][Wolters, 2003] and the mathematical dipole [Wolters, 2003]. This is indicated in column 5 of Table 1 by means of the number of non-zeros of the FE Right-Hand Side (RHS) being c_{nz} for the blurred and N for the mathematical dipole. We also indicate the maximal memory necessary for the current implementation in our FE based source reconstruction software NeuroFEM (http://www.simbio.de).

Table 1 Comparing the computational complexity between the lead field bases- and the standard- approach in an influence matrix computation

Platform	Method	Solver method	Setup time		RHS:#nonzeros	Influence matrix		Max.memory (in MB)	
			MEG	EEG		MEG	EEG	MEG	EEG
Pentium 4 PC	New Lead field bases approach	3RHS-AMG-CG	2.6min	1.3min	$c_{nz} \approx 20$	18 sec	21 sec	654	405
			5min	3.1min	$N = 147287$	3 h	2.2 h	654	405
	Standard	symIC(0)-CG	0.2sec	0.2sec	$c_{nz} \approx 20$	52 h	59 h	432	219
Mac G4	New Lead field bases approach	3RHS-AMG-CG	14min	6.5min	$c_{nz} \approx 20$	63 sec	56 sec	654	405
			23min	11.3min	$N = 147287$	8.4 h	6 h	654	405
	Standard	symIC(0)-CG	0.5sec	0.5sec	$c_{nz} \approx 20$	230 h	302 h	432	219

DISCUSSION

In this paper we presented a new approach to strongly reduce the algorithmic complexity of EEG/MEG inverse source reconstruction algorithms, which are based on FE volume conductor modeling of the human head. The FE computational complexity of the standard approach can be seen as the main disadvantage of FE compared to Multi-Layer Sphere or BE head modeling. Our approach turns out to be very effective if the number of EEG/MEG sensors is much smaller than the number of sources for which a forward computation has to be carried out. This is the case in most applications, since the number of sensors is about 10^2, while the number of necessary forward computations is often beyond 10^4. Our approach opens new possibilities concerning the resolution of FE head modeling. The number of large sparse linear systems that have to be solved per head geometry is now limited to the number of EEG/MEG sensors in order to compute the lead field basis, a matrix with ``number of sensors" rows and ``number of FE nodes" columns. The algebraic multigrid preconditioned conjugate gradient method with simultaneous treatment of multiple right-hand sides is an efficient solver for this setup phase, as shown in [Wolters, 2004/2]. Each FE forward computation within inverse methods on both continuous and discrete source parameter space is then reduced to the multiplication of the FE right-hand side with the lead field basis. In combination with the blurred dipole model, a FE forward solution is then limited to $2 * s * c_{\text{nz}}$ operations with s the number of sensors and c_{nz} the number of neighbours to a FE node. For the mathematical dipole model we show in [Wolters, 2004/1] how to further speedup the influence matrix computations by using the data-sparse H-matrix format. In combination with [Wolters, 2002], the parallelization of our new lead field approach is straight forward, important especially for higher resolutions where the memory has to be distributed.

ACKNOWLEDGEMENTS

This work has been supported by the MPI for Mathematics in the Sciences Leipzig, by the IST-program of the European Commission, project No.10378, http://www.simbio.de and by the NIH NCRR center for Bioelectric Field Modeling, Simulation and Visualization.

REFERENCES

Buchner, H., Knoll, G., Fuchs, M., Rienaecker, A., Beckmann, R., Wagner, M., Silny, J. and Pesch, J. Inverse localization of electric dipole current sources in finite element models of the human head. Electroenc.Clin.Neurophysiol., 102, 1997, p.267-78.

Marin, G., Guerin, C., Baillet, S., Garnero, L., Meunier G. Influence of skull anisotropy for the forward and inverse problem in EEG: simulation studies using the FEM on realistic head models. Human Brain Mapping, 6, 1998, p.250-69.

Haueisen, J., Tuch, D.S., Ramon, C., Schimpf, P.H., Wedeen, V.J., George, J.S., Belliveau, J.W.. The Influence of Brain Tissue Anisotropy on Human EEG and MEG. Neuroimage, 15, 2002, p.159-66.

Nolte, G.. The magnetic lead field theorem in the quasi-static approximation and its use for magnetoencephalography forward calculation in realistic volume conductors. Phys. Med. Biol., 48 (22), 2003, p.3637-52.

Pohlmeier, R. Lokalisation elektrischer Gehirnaktivitaet durch inverse Analyse des Magnetoenzephalogramms (MEG) mit Finite-Elemente-Modellen des Kopfes. Diplomarbeit in Elektrotechnik, RWTH Aachen, 1996.

Vamrunste, B., Van Hoey, G., Van de Walle, R., D'Have, M.R.P., Lemahieu, I.A., Boon, P.A.J.M. The Validation of the Finite Difference Method and Reciprocity for Solving the Inverse Problem in EEG Dipole Source Analysis. Brain Topography, 14 (2), 2001, p. 83--92.

Weinstein, D., Zhukov, L., Johnson, C. Lead-field bases for EEG source imaging. Annals of Biomed.Eng., 28 (9), 2002, p. 1059--66.

Wolters, C.H., Kuhn, M., Anwander, A., Reitzinger, S. A Parallel Algebraic Multigrid Solver for Finite Element Method based Source Localization in the Human. Comp.Vis.Sci., 5, 2002, p.165-77.

Wolters, C.H. Influence of Tissue Conductivity Inhomogeneity and Anisotropy on EEG/MEG based Source Localization in the Human Brain. MPI Series in Cognitive Neuroscience, No. 39, Leipzig: MPI of Cognitive Neuroscience, also: Dissertation, University of Leipzig, http://dol.uni-leipzig.de/pub/2003-33, 2003.

Wolters, C.H., Grasedyck, L., Hackbusch, W. Efficient Computation of Lead Field Bases and Influence Matrix for the FEM-based EEG and MEG Inverse Problem. Inverse Problems, 20 (4), 2004, p.1099-116.

Wolters, C.H., Anwander, A., Reitzinger, S., Haase G. Algebraic MultiGrid with Multiple Right-Hand Side Treatment for an Efficient Computation of EEG and MEG Lead Field Bases. in Biomag2004, Proc. 14th Int. Conf. on Biomagnetism, Boston, USA, August 2004.