Applying Constraints to the Electrocardiographic Inverse Problem

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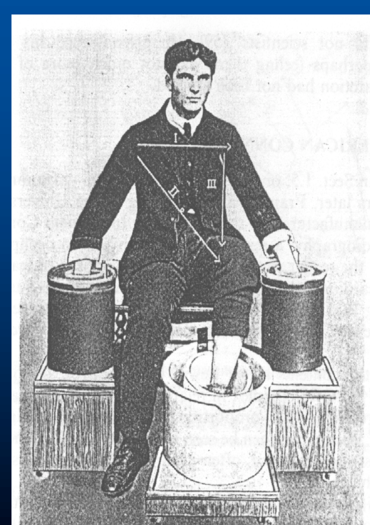
Electrocardiography



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Electrocardiographic Mapping

- Bioelectric Potentials
- Goals
  - Higher spatial density
  - Imaging modality
- Measurements
  - Body surface
  - Heart surfaces
  - Heart volume

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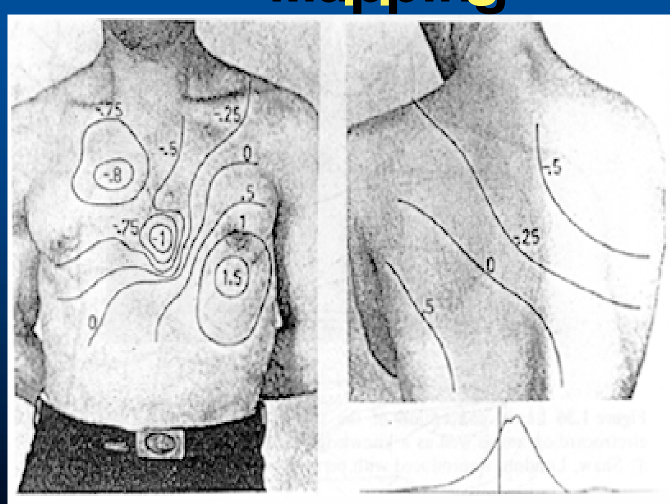
Body Surface Potential Mapping



