Multiple Current Dipole Estimation Using Simulated Annealing

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***Abstract*—A method for estimating electrical current distribution in the human brain using a multiple current dipole model is presented. A cost function for estimating multiple dipoles is proposed and a simulated annealing algorithm is used to obtain an acceptable solution. Computer simulation is used to evaluate the effectiveness of this method.**

I. Introduction

A METHOD based on magnetic field measurement for imaging electrical current distribution in the human brain has been studied in recent years. The magnetic field induced by current distribution is measured using a highly sensitive detector and current distribution is estimated from the measured data. This problem estimating current distribution, therefore, can be seen as an inverse problem.

Until recently, most research has focused on simple external magnetic fields created by, for example, audio or visual stimuli. In this estimation, a single current dipole has been used as the current source model because of its simplicity [1]. When the source current is localized in one small region of the brain, this model is reasonable. All of the fields created by external stimuli, however, are not always explained by a single current dipole. Of course, a single dipole poorly represents the current distribution caused by higher brain functions.

Two- and three-dimensional reconstruction of current distribution has also been tried [2]–[7]. This approach, however, still has some problems in terms of the signal-to-noise ratio, the total amount of measured data, and so forth.

We therefore propose using a multiple dipole model as an intermediate model. While a more complicated field can be expressed using this method, the problem to be solved becomes more difficult because of the existence of suboptimal solutions [7]. To overcome this, we also propose using a stochastic algorithm which is called simulated annealing.

In Section II, the multiple dipole model is introduced and the inverse problem is formulated. We treat this problem as an optimization problem and design a cost function which is minimized for the solution. Particularly, we propose a cost function that can handle an unknown number of dipole. In Section III, the simulated annealing algorithm is shown. In Section IV, the proposed method is evaluated using computer simulation.

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II. Formulation of the Problem

In neuromagnetic imaging, magnetic field components, normal to the surface of the head, are often measured. For dipole estimation, the head is often modeled by a spherically symmetric conductor or a homogeneous conductor with spherical shape [8]. Since the volume current does not induce a radial magnetic field component on the surface of the head in such models, it is easy to estimate the current dipole (impressed current) from data measured on the surface of the head. In this paper, we assume that the head can be modeled by a homogeneous conductor with a spherical shape and that the magnetic field component normal to the surface can be used for the dipole estimation.

The center of the sphere is defined as the coordinate origin. Let $\mathbf{r}_n$ be a position vector of the n-th dipole, $\mathbf{q}_n$ be a moment of the dipole, and $\mathbf{r}_m$ a position vector of the m-th measuring point. The magnetic field induced by N dipoles at the measuring position $\mathbf{r}_m$ is subject to Biot-Savart's law and is expressed as

$$\mathbf{B}(\mathbf{r}_m) = \frac{\mu_0}{4\pi} \sum_{n=1}^{N} \frac{\mathbf{q}_n \times (\mathbf{r}_m - \mathbf{r}_n)}{|\mathbf{r}_m - \mathbf{r}_n|^3} \quad \text{for } m = 1, \ldots, M \tag{1}$$

where M is the total number of measuring points. The magnetic permeability μ of the medium is approximately expressed by the permeability μ_0 of free space and $\times$ means the vector product. The radial component measured by the detector is written as follows

$$\begin{aligned} g(\mathbf{r}_m) &= \mathbf{B}(\mathbf{r}_m) \cdot \frac{\mathbf{r}_m}{|\mathbf{r}_m|} \\ &= \frac{\mu_0}{4\pi} \sum_{n=1}^{N} \frac{\mathbf{q}_n \cdot (\mathbf{r}_m \times \mathbf{r}_n)}{|\mathbf{r}_m - \mathbf{r}_n|^3} \quad \text{for } m = 1, \ldots, M \end{aligned} \tag{2}$$

where the dot means the scalar product. The objective of this inverse problem is to find both the position and the moment of each current dipole from the measured data set $\{g(\mathbf{r}_m)\}$.

Under the assumptions that the head can be modeled by a homogeneous conductor with a spherical shape and that the magnetic field component normal to the surface can be used for the dipole estimation, the radial component of the current dipole $\mathbf{q}_n$ produces no magnetic field outside the conductor.

