equate short term stability, we have observed long term instability during previous chronic studies. A blood pressure simulator was designed and calibrated against aortic pressure (for systolic LV pressure estimate) and left atrial pressure (for end-diastolic LV pressure estimate). Recalibration of the pressure simulator was performed once a week. Calibration of the dimension channels (Sono1 and 2) was performed by a dimension-vs-demodulator output relationship that was obtained for each channel.

Small size and low weight are important design criteria since miniswine carry the biotelemetry device in a pouch which is secured to their back. The entire biotelemetry system, including 6 AA batteries, is enclosed in a 4 × 6 × 12 cm plastic container, and weighs approximately 200 g. We observed that the animals quickly adapted to the backpack containing the biotelemetry system and that it did not cause any noticeable discomfort.

To summarize, we have developed a biotelemetry system that has proven useful in chronically recording regional left-ventricular wall thickness dimensions and left ventricular blood pressure from unrestrained conscious miniswine. Our design features a high-voltage dc-dc converter for improved ultrasonic signal strength, a time-to-voltage converter that is highly immune to synchronization frequency variations, and low-power consumption. Moreover, the small size and low weight of the system enhances its suitability for application using unrestrained conscious animals. Using surface mounted components the biotelemetry system could be miniaturized further, thus becoming suitable for application using smaller animals such as rabbits and rats.

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A Linear Discretization of the Volume Conductor Boundary Integral Equation Using Analytically Integrated Elements

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***Abstract*—A method is presented to compute the potential distribution on the surface of a homogeneous isolated conductor of arbitrary shape. The method is based on an approximation of a boundary integral equation as a set linear algebraic equations. The potential is described as a piecewise linear or quadratic function. The matrix elements of the discretized equation are expressed as analytical formulas.**

I. Introduction

For analysis of electrophysiologic data and for simulation studies it is required to compute the potential distribution on the surface of an isolated conductor of arbitrary shape. An example is the computation of the potential caused by a current dipole embedded in a head shaped conductor, simulating the spatial distribution of the electroencephalogram [1]-[3], [8], [9]. Another biophysical example is the potential distribution at the torso, generated by cardiac

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activity [4], [5]. Similar computational problems arise in electrical and mechanical engineering. If the medium is isotropic and consists of compartments having different physical properties, the boundary element method (BEM) is often appropriate to compute the fields. Although the different applications of the BEM have many theoretical aspects in common (modeling the surfaces, formulations in terms of integral equations, computing the matrix elements of the discretized system [6]), there are also important differences in the practical implementations (boundary conditions, source term, physical parameters), giving rise to a rapidly growing field of applied mathematical research. It seems, however, that the application of the BEM in biophysics has been developed in a somewhat isolated way.

In this paper we consider the computation of the potential ψ caused by a current source inside a conductor with a constant and isotropic conductivity and outer surface S. Within biophysics the corresponding boundary integral equation has been solved through a description of the surface in terms of flat triangles on which the potential is assumed to be constant. This leads to a linear system of equations, of which the matrix elements represent the solid angles of the triangles. The matrix elements can therefore be computed with a simple analytical formula [10]. It will be shown that more general choices can be made to discretize the boundary integral equation, without destroying the analytical computation of the matrix elements.

The potential $\psi(\vec{x})$ is given by a solution of the following boundary integral equation:

$$\frac{\Omega(S;\vec{x})}{2\pi}\psi(\vec{x}) = 2\psi^{\infty}(\vec{x}) - \frac{1}{2\pi}\oint\oint_{S\setminus\exists(\vec{x})}\psi(\vec{x}\,')\nabla'\frac{1}{|\vec{x}\,'-\vec{x}|}\cdot d\vec{S}\,'. \tag{1}$$

