Wavefront-based models for inverse electrocardiography

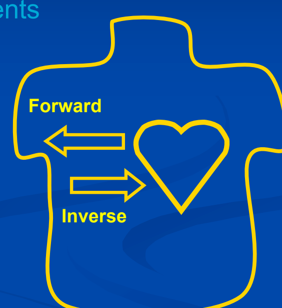
Alireza Ghodrati (*Draeger Medical*)

Dana Brooks, Gilead Tadmor (*Northeastern University*)

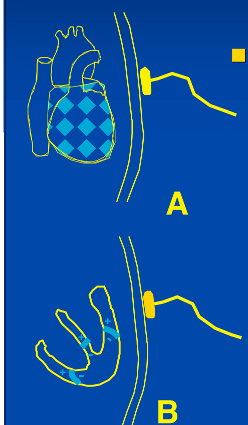
Rob MacLeod (*University of Utah)*

Inverse ECG Basics

- Problem statement:
  - Estimate sources from remote measurements
- Source model:
  - Potential sources on epicardium
  - Activation times on endo- and epicardium
- Volume conductor
  - Inhomogeneous
  - Three dimensional
- Challenge:
  - Spatial smoothing and attenuation
  - An ill-posed problem

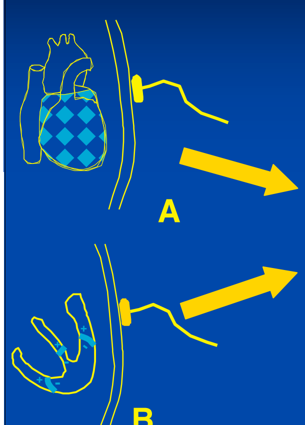


Source Models



- A) Epi (Peri)cardial potentials
  - Higher order problem
  - Few assumptions (hard to include assumptions)
  - Numerically extremely ill-posed
  - Linear problem
- B) Surface activation times
  - Lower order parameterization
  - Assumptions of potential shape (and other features)
  - Numerically better posed
  - Nonlinear problem

Combine Both Approaches?

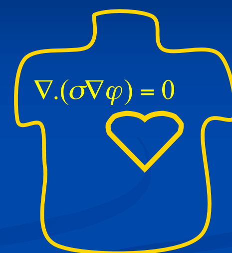


- WBCR: wavefront based curve reconstruction
  - Surface activation is time evolving curve
  - Predetermined cardiac potentials lead to torso potentials
  - Extended Kalman filter to correct cardiac potentials
- WBPR: wavefront based potential reconstruction
  - Estimate cardiac potentials
  - Refine them based on body surface potentials
  - Equivalent to using estimated potentials as a constraint to inverse problem

Use phenomenological data as constraints!

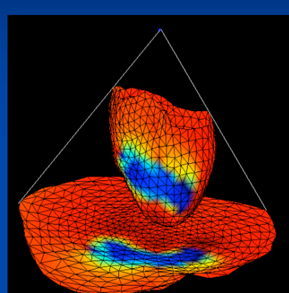
The Forward model

- Laplace's equation in the source free medium
- $y(k) = Ax(k)$ where
  - A forward matrix created by *Boundary Element Method or Finite Element Method*
  - $y(k)$ torso potentials
  - $x(k)$ heart surface potentials

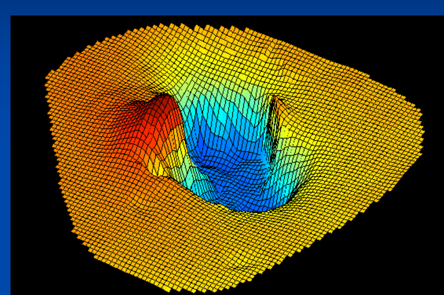


Spatial Assumptions

Projection to a plane

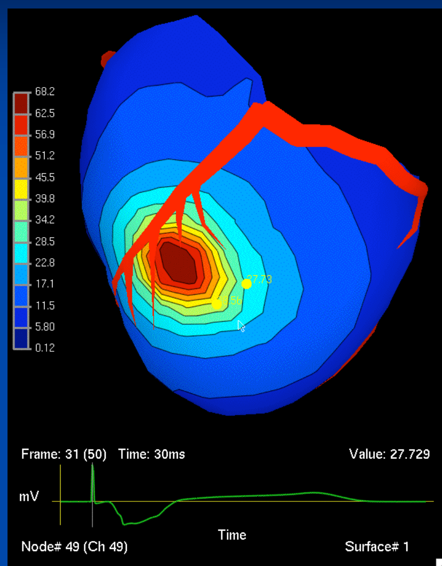


Potential surface



- Three regions: activated, inactive and transition
- Potential values of the activated and inactive regions are almost constant
- Complex transition region

Temporal Assumptions



- Propagation is mostly continuous
- The activated region remains activated during the depolarization period

WBCR Formulation

$$c_{n+1} = f(c_n) + u_{n+1}$$

$$y_{n+1} = Ag(c_{n+1}) + w_{n+1}$$

n : time instant
c : activation wavefront which is state variable (continuous curve)
y : measurements on the body
f : state evolution function
g : potential function
u : state model error (Gaussian white noise)
w : forward model error (Gaussian white noise)

Curve Evolution Model, *f( )*

Speed of the wavefront:

$$v(s,t) = \delta(s)(a(t)\cos^2(\eta) + b(t))$$

$v(s,t)$: speed in the normal direction at point s on the heart surface and time t.