Thus, it is theoretically impossible to estimate this component from the measured data. The parameters which we can estimate are two tangential components of the moment vector and three components of the position vector.

We will now look at the cost function needed to solve the inverse problem. For simplicity, we will first consider a case where the number of current dipoles is known. The cost function to be minimized is the squared error between the measured data and the magnetic field induced by the estimated dipoles. If $\{\widehat{g}(\mathbf{r}_m)\}$ is the field induced by the estimated dipoles, the cost function is

$$E_1 = \sum_{m=1}^{M} \{g(\mathbf{r}_m) - \widehat{g}(\mathbf{r}_m)\}^2. \tag{3}$$

This function can also be expressed using vector notation as

$$E_1 = |\mathbf{g} - \widehat{\mathbf{g}}|^2 \tag{4}$$

where

$$\mathbf{g} = [g(\mathbf{r}_1), g(\mathbf{r}_2), \ldots, g(\mathbf{r}_M)]^T \tag{5}$$

and

$$\widehat{\mathbf{g}} = [\widehat{g}(\mathbf{r}_1), \widehat{g}(\mathbf{r}_2), \ldots, \widehat{g}(\mathbf{r}_M)]^T. \tag{6}$$

Next, consider a much more practical case where the number of current dipoles is unknown. The upper limit on the number of dipoles is based on the total number of measuring points. For example, if the total number of measuring points is 32, no more than six dipoles ($5 \times 6 = 30$ parameters) can be originally identified. This limit of roughly six dipoles is, however, only an estimate and is not always representative of the actual number.

For example, even if six dipoles are estimated, the actual number might be two. In this case, the ideal solution is for only two of the dipole estimates to have actual values and for the other four dipole estimates to become zero. The squared error criterion (4) however, does not always provide such a solution. There exist infinite combinations of estimated six dipoles that induce the same magnetic field as the actual field induced by true dipoles. In other words, the cost function has a wide flat bottom where the solution provides the minimum value for (4). This ambiguity of solution is caused by the ill-posed nature of this inverse problem.

We therefore modified the cost function to provide one reasonable answer from an infinite number of solutions. The criterion used to select a solution is "the solution which is composed of least number of dipole is the best solution." This criterion should provide a reasonable solution when the current distribution is caused by some sensory stimulation, since it tends to be localized in a small region. Even when we have no *a priori* knowledge about the current distribution, this criterion is appealing since it can provide the most reasonable solution in the sense that the resulting solution corresponds to the least brain activity which explains the measured data [4].

Taking the above criterion into account, we propose the following revised cost function

$$E_2 = |\mathbf{g} - \widehat{\mathbf{g}}|^2 + w \sum_{i=1}^{N} |\widehat{\mathbf{g}}_i|^{\alpha} \tag{7}$$

where α is a real number satisfying $0 < \alpha < 1$ and the constant w is a proper weighting factor. In the second term, $\widehat{\mathbf{g}}_i$ represents the contribution by the i-th estimated dipole only. Each element of $\widehat{\mathbf{g}}_i$ can be expressed as

$$\widehat{g}_i(\mathbf{r}_m) = \frac{\mu_0}{4\pi} \frac{\widehat{\mathbf{q}}_i \cdot (\mathbf{r}_m \times \widehat{\mathbf{r}}_i)}{|\mathbf{r}_m - \widehat{\mathbf{r}}_i|^3}. \tag{8}$$

It should be noted that the norm for the magnetic field vector $\widehat{\mathbf{g}}_i$ is calculated rather than dipole vector $\widehat{\mathbf{q}}_i$. This is done to avoid a position shift; if the norm of $\widehat{\mathbf{q}}_i$ was used, each dipole would tend to shift towards the detector [4]. This is understood intuitively through the following simple example. There are two dipoles; one is an arbitrary dipole and the other is closer to the detector than the first dipole and has lesser magnitude. These two dipoles may induce almost the same magnetic field. If the norm of a moment is added to the cost function, the dipole near the detector is selected. This example explains why the dipoles tend to shift to the detector side. If $\widehat{\mathbf{g}}_i$ is used, there is no bias based on the position; consequently, a more exact solution is determined.