Here, $\psi^{\infty}(\vec{x})$ is the potential generated in a medium of infinite extent and $\Omega(S;\vec{x})$ is the solid angle of the surface S, viewed from the point $\vec{x}$. If $\vec{x}$ is at a regular point of S the solid angle equals 2π. The symbol $S\setminus\exists(\vec{x})$ denotes that the integral is taken over the surface S within which an environment of $\vec{x}$ is excluded, and that subsequently the limit for $\hat{E}(\vec{x}) \rightarrow \{\vec{x}\}$ is taken. When it is taken into account that the conductor consists of different compartments having different conductivities, (1) has to be modified accordingly [7]:

$$(\sigma_j^-\Omega^-(S_j;\vec{x}) + \sigma_j^+\Omega^+(S_j;\vec{x}))\psi(\vec{x}) = 4\pi\psi^{\infty}(\vec{x}) - \sum_{i=1}(\sigma_i^- - \sigma_i^+)\oint\oint_{S_i\setminus\exists(\vec{x})}\psi(\vec{x}\,')\nabla'\frac{1}{|\vec{x}\,'-\vec{x}|}\cdot d\vec{S}\,'. \tag{2}$$

Here $\psi^{\infty}(\vec{x})$ is the potential in an infinite medium with unit conductivity, σ_i^- and σ_i^+ are the conductivities just inside and outside the interface S_i and Ω^- and Ω^+ are the solid angles of the surface S viewed from inside and outside S, respectively (thus $\Omega^- + \Omega^+ = 4\pi$). However, in the present paper we shall restrict ourselves to (1) because the generalization to (2) is rather straight forward.

Equation (1) can be solved approximately by choosing a set of functions $\{h_n(\vec{x})\}_{n=1}^N$ and a set of discretization points $\{\vec{x}_m\}_{m=1}^N$ such that

$$h_n(\vec{x}_m) = \delta_{nm}. \tag{3}$$

The unknown potential $\psi(\vec{x})$ is expanded in terms of $h_n(\vec{x})$:

$$\psi(\vec{x}) = \sum_{n=1}^{N}\psi_n h_n(\vec{x}). \tag{4}$$

With these definitions (1) is approximately equivalent to the following set of linear equations with N unknowns ψ_n:

$$\sum_{n=1}^{N} B_{mn}\psi_n = 2\psi^{\infty}(\vec{x}_m) \tag{5}$$

where the matrix elements B_{mn} are given by

$$B_{mn} = \frac{1}{2\pi}\left(\oint\oint_{S\setminus\exists(\vec{x}_m)} h_n(\vec{x}\,')\nabla'\frac{1}{|\vec{x}\,'-\vec{x}_m|}\cdot d\vec{S}\,' + \delta_{nm}\Omega(S;\vec{x}_m)\right).. \tag{6}$$

The simplest choice for the functions $h_n(\vec{x})$ is obtained from a triangularization of the surface S and taking $h_n(\vec{x})$ equal to 1 on the nth triangle and zero elsewhere. The discretization points $\vec{x}_m$ are given by the centers of the triangles. In this approach the potential is described as piecewise constant function and therefore many triangles are required to represent the potential accurately. Here we propose to interpolate the potential linearly and demonstrate how the corresponding matrix elements can be calculated analytically.

II. Calculation of the Matrix Elements for the Linear Interpolation Approach

In the linear interpolation approach the surface S is also subdivided in small triangles, but contrary to the constant potential approach, the discretization points are given by the vertices of the grid. The interpolation functions are given by

$$h_n(\vec{x}) = \begin{cases} \dfrac{\det(\vec{x}_k,\vec{x}_l,\vec{x})}{\det(\vec{x}_k,\vec{x}_l,\vec{x}_n)} & \text{if } \vec{x}\in\Delta_{\text{kln}} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

where $\vec{x}_k$, $\vec{x}_l$, and $\vec{x}_n$ are the corner points of the triangle Δ_{kln}. Obviously, $h_n(\vec{x})$ is a piecewise linear function of $\vec{x}$ and it satisfies condition (3).

To compute the matrix elements the integration area will be partitioned into triangles adjacent to $\vec{x}_n$. On each triangle, the normal $\vec{n}$ is independent of $\vec{x}$. Furthermore, the diagonal elements will be treated separately, so that we may assume that $n \neq m$. It is found that

$$B_{mn} = \frac{-1}{2\pi}\left(\sum_{\Delta kln}\frac{\det(\vec{x}_k-\vec{x}_m,\vec{x}_l-\vec{x}_m,\vec{x}_n-\vec{x}_m)}{A_{kln}}\cdot\iint_{\Delta kln}\frac{h_n(\vec{x}\,')}{|\vec{x}\,'-\vec{x}_m|^3}\,dS'\right) \tag{8}$$

where the summation is over all triangles adjacent to $\vec{x}_n$ and where A_{kln} is the area of triangle Δ_{kln}. Note that in the evaluation of (8) the order of $\vec{x}_k$, $\vec{x}_l$, and $\vec{x}_n$ has to be such that the permutation $\vec{x}_k \rightarrow \vec{x}_l \rightarrow \vec{x}_n \rightarrow \vec{x}_k$ corresponds by the right-hand rule to the outward normal of the surface.