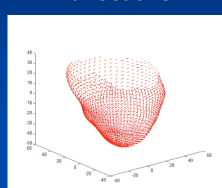
$\delta(s)$: spatial factor

$a\ b$: coefficients of the fiber direction effect

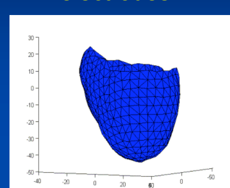
η : angle between fiber direction and normal to the wavefront

Surface fiber directions

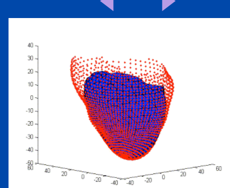
Auckland heart fiber directions



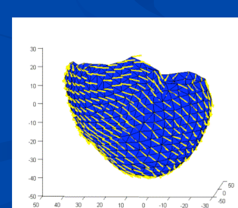
Utah heart electrodes



Geometry matching

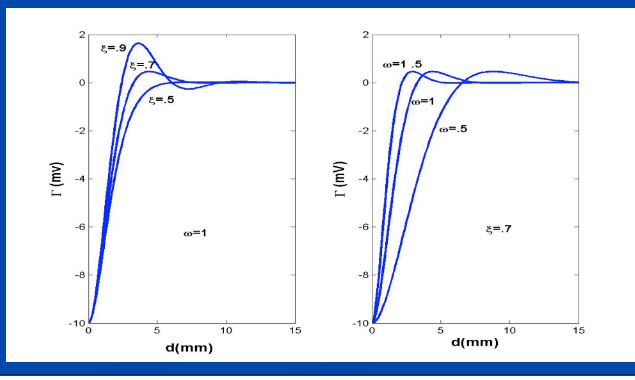


Utah heart with approximated fiber directions



Wavefront Potential, *g( )*

- Potential at a point on the heart surface
- Second order system step response
- Function of distance of each point to the wavefront curve
- Negative inside wavefront curve and zero outside plus reference potential



Setting Model Parameters

- **Goal :** Find rules for propagation of the activation wavefront
- Study of the data
  - Dog heart in a tank simulating human torso
  - 771 nodes on the torso, 490 nodes on the heart
  - 6 beats paced on the left ventricle & 6 beats paced on the right ventricle

Filtering the residual (Extended Kalman Filter)

- Error in the potential model is large
- This error is low spatial frequency
- Thus we filtered the low frequency components in the residual error

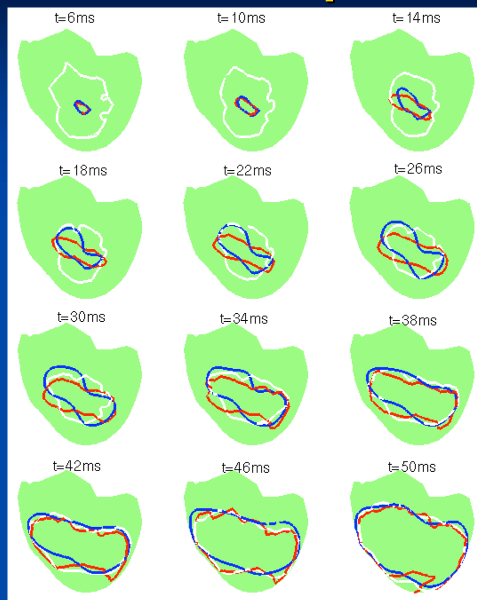
$$\min \| U_k^T (y_n - Ag(c_n)) \|$$

$A = USV^T$ $\quad U_k$ contains column k+1 to N of U

Implementation

- Spherical coordinate (θ, ϕ) to represent the curve.
- B-spline used to define a continuous wavefront curve.
- Distance from the wavefront approximated as the shortest arc from a point to the wavefront curve.
- Torso potentials simulated using the true data in the forward model plus white Gaussian noise (SNR=30dB)
- Filtering : k=3
- Extended Kalman Filtering

Sample Results



Red : wavefront from true potential

White: wavefront from Tikhonov solution

Blue: wavefront reconstructed by WBCR

Wavefront-based Potential Reconstruction Approach (WBPR)