Furthermore, the exponent index α, which is less than one, helps to select a lesser number of dipoles. The reason for this is explained in the Appendix.

III. Estimation Procedure

In Section II, the cost function to be minimized was established. The rest of our task is to construct an algorithm that provides the globally optimum solution. As mentioned above, and as we shall see in the computer simulation, the cost function may have local minima. We therefore apply simulated annealing to this problem.

Simulated annealing, which is analogous to thermodynamics, was originally proposed by Kirkpatrick *et al.* [9] to solve optimization problems. One of the most important characteristics of this algorithm is that it can estimate the globally optimum solution even when there are local minima. The following is a brief review of this algorithm.

Fig. 1 shows a flow chart of the simulated annealing algorithm for the current dipole estimation. First, an initial estimate of the current dipoles are arbitrarily made. Next, perturbations are introduced to each parameter for each dipole by adding or subtracting a small value. Then, the change in the cost function, ΔE, caused by this perturbation is calculated. If $\Delta E < 0$, the perturbation is accepted as a favorable change which decreases the cost function and the value of the parameter is updated. If $\Delta E > 0$, the perturbation is accepted, subject to the Boltzmann probability statistics $p(\Delta E) = \exp(-\Delta E/kT)$, where T is a factor associated with the thermodynamic temperature.

This stochastic acceptance is essential in avoiding a trap at a local minima. These trials and updates over all parameters are repeated until thermal equilibrium is reached. The thermal equilibrium is defined as the condition where the number of accepted perturbations that increase the cost function is equal to that of accepted perturbations that decrease the cost function. When thermal equilibrium is reached, the temperature T is decreased according to a predetermined schedule. The same

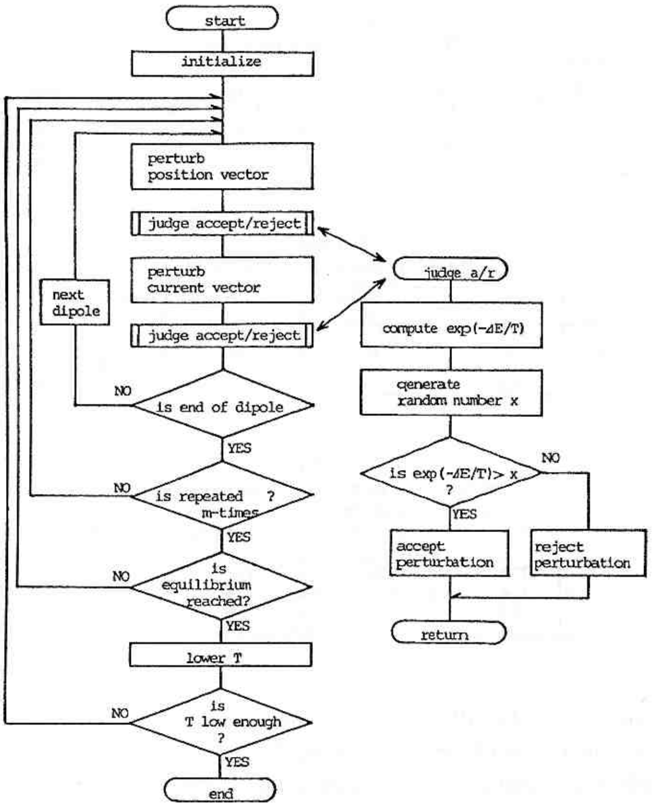


Fig. 1. Simulated annealing algorithm for dipole estimation.

procedure is repeated until the next thermal equilibrium is obtained. This process continues until the temperature becomes so low that no more updating of parameters occur. The whole process described above enables the estimates to reach the global minimum.

IV. Computer Simulations and Discussions

We investigated both the ability of the proposed cost function and the need for annealing by using computer simulation. The human head was modeled as a spherical homogeneous conductor with a radius of 80 mm. The magnetic field component normal to the spherical surface was measured at 64 points evenly distributed over the upper hemispherical surface along the head. For the measurement, a noise-free environment was assumed in order to focus on the ill-posed nature of this inverse problem and the treatment for the existence of local minima.