We will next consider the integral over each triangle in (8). For convenience we assume that the triangle is given by the corner points $\vec{y}_1$, $\vec{y}_2$, and $\vec{y}_3$ and the view point $\vec{x}_m$ will be shifted to the origin. In the constant potential approach the off-diagonal matrix elements are given by $\Omega/(2\pi)$, with

$$\Omega = 2 \arctan \frac{d}{|\vec{y}_1||\vec{y}_2||\vec{y}_3| + |\vec{y}_1|(\vec{y}_2 \cdot \vec{y}_3) + |\vec{y}_3|(\vec{y}_1 \cdot \vec{y}_2) + |\vec{y}_2|(\vec{y}_3 \cdot \vec{y}_1)}, \tag{9}$$

where $d = \det(\vec{y}_1, \vec{y}_2, \vec{y}_3) = \vec{y}_1 \cdot (\vec{y}_2 \times \vec{y}_3)$.

In the linear potential approach, the off-diagonal matrix elements consist of the following integrals:

$$\Omega_i = \frac{1}{A} \iint_{\Delta_{123}} \frac{\vec{z}_i \cdot \vec{x}'}{|\vec{x}'|^3} dS' \tag{10}$$

where

$$\vec{z}_i = \vec{y}_{i+1} \times \vec{y}_{i-1}, \quad i = 1, 2, 3. \tag{11}$$

In (11) the triangular points $\vec{y}_i$ are defined such that $\vec{y}_4 \equiv \vec{y}_1$ and $\vec{y}_0 \equiv \vec{y}_3$. To find an analytical expression for (10) we will first consider

$$\vec{\Omega} = \iint_{\Delta_{123}} \nabla' \frac{1}{|\vec{x}'|} \times d\vec{S}'. \tag{12}$$

Here, $\vec{\Omega}$ can be interpreted as the magnetic field generated at the origin, caused by a constant distribution of current dipoles on triangle Δ_{123}. Equation (12) will be evaluated in two different ways. One way is to apply Stokes' theorem, and the other way is to apply the ∇'-operator. Both options yield a linear combination of $\vec{y}_1$, $\vec{y}_2$ and $\vec{y}_3$ of which the coefficients depend on Ω_1, Ω_2, and Ω_3. By comparing the coefficients of both evaluations, a system of three linear equations is obtained with the integrals as unknowns.

The first way of evaluating (12) yields:

$$\vec{\Omega} = \oint_\Delta \frac{d\vec{\gamma}'}{|\vec{x}'|} = \sum_{i=1}^{3} \int_0^1 \frac{ds}{|\vec{y}_i + s(\vec{y}_{i+1} - \vec{y}_i)|^2} (\vec{y}_{i+1} - \vec{y}_i)$$
$$= \sum_{i=1}^{3} (\gamma_{i-1}^0 - \gamma_i^0)\vec{y}_i \tag{13}$$

with

$$\gamma_i^0 = \frac{-1}{|\vec{y}_{i+1} - \vec{y}_i|} \cdot \ln \frac{|\vec{y}_i||\vec{y}_{i+1} - \vec{y}_i| + \vec{y}_i \cdot (\vec{y}_{i+1} - \vec{y}_i)}{|\vec{y}_{i+1}||\vec{y}_{i+1} - \vec{y}_i| + \vec{y}_{i+1} \cdot (\vec{y}_{i+1} - \vec{y}_i)}. \tag{14}$$

The symbol γ_i^0 can be interpreted as the potential at the origin, caused by a homogeneous distribution of charge on the line segment from $\vec{y}_i$ to $\vec{y}_{i+1}$.