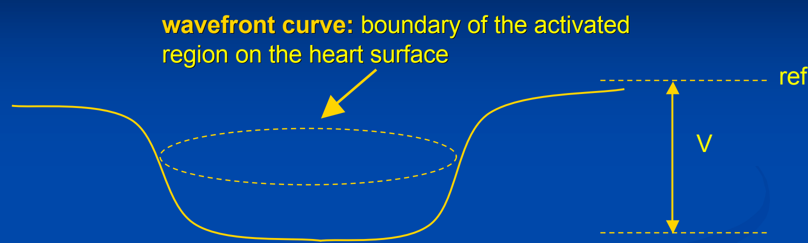
Tikhonov (Twomey) solution:

$$\hat{x}_x = argmin\|Ax_k - y_k\|_2^2 + \lambda^2\|R(x_k - \bar{x}_k)\|_2^2$$

$$\hat{x}_x = \bar{x}_k(A^tA + \lambda^2R^tR)^{-1}A^t(y_k - A\bar{x}_k)$$

- Previous reports were mostly focused on designing R, leaving $\bar{x}_k = 0$
- We focus on approximation of an initial solution $(\bar{x}_k)$ while R is identity

Potentials from wavefront



$$\bar{x}_{k,i} = \alpha_{k,i}\frac{V}{2}(1 - e^{-0.5d_{k,i}}) + \frac{V}{2} + \phi_{ref}$$

$\bar{x}_{k,i}$: Potential estimate of node i at time instant k
$d_{k,i}$: Distance of the node i from the wavefront
V : Negative value of the activated region
ϕ_{ref} : reference potential
$\alpha_{k,i}$: -1 inside the wavefront curve, 1 outside the wavefront curve

WBPR Algorithm

Step 1: $c_k = f(\hat{x}_{k-1})$ Wavefront from thresholding previous time step solution

Step 2: $\bar{x}_k = g(c_k)$ Initial solution from wavefront curve

Step 3: $\hat{x}_k = \bar{x}_k + (A^T A + \ddot{e}^2 I)^{-1} A^T (y_k - A\bar{x}_k)$

Simulation study

- 490 lead sock data (Real measurements of dog heart in a tank simulating a human torso)
- Forward matrix : 771 by 490
- Measurements are simulated and white Gaussian noise was added (SNR=30dB)
- The initial wavefront curve: a circle around the pacing site with radius of 2cm

