Reduced Dual-Formulation for Analytical Anisotropic Single Scattering

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Abstract

An analytical approach to solving anisotropic single scattering from point light sources in homogeneous media was recently derived via a dual-formulation of the air-light integral. In this paper, we demonstrate how to reduce the evaluation of the terms involved in the solution and provide an efficient and practical implementation substantially increasing the real-time performance characteristics.

CR Categories: I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism—Color, shading, shadowing, and texture Keywords: participating media, analytical integration

1 Introduction & Related Work

[Pegoraro and Parker 2009] recently derived a closed-form solution to single scattering in homogeneous isotropic media. They subsequently extended the analytical approach to anisotropic phase functions and light distributions via domain partitioning of the air-light integral [Pegoraro et al. 2009]. Solving the resulting dual-formulation however requires multiple evaluations of the computationally costly complex-valued exponential integral function.

In this paper, we reformulate the terms involved in solving the dual-formulation and reduce the computation to only 3 evaluations of the complex-valued exponential integral instead of 4. In addition, we provide an efficient implementation yielding a substantial speed-up of the real-time performance achieved on graphics hardware.

2 Reduced Evaluation & Implementation

Defining u as the variable of integration in the simplified form of the air-light integral as illustrated in figure 1, [Pegoraro et al. 2009] proposed the changes of variable $v = u+\sqrt{1 + u^2}$ and $w = u - \sqrt{1 + u^2}$, and formulated a solution requiring 4 evaluations of the complex-valued exponential integral Ei with the parameters v_a, v_h, w_b and w_h involved by the computation of the following terms

$$I_0(-H, v_h, v_a) = i_0(-H, v_a) - i_0(-H, v_h) \quad (1)$$
$$I_1(-H, v_h, v_a) = i_1(-H, v_a) - i_1(-H, v_h) \quad (2)$$
$$J_0(H, w_h, w_b) = j_0(H, w_b) - j_0(H, w_h) \quad (3)$$
$$J_1(H, w_h, w_b) = j_1(H, w_b) - j_1(H, w_h) \quad (4)$$

$$J_e(H, w_h, w_b) = \mathrm{Ei}\left(\frac{H}{w_b}\right) - \mathrm{Ei}\left(\frac{H}{w_h}\right) \quad (5)$$

with H the optical distance from the light to the ray and where

$$i_0(a, v) = \sin(a)\Re(\mathrm{Ei}(av + \imath a)) - \cos(a)\Im(\mathrm{Ei}(av + \imath a))$$
$$i_1(a, v) = \cos(a)\Re(\mathrm{Ei}(av + \imath a)) + \sin(a)\Im(\mathrm{Ei}(av + \imath a))$$
$$j_0(a,w) = -\sin(a)\Re\left(\mathrm{Ei}\left(\frac{a}{w} + \imath a\right)\right) + \cos(a)\Im\left(\mathrm{Ei}\left(\frac{a}{w} + \imath a\right)\right)$$
$$j_1(a,w) = \cos(a)\Re\left(\mathrm{Ei}\left(\frac{a}{w} + \imath a\right)\right) + \sin(a)\Im\left(\mathrm{Ei}\left(\frac{a}{w} + \imath a\right)\right) - \mathrm{Ei}\left(\frac{a}{w}\right).$$

We here highlight that $1/w = -v$, that $\cos(-a) = \cos(a)$ and $\sin(-a) = -\sin(a)$, and that $\mathrm{Ei}(\bar{z}) = \overline{\mathrm{Ei}(z)}$ yielding the identities

$$j_0(a,w) = i_0(-a, v) \quad (6)$$
$$j_1(a,w) = i_1(-a, v) - \mathrm{Ei}(-av) \quad (7)$$

from which directly follows that the terms involving the complex-valued exponential integral at v_h and w_h are identical. This reduces the computation to only 3 evaluations of the latter function as illustrated in figure 2 which additionally explicitly sets $u_h = 0$.

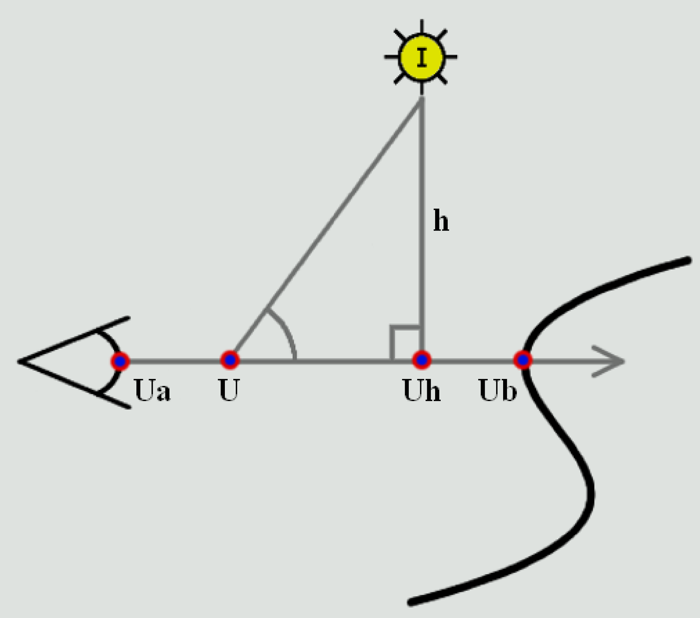

Figure 1: Illustration of the terms involved in the computation of the air-light integral.