The simulated annealing temperature was decreased according to the decrement rule [10], [11]

$$T_k = \begin{cases} T_0/(1+k) & k \le K_{\lim} \\ 0.8T_{k-1} & k > K_{\lim} \end{cases}, \tag{9}$$

where Tk is the k-th temperature and T_0 is the initial temperature. The decrement rule is switched from $T_k = T_0/(1+k)$ to a much faster decrement according to $T_k = 0.8T_{k-1}$ at $K_{\lim}$. In our simulation, $K_{\lim}$ was determined empirically as 300.

In the modified cost function, the exponent index α in the second term was set at 1/2.

TABLE I
ORIGINAL SOURCES (UNIT OF POSITION VECTOR IN [MM]. UNIT OF CURRENT VECTOR IS ARBITRARY.)

	Position vector			Current vector		
dipole	px	py	pz	qu	qv	s
#1	18.5	29.9	15.5	9.8	-28.4	30.0
#2	0.1	3.3	47.7	27.5	-11.9	30.0
#3	29.3	3.8	5.3	-3.1	29.8	30.0

TABLE II
INITIAL ESTIMATE

	Position vector			Current vector		
dipole	px	py	pz	qu	qv	s
#1	20.0	40.0	40.0	8.9	3.0	9.4
#2	-20.0	40.0	40.0	26.8	-8.9	28.2
#3	0.0	20.0	40.0	20.0	8.9	21.9
#4	0.0	40.0	40.0	20.0	0.0	20.0

TABLE III
FINAL ESTIMATE WHEN $w = 0$

	Position vector			Current vector				
dipole	px	py	pz	qu	qv	d	s	E1
#1	23.1	32.1	15.4	10.0	-17.8	5.1	20.4	
#2	-0.9	3.9	48.0	23.5	-17.5	1.2	33.9	0.58
#3	26.5	0.3	15.6	-9.7	29.0	11.2	30.6	
#4	8.6	30.8	22.9	3.6	-9.3	---	10.0	

A. Effectiveness of the Modified Cost Function

Ability of the modified cost function to estimate an unknown number of dipoles was investigated. In the first example, we assumed an original source to be composed of three dipoles and prepared four dipoles for the estimation. Although the upper limit, based on the number of measuring points, is greater than four, four dipoles were sufficient to show the ambiguity of solution.

Table I shows the original sources. The values p_x, p_y and p_z denote the Cartesian coordinates of the position of the dipole in [mm]. The values q_u and q_v denote the two tangential components of a moment of the dipole in an arbitrary unit. The magnitude of the dipole in a tangential plane, s, is defined as

$$s = \sqrt{q_u^2 + q_v^2}. \tag{10}$$

Table II shows the initial values for each parameter in the estimating procedure. Table III shows the final estimate when the weighting factor, w, in the cost function, (7), is zero, which corresponds to (4). In this table, d is the localization error for each dipole estimate corresponding to an original dipole, and is defined as

$$d = \sqrt{(\widehat{p}_x - p_x)^2 + (\widehat{p}_y - p_y)^2 + (\widehat{p}_z - p_z)^2}. \tag{11}$$

The E_1 in the table is the squared error of the magnetic field in arbitrary unit calculated by (4). While there is a good estimate for dipole #2, a localization error of over 10 mm occurs for dipole #3. Furthermore, all four dipoles have nonzero values for dipole magnitude. This leads to a misunderstanding that there exists four dipoles.

E_1 in the final estimates is very small compared with that of the initial estimate which was 8052.06, but still not zero. This means that the estimate does not completely reach the global minimum. We believe the reason for this is as follows. Theoretically, the cooling temperature inversely proportional to a logarithmic function of time is necessary to reach the global minimum [12]. The actual schedule used

TABLE IV
FINAL ESTIMATE WHEN $w = 0.5$

	Position vector			Current vector				
dipole	p_x	p_y	p_z	q_u	q_v	d	s	E_1
#1	18.2	28.6	15.3	11.2	-29.5	1.4	31.6	
#2	0.2	3.2	47.7	28.0	-11.3	0.1	30.2	0.17
#3	31.0	3.9	7.3	-4.0	26.5	2.6	26.8	
#4	15.9	27.4	0.3	0.0	0.0	---	0.0	

