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Radial Anisotropy Added to a Spherically Symmetric Conductor Does Not Affect the External Magnetic Field Due to Internal Sources.

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Abstract. – It has been shown previously that the magnetic field outside a spherically symmetric conducting body due to internal current sources does not depend on the profile of conductivity along the radius. It is demonstrated here that an arbitrary distribution of radial anisotropy superimposed on the symmetric conductivity, even if the spherical symmetry is broken, does not alter the magnetic field outside. This result simplifies the analysis of magnetic fields produced by the human brain.

In magnetoencephalography (MEG) [1,2], the brain is studied by measuring the magnetic field due to electric currents produced by neurons in the cortex. One of the difficulties and uncertainties in the data analysis has been the lack of understanding of how tissue anisotropy modifies the external magnetic field produced by internal sources. In the gray matter, the conductivity is about twice as high in the direction perpendicular to the cortex than along it. In the white matter, the conductivity along the fibres may be up to 10 times higher than in the transverse direction [3,4].

It has been shown previously that the conductivity profile of a spherically symmetric isotropic conductor does not affect the external magnetic field [5]. The conductivity values have been shown to be irrelevant even if the volume conductor has spherically symmetric anisotropy, *i.e.* even if the conductivity in the radial direction differs from that in the tangential directions so that spherical symmetry is maintained [6]. These results are generalized in this paper by showing that the conductivity in the radial direction can be an arbitrary function of location with no consequence on the external magnetic field. This finding simplifies significantly the interpretation of neuromagnetic fields, because the major anisotropies in the human head that are not spherically symmetric need not be explicitly modelled.

Let us consider a conducting body containing an internal primary current distribution $\boldsymbol{J}^{\mathrm{p}}(\boldsymbol{r})$. As a result of $\boldsymbol{J}^{\mathrm{p}}$, an electric field $\boldsymbol{E}(\boldsymbol{r})$ and a volume current $\boldsymbol{J}^{\mathrm{v}}(\boldsymbol{r})$ are produced. The total current is then

$$\boldsymbol{J}(\boldsymbol{r}) = \boldsymbol{J}^{\mathrm{p}}(\boldsymbol{r}) + \boldsymbol{J}^{\mathrm{v}}(\boldsymbol{r}) = \boldsymbol{J}^{\mathrm{p}}(\boldsymbol{r}) + \underline{\underline{\sigma}}(\boldsymbol{r})\boldsymbol{E}(\boldsymbol{r}), \tag{1}$$

where $\underline{\underline{\sigma}}$ is the conductivity tensor(1). If the conductivity has a spherically symmetric isotropic part, $\sigma(\boldsymbol{r}) = \sigma(r)$, plus an arbitrary distribution of radial anisotropy (additional, generally asymmetric, conductivity $\sigma_r(\boldsymbol{r})$ in the radial direction), we have

$$\underline{\underline{\sigma}}(\boldsymbol{r})\boldsymbol{E}(\boldsymbol{r}) = \sigma(r)\boldsymbol{E}(\boldsymbol{r}) + \sigma_r(\boldsymbol{r})E_r(\boldsymbol{r})\boldsymbol{e}_r\,, \tag{2}$$

where $\boldsymbol{e}_r = \boldsymbol{r}/r$ is a unit vector in the radial direction and $E_r(\boldsymbol{r}) = \boldsymbol{E}(\boldsymbol{r})\cdot\boldsymbol{e}_r$. Here, $\sigma_r(\boldsymbol{r})$ is allowed to be negative so that conductivity in the radial direction can be lower than in the tangential directions.

We consider the low-frequency limit, *i.e.* the quasi-static approximation of Maxwell's equations. Then, the electric field can be derived from a potential, $\boldsymbol{E} = -\nabla V$, and the magnetic flux density (magnetic field) $\boldsymbol{B}$ at point $\boldsymbol{r}$ is obtained from $\boldsymbol{J}$ according to the Ampère-Laplace law:

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\int_G \frac{[\boldsymbol{J}^{\mathrm{p}}(\boldsymbol{r}') - \sigma(r')\nabla' V(\boldsymbol{r}') + \sigma_r(\boldsymbol{r}')E_r(\boldsymbol{r}')\boldsymbol{e}_{r'}]\times\boldsymbol{R}}{R^3}\,\mathrm{d}v'\,, \tag{3}$$

where $\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}'$ and $\boldsymbol{r}'$ is the integration variable in region G. Volume G is assumed to contain the conducting body, extending outside it so that $\sigma = 0$ at the boundary A of G. We show next that $-\sigma\nabla' V$ in eq. (3) can be replaced by $V\nabla'\sigma$ without altering the value of the integral. With the vector identities $\boldsymbol{u}\times(\boldsymbol{R}/R^3) = \boldsymbol{u}\times\nabla'(1/R) = (\nabla'\times\boldsymbol{u})/R - \nabla'\times(\boldsymbol{u}/R)$ and $\int_G \nabla\times\boldsymbol{u}\,\mathrm{d}v = \int_A \mathrm{d}\boldsymbol{A}\times\boldsymbol{u}$,

$$\int_G \frac{(\sigma\nabla' V + V\nabla'\sigma)\times\boldsymbol{R}}{R^3}\,\mathrm{d}v' = \int_G \frac{\nabla'\times(\sigma\nabla' V + V\nabla'\sigma)}{R}\,\mathrm{d}v' - \int_G \nabla'\times[(\sigma\nabla' V + V\nabla'\sigma)/R]\,\mathrm{d}v' =$$

a × b = − b × a

$$= \underbrace{\int_G \frac{\nabla'\sigma\times\nabla' V + \nabla' V\times\nabla'\sigma}{R}\,\mathrm{d}v'}_{=0} - \int_A \mathrm{d}\boldsymbol{A}'\times[(\sigma\nabla' V + V\nabla'\sigma)/R] = 0\,. \tag{4}$$

The surface integral above vanishes because $\sigma = 0$ and $\nabla'\sigma = 0$ on A.

Replacing $-\sigma\nabla' V$ in eq. (3) by $V\nabla'\sigma = V(\partial\sigma/\partial r')\boldsymbol{e}_{r'}$ and defining $J_r^{\mathrm{p}} = \boldsymbol{J}^{\mathrm{p}}\cdot\boldsymbol{e}_{r'}$ and $\boldsymbol{J}_t^{\mathrm{p}} =$

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(1) While $\boldsymbol{J}^{\mathrm{v}}$ represents the passive flow of charge in response to $\boldsymbol{E}$, $\boldsymbol{J}^{\mathrm{p}}$ is caused by active mechanisms, diffusion, or structures in conductivity that are not detected at the length scale of observation but influence the current flow. The quantities $\boldsymbol{J}^{\mathrm{p}}$, $\boldsymbol{E}$, and $\underline{\underline{\sigma}}$ are macroscopic and depend on the scale of observation.

$= \boldsymbol{J}^{\mathrm{p}} - J_r^{\mathrm{p}} \boldsymbol{e}_{r'}$, we may now write

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \int_G \frac{[\boldsymbol{J}_t^{\mathrm{p}} + (J_r^{\mathrm{p}} + V\,\partial\sigma/\partial r' + \sigma_r E_r)\boldsymbol{e}_{r'}] \times \boldsymbol{R}}{R^3}\, \mathrm{d}v' . \tag{5}$$

Since $\boldsymbol{e}_{r'} \times \boldsymbol{R} \cdot \boldsymbol{e}_r = \mathbf{r}'/r' \times (\mathbf{r} - \mathbf{r}') \cdot \mathbf{r}/r = 0$, the radial component of the magnetic field is simply

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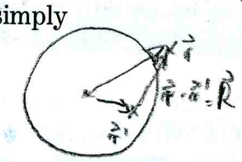

$$B_r(\boldsymbol{r}) = \boldsymbol{B}(\boldsymbol{r}) \cdot \boldsymbol{e}_r = \frac{\mu_0}{4\pi} \int_G \frac{\boldsymbol{J}_t^{\mathrm{p}}(\boldsymbol{r}') \times \boldsymbol{R} \cdot \boldsymbol{e}_r}{R^3}\, \mathrm{d}v' . \tag{6}$$

We notice that B_r is not affected by J_r^{p}, $\sigma(r')$, or σ_r. The tangential components of the magnetic field are also unaffected. This can be shown by integrating two components of $\nabla \times \boldsymbol{B} = 0$, valid in the current-free region outside the conductor, in spherical coordinates [7]: ← habe ich! $rB_\theta = \int_\infty^r (\partial B_r/\partial\theta)\, \mathrm{d}r$ and $r \sin(\theta) B_\phi = \int_\infty^r (\partial B_r/\partial\theta)\, \mathrm{d}r$. Since the tangential components of the magnetic field can be computed from B_r, they are no more influenced by J_r^{p}, $\sigma(r')$, or σ_r than the radial field component.