ComputeIJ(v_a, w_b, H)

$co = \cos(-H)$;
$si = \sin(-H)$;
$\{re, im\} = \mathrm{Ei}(-Hv_a, -H)$;
$I_0 = si * re - co * im$;
$I_1 = co * re + si * im$;
$\{re, im\} = \mathrm{Ei}(H/w_b, H)$;
$J_0 = si * re + co * im$;
$J_1 = co * re - si * im$;
$\{re, im\} = \mathrm{Ei}(-H, -H)$;
$I_0 -= si * re - co * im$;
$J_0 -= si * re - co * im$;
$I_1 -= co * re + si * im$;
$J_1 -= co * re + si * im$;
$J_e = E_1(H) - E_1(-H/w_b)$;
$J_1 -= J_e$;
$return\{I_0, I_1, J_0, J_1, J_e\}$;

Figure 2: Pseudo-code of the reduced evaluation.

3 Results & Conclusion

The method was implemented in a fragment shader using OpenGL and Cg running on an NVIDIA GeForce GTX 280 under Windows Vista 64-bit. The integral was evaluated independently for each color channel, hence 3 times per fragment at a resolution of 512x512 as illustrated in figure 3. While the previous method renders at 63 FPS, our new evaluation scheme achieves a frame rate of 77 FPS and consequently yields a speed-up of 1.22X.

In conclusion, we have shown how to reduce the evaluations involved in analytically solving the dual-formulation of the air-light integral for anisotropic single scattering from point light sources in homogeneous media. Moreover, we have provided a practical implementation and demonstrated substantial gains in performance.

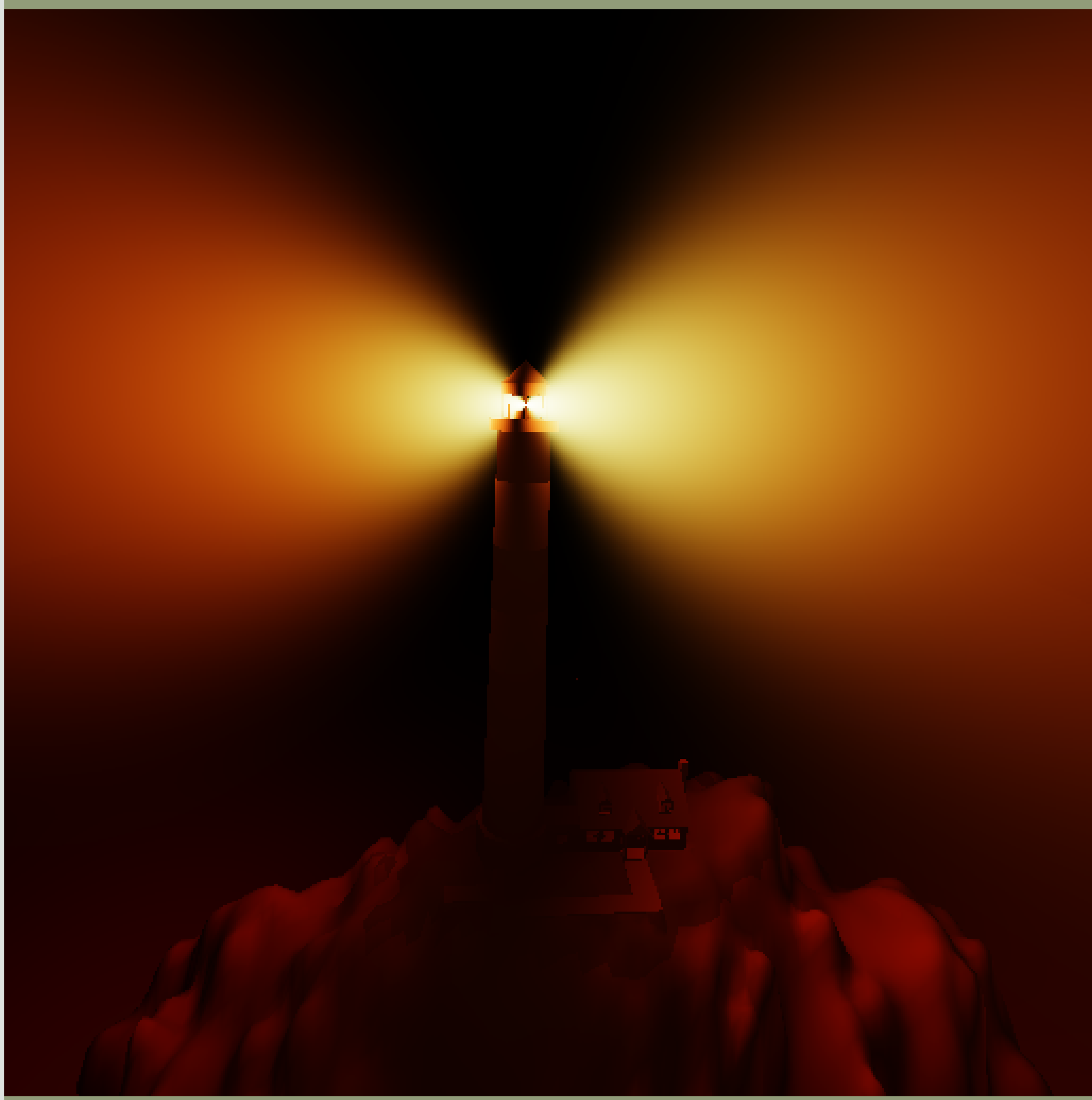

Figure 3: A lighthouse in thick brume with an anisotropic two-lobed spotlight rendered in real-time.

References

PEGORARO, V., AND PARKER, S. G. 2009. An Analytical Solution to Single Scattering in Homogeneous Participating Media. Eurographics (Computer Graphics Forum) 28, 2, 329–335.

PEGORARO, V., SCHOTT, M., AND PARKER, S. G. 2009. An Analytical Approach to Single Scattering for Anisotropic Media and Light Distributions. In Graphics Interface, 71–77.