Results: WBPR with epicardially paced beat

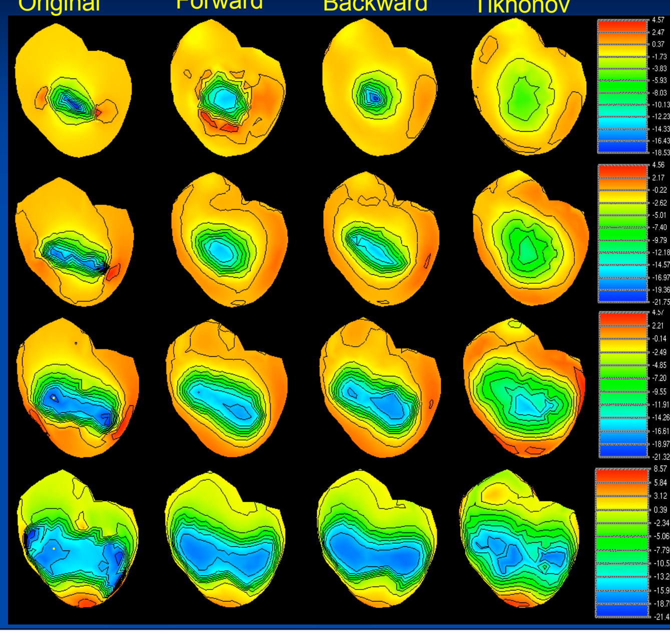
Original Forward Backward Tikhonov

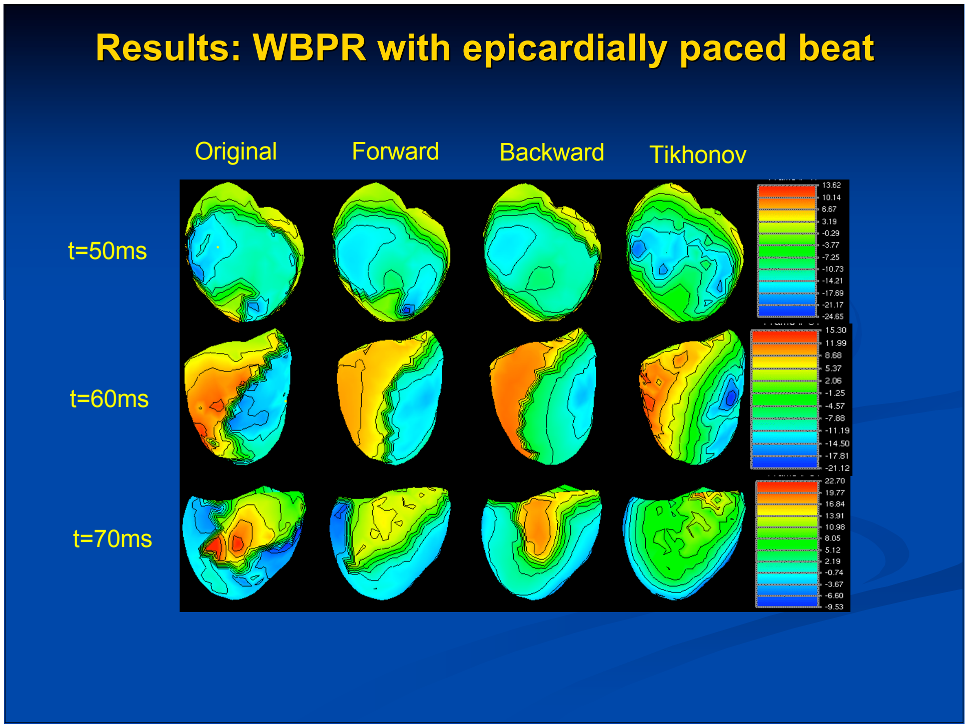
t=10ms

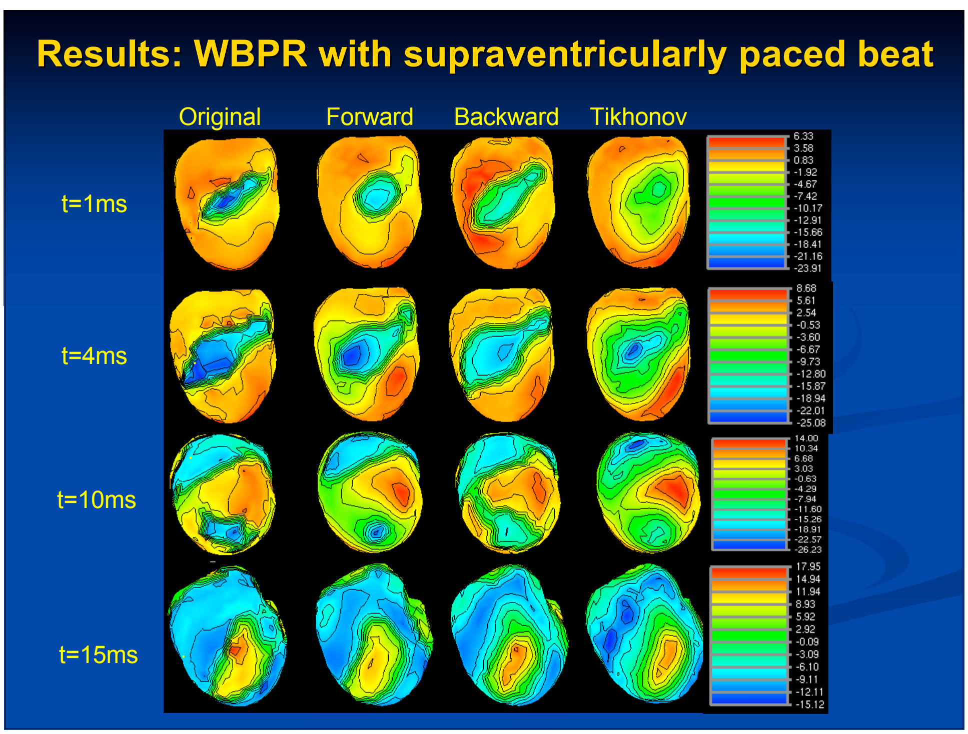
t=20ms

t=30ms

t=40ms




Results: WBPR with epicardially paced beat
Original
Forward
Backward
Tikhonov
t=50ms
t=60ms
t=70ms


Results: WBPR with supraventricularly paced beat
Original
Forward
Backward
Tikhonov
t=1ms
t=4ms
t=10ms
t=15ms

Conclusions

- High complexity is possible and sometimes even useful
- WBCR approach reconstructed better activation wavefronts than Tikhonov, especially at early time instants after initial activation
- WBPR approach reconstructed considerably better epicardial potentials than Tikhonov
- Using everyone's brain is always best

Future Plans

- Employ more sophisticated temporal constraints
- Investigate the sensitivity of the inverse solution with respect to the parameters of the initial solution
- Use real torso measurements to take the forward model error into account
- Investigate certain conditions such as ischemia (the height of the wavefront changes on the heart)
- Compare with other spatial-temporal methods