TABLE V
FINAL ESTIMATE WHEN $w = 1.0$

	Position vector			Current vector				
dipole	p_x	p_y	p_z	q_u	q_v	d	s	E_1
#1	19.5	38.0	19.3	6.2	-16.9	9.0	18.0	
#2	2.0	2.5	42.8	42.9	11.3	5.3	44.4	21.88
#3	-5.9	0.6	31.8	0.0	0.0	---	0.0	
#4	-14.8	26.9	22.7	0.0	0.0	---	0.0	

TABLE VI
FINAL ESTIMATE FOR ORIGINAL SOURCES
COMPOSED OF #1 AND #2 WHEN $w = 0.5$

	Position vector			Current vector				
dipole	p_x	p_y	p_z	q_u	q_v	d	s	E_1
#1	19.1	31.1	15.2	9.1	-26.2	1.4	27.7	
#2	0.4	3.3	47.6	28.4	-10.0	0.3	30.1	0.22
#3	2.5	36.2	31.3	0.0	0.0	---	0.0	
#4	15.2	25.7	4.6	0.0	0.0	---	0.0	

TABLE VII
OTHER ORIGINAL SOURCES

	Position vector			Current vector		
dipole	p_x	p_y	p_z	q_u	q_v	s
#1	7.0	0.8	69.3	-1.2	30.0	30.0
#2	56.2	5.8	39.4	25.9	-15.2	30.0
#3	-0.4	7.0	35.6	26.6	13.8	30.0

TABLE VIII
FINAL ESTIMATE WHEN $w = 0$

	Position vector			Current vector				
dipole	p_x	p_y	p_z	q_u	q_v	d	s	E_1
#1	6.9	0.9	68.7	-1.3	31.6	0.7	31.6	
#2	56.4	5.8	39.6	25.3	-15.0	0.3	29.4	0.77
#3	-3.0	9.2	33.5	31.3	5.8	4.0	31.8	
#4	-16.8	14.0	29.7	-3.4	2.1	---	4.0	

TABLE IX
FINAL ESTIMATE WHEN $w = 0.5$

	Position vector			Current vector				
dipole	p_x	p_y	p_z	q_u	q_v	d	s	E_1
#1	6.5	1.3	67.5	0.4	34.3	3.9	34.3	
#2	56.5	6.0	39.4	25.4	-14.7	0.1	29.3	3.13
#3	-2.4	9.7	29.6	25.0	9.4	5.5	26.7	
#4	13.8	19.7	19.8	0.0	0.0	---	0.0	

here, however, is a modified version of the ideal schedule. Though the schedule used takes much less time to converge than the ideal schedule, the probability of reaching the global minimum decreases a little. The result is therefore due to the suboptimal annealing schedule. The final value of the cost function, however, is sufficiently small and we feel that there is no problem with practical use.

Table IV shows the final estimate when w is 0.5. Dipole estimates #1, #2, and #3 are very close to the original dipoles #1, #2, and #3. Furthermore, it should be noted that dipole estimate #4 disappears, i.e., the magnitude of the current dipole becomes zero. This confirms the effectiveness of selecting a solution based of the minimum number of dipoles. The squared error of the magnetic field remains a small value.

Table V shows the final estimate when w is set to 1.0. The localization errors of the #1 and #2 dipole estimates are greater than the those in the case where $w = 0.5$. Furthermore, both the #3 and #4 dipole estimates disappear. This is due to the excess effect of the second term in the cost function.

A computer simulation where two original dipoles were estimated using four dipoles was also performed. The #1 and #2 dipoles in Table I were used as the two original dipoles, and the four dipoles shown in Table II were again used as the initial guess. Table VI shows the results when $w = 0.5$. A very good estimate, close to the original dipoles, was obtained. It was found from the above simulations that the proposed method, using the cost function with an appropriate w, successfully provides a good estimate for current source, even if the number of dipoles is unknown.