Since for any $\vec{x}$ we have

$$\vec{x} = d^{-1} \sum_{k=1}^{3} (\vec{z}_k \cdot \vec{x})\vec{y}_k, \tag{15}$$

the second way of evaluating (12) yields

$$\vec{\Omega} = \frac{\vec{n}}{d} \times (\Omega_1\vec{y}_1 + \Omega_2\vec{y}_2 + \Omega_3\vec{y}_3) \tag{16}$$

where $\vec{n}$ is a normal vector of the triangle,

$$\vec{n} = \vec{z}_1 + \vec{z}_2 + \vec{z}_3. \tag{17}$$

A direct comparison of the coefficients of (13) and (16) yields a system of three linear equations in the unknown integrals. However, since $\Sigma(\gamma_{i-1}^0 - \gamma_i^0) = 0$, this sytem is singular. Any point $\vec{x}$ in the plane through $\vec{y}_1$, $\vec{y}_2$ and $\vec{y}_3$ satisfies the equation $\vec{n} \cdot \vec{x} = d$, and therefore the following identity can be obtained from (10):

$$\Omega_1 + \Omega_2 + \Omega_3 = \Omega. \tag{18}$$

Finally, using this identity, we arrive at

$$\Omega_i = A^{-2}\left(\vec{z}_i \cdot \vec{n}\Omega + d(\vec{y}_{i+1} - \vec{y}_{i-1}) \cdot \sum_{k=1}^{3} (\vec{y}_k - \vec{y}_{k+1})\gamma_k^0\right) \tag{19}$$

where A is twice the area of the triangle, $A = |\vec{n}|$.

To find the diagonal matrix elements it is noted that since $\Sigma_n h_n(\vec{x}) = 1$ (7) the vector $(1, 1, \cdots, 1)$ is an eigenvector of B, with eigenvalue zero:

$$\sum_{n=1}^{N} B_{mn} = \frac{1}{2\pi}\left(\oint\oint_{S \setminus \exists(\vec{x}_m)} \nabla' \frac{1}{|\vec{x}' - \vec{x}_m|} \cdot d\vec{S}' + \Omega(\vec{x}_m)\right) = 0. \tag{20}$$

This property of B has been demonstrated before for the discretization of smooth surfaces by the constant potential approximation [11]. It is a related to the fact that ψ in (1) and (5) is determined only up to an additive constant. Similar to the constant potential approach, this ambiguity in the potential can be removed by using the deflation technique.

A useful consequence of (20) is that the diagonal elements of B can be expressed in terms of the other matrix elements:

$$B_{mm} = -\sum_{n \neq m}^{N} B_{mn}. \tag{21}$$

Note that (20) is only true when the matrix elements are expressed as analytical formulas and that numerical approximations might spoil its application in (21). Also the use of the multiple deflation technique is restricted to the case that (20) holds exactly.

III. Quadratic Expansion Functions

The integration technique presented here can similarly be applied to compute the matrix elements for quadratic interpolation on flat triangles. The integrals that need to be computed are of the form:

$$\Omega_{ij} = \frac{1}{dA} \iint_{\Delta_{123}} \frac{(\vec{z}_i \cdot \vec{x}')(\vec{z}_j \cdot \vec{x}')}{|\vec{x}'|^3} dS'. \tag{22}$$

Instead of (16) we evaluate the following in two different ways:

$$\vec{\Omega}_i = d^{-1} \iint_{\Delta_{123}} (\vec{z}_i \cdot \vec{x}')\nabla'|\vec{x}'|^{-1} \times d\vec{S}'. \tag{23}$$

Stokes' theorem cannot directly be applied on (23); the gradient has to be moved in front first. So we obtain,

$$\vec{\Omega}_i = d^{-1} \sum_{k=1}^{3} \int_{\vec{y}_k}^{\vec{y}_{k+1}} \frac{\vec{z}_i \cdot \vec{x}'}{|\vec{x}'|} d\vec{\gamma}' - \frac{\Upsilon}{dA} \vec{z}_i \times \vec{n} \tag{24}$$

with

$$\Upsilon = \iint_\Delta \frac{dS'}{|\vec{x}'|} = \frac{|d|}{A}\left(\Omega - \sum_{k=1}^{3} \text{sign}(d\vec{z}_{k-1} \cdot \vec{n}) \cdot \sqrt{|\vec{z}_{k-1}|^2 A^2 - |\vec{z}_k - \vec{z}_{k-1}|^2}\, \gamma_k^0\right). \tag{25}$$

This formula has been derived in [12], [13].