Taccardi et al, Circ., 1963

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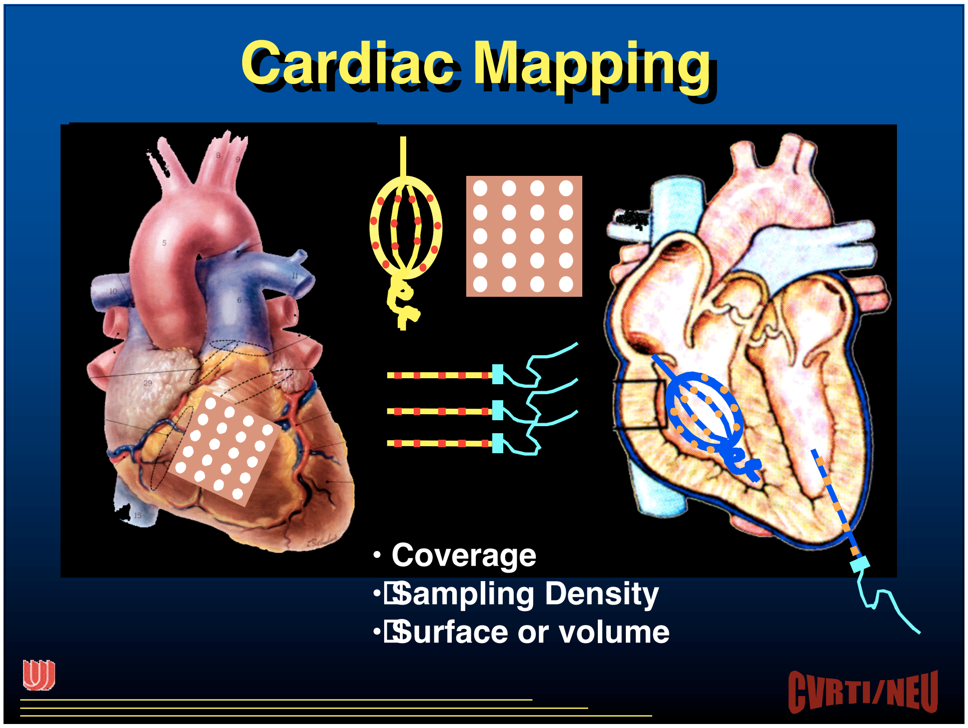
Cardiac Mapping
• Coverage
• Sampling Density
• Surface or volume
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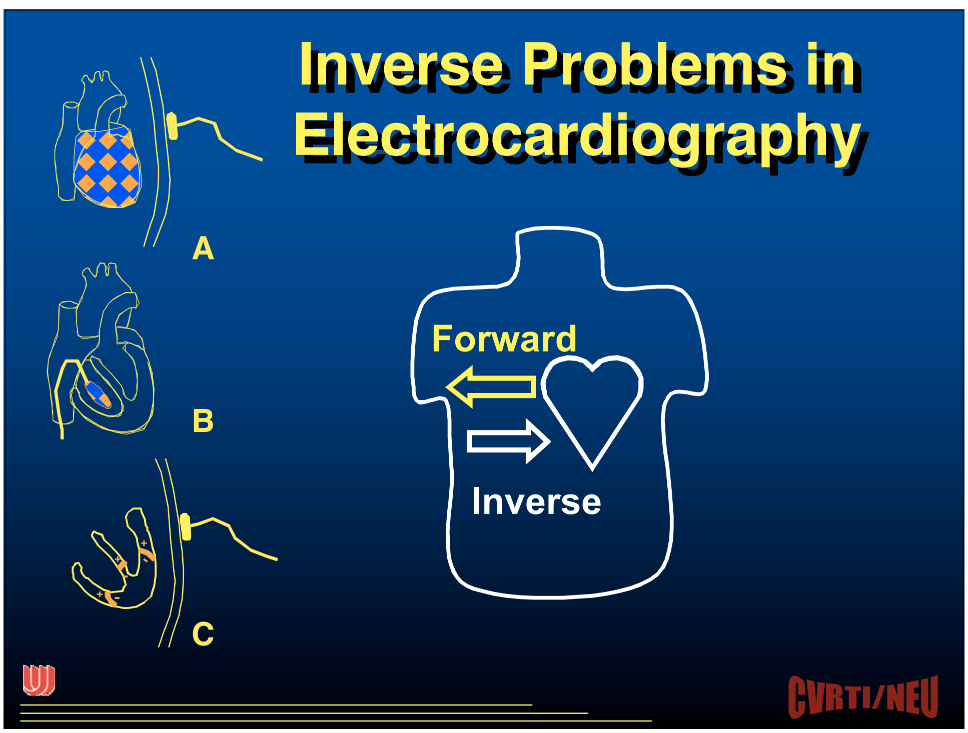
Inverse Problems in Electrocardiography
A
B
C
Forward
Inverse
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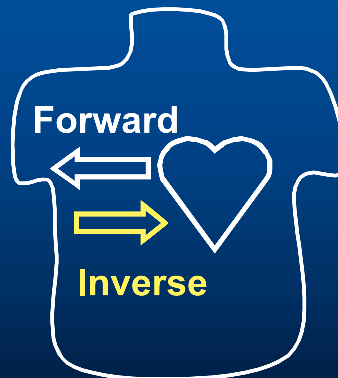
Epicardial Inverse Problem

- Definition
  - Estimate sources from remote measurements
- Motivation
  - Noninvasive detection of abnormalities
  - Spatial smoothing and attenuation



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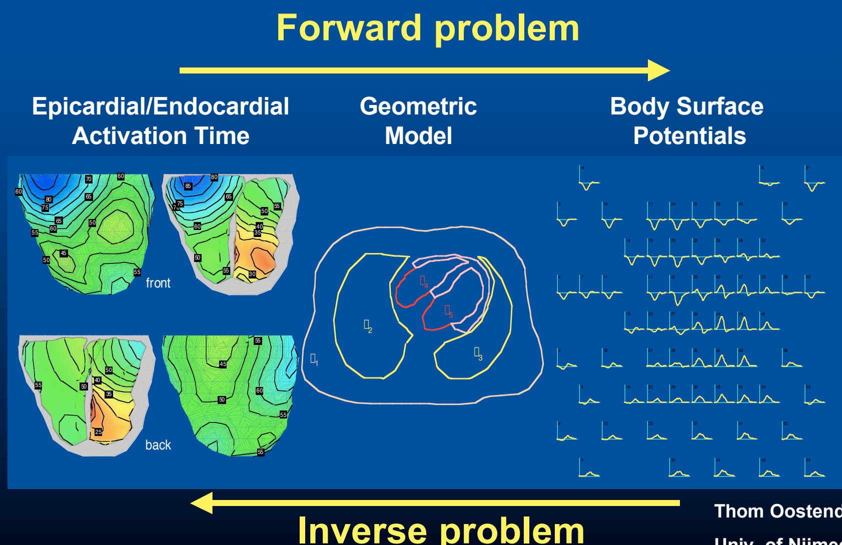
Forward/Inverse Problem