Thus, an arbitrary distribution of σ_r has no effect on the external magnetic field. The well-known fact [5] that J_r^{p} is silent in the spherical geometry was obtained simultaneously. Note that our result is not an obvious consequence of eq. (3), because σ_r and J_r^{p} do affect V and could thereby influence the magnetic field as well. It was therefore necessary to show that the effect via V vanishes.

Figure 1 depicts the spherical model of the human head. The inner and outer compartments represent the brain and the scalp, respectively. The layer between them is the poorly

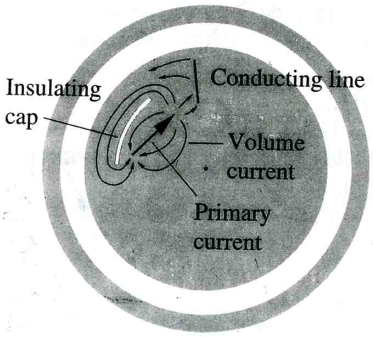

Fig. 1.

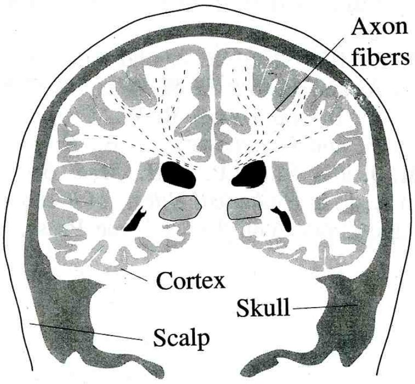

Fig. 2.

Fig. 1. – Two basic types of anomalies in the conductivity of a spherically symmetric head model that do not affect the external magnetic field. A radial line conductor can model fibre bundles in the white matter (see fig. 2); the poorly conducting or insulating thin cap can approximate a sheet of bone or a poorly conducting membrane. Some flow lines of volume current are shown schematically; the large arrow depicts an element of primary current (the internal source).

Fig. 2. – The human brain in cross-section. The dashed lines indicate schematically the orientation of some major axon bundles in the white matter. The conductivity is up to 10 times as high along the fibres than in the perpendicular direction. The dominant anisotropy direction is radial.

conducting skull. The examples of radially anisotropic anomalies are shown: a radial line conductor and a cap-shaped insulator. These certainly change the pattern of return current flow produced by the element of primary current (large arrow) but do not affect the external magnetic field.

The well-conducting thin line represents radial anisotropy, because it affects the radial but not the tangential conductivity. Likewise, the insulating cap, being infinitely thin and tangential, does not hinder the tangential current flow at all. It should be noted that an arbitrary distribution of radial anisotropy can be constructed by superposition from these two kinds of structures.

Figure 2 shows a cross-section of the human brain, illustrating the orientation of fibres in the white matter. One can readily see that a major part of the anisotropy, particularly near the surface of the brain, is radially oriented. This partly explains why the accuracy in determining locations of brain activity from MEG data has been remarkably good, even though the anisotropies have not been taken into account.

It was shown above that an arbitrary spatial distribution of anisotropy of the radial conductivity in an otherwise spherically symmetric conductor does not affect the external magnetic field produced by internal current sources. This quite unexpected result simplifies the analysis of neuromagnetic data and may find applications elsewhere as well. A similar simplification is not possible in the analysis of electroencephalographic data.

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REFERENCES

[1] Hämäläinen M., Hari R., Ilmoniemi R. J., Knuutila J. and Lounasmaa O. V., *Rev. Mod. Phys.*, **65** (1993) 413.
[2] Näätänen R., Ilmoniemi R. J. and Alho K., *Trends Neurosci.*, **17** (1994) 389.
[3] Geddes L. A. and Baker L. E., *Med. Biol. Eng.*, **5** (1967) 271.
[4] Hoeltzell P. B. and Dykes R. W., *Brain Res.*, **14** (1979) 61.
[5] Grynszpan F. and Geselowitz D. B., *Biophys. J.*, **13** (1973) 911.
[6] Peters M. J. and Elias P. J. H., *Med. Biol. Eng. Comput.*, **26** (1988) 617.
[7] Ilmoniemi R. J., in *Biological Effects of Electric and Magnetic Fields*, edited by D. O. Carpenter and S. Ayrapetyan, Vol. 2 (Academic Press, San Diego) 1994, p. 49.