Equation (23) can alternatively be expressed as a linear combination of the unknown integrals in (22). Using (15) we find

$$\vec{\Omega}_i = \frac{\vec{n}}{d} \times \sum_{k=1}^{3} \Omega_{ik}\vec{y}_k. \tag{26}$$

When (24) and (26) are both expressed as a linear combination of $\vec{y}_1$, $\vec{y}_2$ and $\vec{y}_3$ and the coefficients are compared, we find, using the identity $\Omega_i = \Omega_{i1} + \Omega_{i2} + \Omega_{i3}$ the following explicit expression for Ω_{ij}:

$$\Omega_{ij} = \frac{-d(\vec{y}_{j+1} - \vec{y}_{j-1}) \cdot \left(\sum_k \beta_{ik}\vec{y}_k + A^{-1}/\Upsilon(\vec{y}_{i+1} - \vec{y}_{i-1})\right) + \vec{z}_j \cdot \vec{n}\Omega_i}{A^2} \tag{27}$$

where β_{ik} can be found from

$$\beta_{ik} = \begin{pmatrix} \gamma_3^1 + \gamma_1^1 - \gamma_1^0 & \gamma_1^0 - \gamma_1^1 & -\gamma_3^1 \\ -\gamma_1^1 & \gamma_1^1 + \gamma_2^1 - \gamma_2^0 & \gamma_2^0 - \gamma_2^1 \\ \gamma_3^0 - \gamma_3^1 & -\gamma_2^1 & \gamma_2^1 + \gamma_3^1 - \gamma_3^0 \end{pmatrix}_{ik} \tag{28}$$

and

$$\gamma_i^1 \equiv \int_0^1 \frac{s\,ds}{\sqrt{|\vec{y}_i + s(\vec{y}_{i+1} - \vec{y}_i)|^2}} = \frac{|\vec{y}_{i+1}| - |\vec{y}_i| - (\vec{y}_{i+1} - \vec{y}_i) \cdot \vec{y}_i\gamma_i^0}{|\vec{y}_{i+1} - \vec{y}_i|^2}. \tag{29}$$

V. Results and Discussion

The main topic of this paper is the computation of the matrix elements using analytical formulas. To find out the computational advantage of using analytical formulas we considered a triangle Δ_{123} in the y-z-plane: $\vec{x}_1 = (0, -1/a, -a)$, $\vec{x}_2 = (0, -1/a, a)$, and $\vec{x}_3 = (0, 1/a, 0)$, and the viewpoint $\vec{x}$ on the x-axis $\vec{x} = (d, 0, 0)$. The parameter a determines the shape of the triangle and d is the distance of the viewpoint from the origin. We compared the speed and accuracy of the analytic formula for Ω_i with two numerical methods: a six-point formula and a 16-point formula [17].

From Table I it follows that for distances much larger than the size of the triangle, numerical integration performs well for rectangular triangles ($a = 1$) and slightly worse for others. For distances comparable to the size of the triangle however, the numerical accuracy is not acceptable. It also followed from the simulation that analytical integration is more than two times as fast than the 16-points method, and slightly faster than the 6-points method. Although methods have been developed to increase the accuracy of numerical R^{-1}-integrations [14]–[16], these methods will reduce the computation speed. Therefore analytical formulas are advantageous, when dealing with flat surface elements.

One might object that the analytical formulas developed in this paper only work when the surfaces are described with flat triangles. However, we believe that the analytical formulas are also helpful to the case when curved elements are used. In that case the integrals are of the following form [compare with (6)]:

$$I(\vec{x}_m) = \oint_\Delta\oint g(\vec{x}')\nabla' \frac{1}{|\vec{x}' - \vec{x}_m|} \cdot d\vec{S}' \tag{30}$$

where Δ is a curved (triangular) surface element, and $g(\vec{x})$ is a weight function. If Δ is partitioned into a set of small flat triangles

TABLE I
Error %

d		10			1	
a	1	3	0.33	1	3	0.33
6-points	0.003	0.21	0.12	7.5	21.0	34.8
16-points	0.001	0.21	0.036	0.23	4.2	5.8

$g(\vec{x})$ can be approximated by a piecewise constant, a piecewise linear or a piecewise quadratic function. Next the formulas derived in the previous sections can be applied to approximate the integral in (30). In this way R^{-1}-singularity is dealt with by an analytical formula, whereas the regular part $g(\vec{x})$ is approximated by spline interpolation.