We conducted a second computer simulation with another dipole set for the original sources. Table VII shows the original sources. Four dipoles were prepared for estimation, and the initial estimate shown in Table II was used again. Table VIII shows the final estimate when $w = 0$. In this case, #1 to #3 are very close to the original dipoles. The magnitude of dipole #4 is relatively small. This is considered to be a case that the estimate occasionally reached to one optimum solution among the wide solution space.

Table IX shows the final estimate when $w = 0.5$. Although the results are not as good as those shown in Table VIII, the obtained estimates are acceptable. Estimate #4 vanished again as before.

From the several computer simulations, it was found empirically that the best value of w is about 0.5, in the noise-free case. We have not yet found how to determine the optimum value for w. The first term of the cost function has a wide and flat bottom in the case of an ill-posed and noise free condition. On the other hand, the second term causes the bottom to slant. Therefore, theoretically, w should be nonzero, but so small that the minimum point produced by the addition of the second term does not move away from the bottom. In the case of noisy conditions, there seems to be an optimum value for w dependent on the noise statistics. In any case, further investigation is needed to determine the optimal value for w.

B. The Necessity of Annealing

A dipole estimation using quenching instead of annealing was also done. During estimation, the temperature was fixed at zero degrees. As the cost function, (7) was used. The result of quenching is shown in Table X. The final estimate does not provide an acceptable estimate. Also the final value of the squared error remains high compared with the annealing case. This means a local minimum trap in the solution. In this sense, the quench method is identical to conventional optimization algorithms, such as the steepest descent method. This clearly shows the superiority of simulated annealing to conventional optimization algorithms.

Since the algorithm for simulated annealing is simple, its programming is very easy compared to the conventional methods. It is true, however, that the computational performance of simulated annealing is not as good because of slow cooling. In our simulation, it takes about five hours on average to estimate four dipoles using a mini-computer capable of 1

TABLE X
FINAL ESTIMATE BY QUENCHING (THE LOCALIZATION ERROR d COULD NOT BE CALCULATED BECAUSE ANY DIPOLE ESTIMATE DOES NOT CORRESPOND TO THE ORIGINAL DIPOLES.)

	Position vector			Current vector				
dipole	px	py	pz	qu	qv	d	s	E_1
#1	26.2	38.5	22.3	5.6	-10.4	---	11.8	
#2	-3.9	9.4	69.2	0.2	-1.3	---	1.3	13.80
#3	2.4	2.2	41.5	39.6	18.2	---	43.6	
#4	3.4	38.1	22.7	-0.8	-7.7	---	7.7	

MIPS. This time could be reduced considerably by using higher performance computer. Furthermore, parallel estimation of multiple dipoles could make even higher speed processing possible [11].

V. CONCLUSION

A conventional single dipole model is a poor match for the complicated current sources in the human brain. We introduced a multiple dipole model which can overcome this problem. The results, however, can be ambiguous. We thus proposed a cost function which determines a solution with a minimum number of dipoles. Since the proposed cost function generally has local minima, the conventional optimization algorithm using a gradient is not applicable. This difficulty was overcome by using a stochastic algorithm called simulated annealing. Through computer simulation, the effectiveness of the proposed method was demonstrated. Although the fundamental characteristics of the proposed method have been shown in this paper, further research, including the problem of how to determine the weighting factor, is needed.

APPENDIX

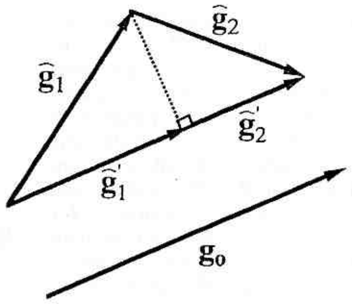
Equation (7) with exponent index α greater than zero and less than one determines a solution with a minimum number of dipoles from among the solutions which minimize (4). This is shown in the following simple case.

First, the true number of dipoles is assumed to be one, and two dipoles are used for estimation. The magnetic field induced by the true dipole is $\mathbf{g}_0$ and the magnetic fields induced by the two estimated dipoles independently are $\widehat{\mathbf{g}}_1$ and $\widehat{\mathbf{g}}_2$. The total field of dipole estimates is therefore $\widehat{\mathbf{g}}_1 + \widehat{\mathbf{g}}_2$. After completing the estimation process when the value of (4) becomes small enough, the equation

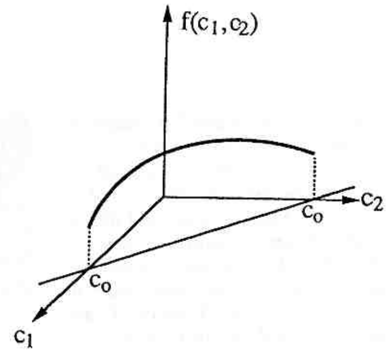
$$\mathbf{g}_0 = \widehat{\mathbf{g}}_1 + \widehat{\mathbf{g}}_2 \tag{A1}$$

must be satisfied. There exist, however, an infinite number of solutions for (A1). Fig. 2(a) shows this graphically. The pair $(\widehat{\mathbf{g}}_1, \widehat{\mathbf{g}}_2)$ is an example of vectors whose summation is equal to $\mathbf{g}_0$ and whose directions are different from each other. It is clear that there are an infinite number of such examples. Now consider one more pair $(\widehat{\mathbf{g}}'_1, \widehat{\mathbf{g}}'_2)$ whose directions are both the same as $\mathbf{g}_0$, as in Fig. 2(a), and then evaluate the second term of (7) for these examples. For $0 < \alpha$, since $|\widehat{\mathbf{g}}'_1|^\alpha < |\widehat{\mathbf{g}}_1|^\alpha$ and $|\widehat{\mathbf{g}}'_2|^\alpha < |\widehat{\mathbf{g}}_2|^\alpha$, the following relation is obtained

$$|\widehat{\mathbf{g}}'_1|^\alpha + |\widehat{\mathbf{g}}'_2|^\alpha < |\widehat{\mathbf{g}}_1|^\alpha + |\widehat{\mathbf{g}}_2|^\alpha. \tag{A2}$$



(a)



(b)

Fig. 2. (a) Two examples of pair $(\widehat{\mathbf{g}}_1, \widehat{\mathbf{g}}_2)$ whose summation $\widehat{\mathbf{g}}_1 + \widehat{\mathbf{g}}_2$ is equal to $\mathbf{g}_0$. (b) Curve of the cost function $f(c_1, c_2) = c_1^{1/2} + c_2^{1/2}$ along $c_0 = c_1 + c_2$.

Therefore, by minimizing (7), the pair $(\mathbf{g}'_1, \mathbf{g}'_2)$ with the same direction, rather than the pair $(\widehat{\mathbf{g}}'_1, \widehat{\mathbf{g}}'_2)$ with the different directions, is selected.

An ambiguity in the magnitudes of the vectors $\widehat{\mathbf{g}}'_1$ and $\widehat{\mathbf{g}}'_2$, however, still remains. If we denote a unit vector $\mathbf{g}_0/|\mathbf{g}_0|$ as $\mathbf{e}_0$ for the pair $(\widehat{\mathbf{g}}'_1, \widehat{\mathbf{g}}'_2)$ with the same direction, there exist some positive scalars c_0, c_1, c_2 which satisfy the following:

$$\begin{aligned} \mathbf{g}_0 &= c_0 \mathbf{e}_0 \\ \widehat{\mathbf{g}}'_1 &= c_1 \mathbf{e}_0 \\ \widehat{\mathbf{g}}'_2 &= c_2 \mathbf{e}_0 \end{aligned} \tag{A3}$$

where

$$c_0 = c_1 + c_2. \tag{A4}$$

Minimizing the second term of (7) corresponds to minimizing the following function

$$f(c_1, c_2) = c_1^\alpha + c_2^\alpha. \tag{A5}$$

If $\alpha < 1$, then the curve of the function $f(c_1, c_2)$ is as in Fig. 2(b) and consequently, the solution is

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} c_0 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 0 \\ c_0 \end{pmatrix}. \tag{A6}$$

In these cases, one vanishes and the other agrees with the true c_0. This means that the solution composed of a minimum number of dipoles is selected.

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