In [12], [13] an alternative way of calculating the higher-order integrals is presented. This alternative method is based on a geometrical reduction of the general case to the case of rectangular triangles. The integrals for the rectangular triangles can be performed by a simple parameterization of the triangle. Since this method requires some intermediate steps, relatively many auxiliary variables are needed to obtain the result. With the method presented here the use of local parameterizations of the boundary elements is circumvented, yielding more compact formulas which express the integrals directly in the corner points of the triangles.

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A PC-Based Imaging System for Automated Platelet Identification

J.-S. Lin, C.-C. Tai, C.-W. Mao, C.-J. Jen, and K.-S. Cheng

***Abstract*—In this communication, a PC-based imaging system was developed for automatically identifying fluroscence-labeled individual platelets adherent to protein-coated surface under flow conditions. It is to eliminate the laborious and time-consuming task, and the subjective error of manual measurements. Based upon the features of adherent platelets, three passes of the image processing were developed for platelet identification. From the results, 90–95% accuracy could be routinely obtained. The platelet distribution and other related parameters could be easily extracted and investigated.**

Introduction

With the advance of the digital imaging microscopic technique, the studies of complex cellular functions and higher order structure analysis become easier, more objective, and more accurate [1]. A good survey of the objectives of microscopic image analysis appeared previously [2]. Most systems in the past were implemented with the minicomputers. Nowadays, with the rapid developments in VLSI and computer technologies, the PC-based imaging systems are available with low cost.

The microscopic visual inspection process using digital image processing plays an important role in hematological investigations, such as the tracking, counting, classification, and analysis of the white blood cells [3]–[5]. Other applications include the template matching for analyzing the intramembraneous particle distribution [6], and the interactive tracking of the platelets and polymorphonuclear leukocytes on biomaterial surfaces [5], [7]. Epifluorescence video microscopy, in particular, is a necessary tool for investigating the characteristics of adherent platelets in mural thrombogenesis [8]–[10]. In the study of the aggregation of platelets [9], the off-line digital image processing technique was developed for local measurements of the multiplatelet thrombi growth and distribution. However, due to the lack of the morphological information, there is only very limited success in automated identification of the fluorescence image of individual platelets. For all the analysis of platelet images, the identification is inevitably the first and most important step in digital image processing.

In this communication, a PC-based imaging system is employed for automated platelet identification using the image processing and pattern recognition techniques. The platelets adhered to the protein-coated surface under flow conditions could be identified and tracked dynamically. The interesting parameters, such as number of platelets per unit area, accumulation rate, adhesion status, and sustaining period, could also be extracted automatically.

System Description

An imaging system with a 80286-based personal computer (ARC Turbo-12) is used for automated platelet identification. The block-diagram of the whole experimental setup is shown in Fig. 1, which is similar to the one described by Hubble and McIntire [9]. A typical image grabbed from the tape of this imaging system is shown in Fig. 2. The flow direction is from right to left. The imaging card used for grabbing the image from either the video camera or video cassette recorder has the resolution of 512 × 512 with 256 gray levels (VFG-512, Visionetics, Taiwan, ROC). Other instruments included in the system are an epi-fluorescence microscope of Olympus (BH-2RFL), a video camera of Hamamatsu (C-2400-08), a flow chamber, a syringe pump, a dynamic image tracing system of Sony (BNU-820), and a time base corrector of Sony (BVT-800).

In this study, all the identification algorithms and subsequent analysis programs were implemented using the C language, and easily run on an IBM compatible microcomputer.

The whole blood drawn from a nonsmoking, nonmedicated healthy subject was used as the sample. In which, the fluorescent dye of acridine red (2 μM) and the anticoagulant of sodium cirate (0.32%) were added for specifically labeling the platelets and preventing it from clotting. The flow rate controlled by the syringe pump was set at 0.9 ML/min, with a corresponding surface shear rate of 445 s^{-1} to simulate arterial blood flow conditions.

Automated Platelet Identification

From the grabbed fluorescence image of platelets, most platelets identifiable by human eyes had their gray-level values higher than the neighborhood background by at least 20. As flow time increased, more and more platelets adhered to the protein-coated region but not to the uncoated portion of the glass. Therefore, the coating boundary became more and more distinguishable.

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