Thom Oostendorp, Univ. of Nijmegen

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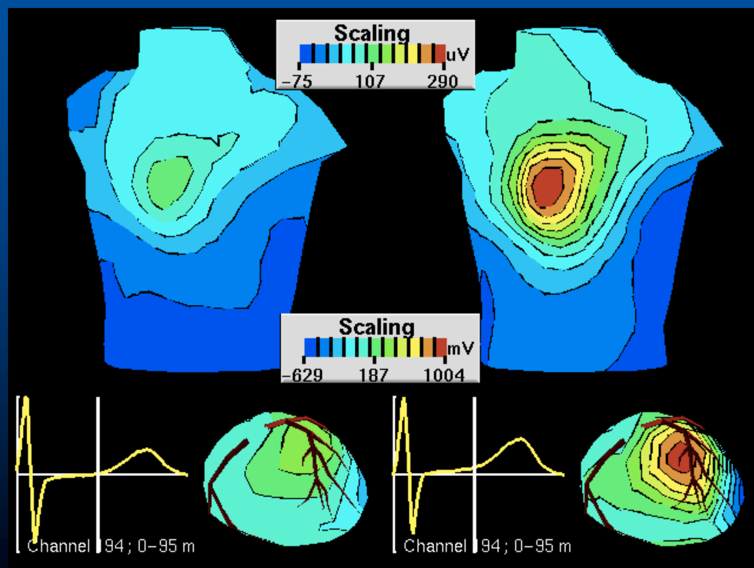
Sample Problem: PTCA



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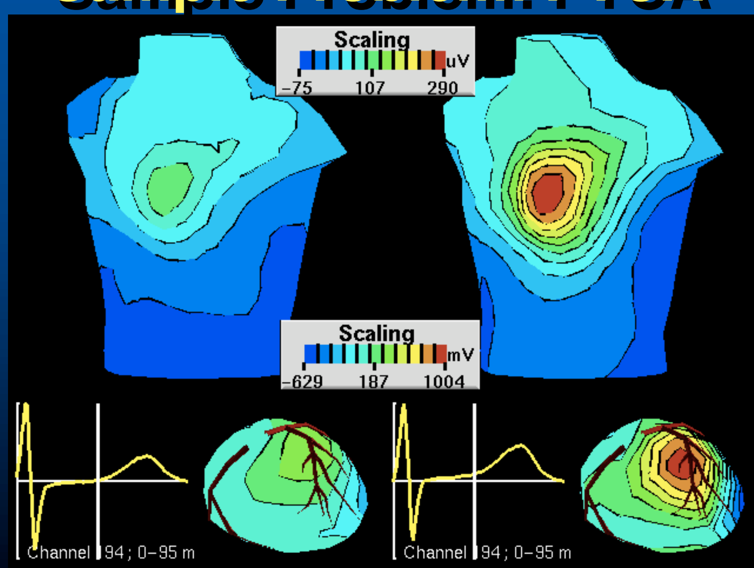
Sample Problem: PTCA



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Elements of the Inverse Problem

- Components
  - Source description
  - Geometry/conductivity
  - Forward solution
  - "Inversion" method (regularization)
- Challenges
  - Inverse is ill-posed
  - Solution ill-conditioned

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Inverse Problem Research

- Role of geometry/conductivity
- Numerical methods
- Improving accuracy to clinical levels
- Regularization
  - *A priori* constraints versus fidelity to measurements

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Regularization

- Current questions
  - Choice of constraints/weights
  - Effects of errors
  - Reliability
- Contemporary approaches
  - Multiple Constraints
  - Time Varying Constraints
  - Tuned constraints
  - Multisource constraints

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Tikhonov Approach

Problem formulation

$$\mathbf{y}(k) = \mathbf{A} \cdot \mathbf{h}(k) + \mathbf{e}(k) \qquad k = 1, 2, \ldots, L$$

Constraint

$$\hat{\mathbf{h}}_\lambda = \arg\min_{\mathbf{x}} \left(\|\mathbf{y} - \mathbf{A}\mathbf{x}\|^2 + \lambda^2 \|\mathbf{R}\mathbf{x}\|^2 \right),$$

Solution

$$\hat{\mathbf{h}}_\lambda = (\mathbf{A}^T\mathbf{A} + \lambda^2\mathbf{R}^T\mathbf{R})^{-1}\mathbf{A}^T\mathbf{y}$$

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Multiple Constraints

For k constraints

$$\hat{\mathbf{h}}_\lambda = \arg\min_{\mathbf{x}} \left(\|\mathbf{y} - \mathbf{A}\mathbf{x}\|^2 + \sum_{i=1}^{k} \lambda_i^2 \|\mathbf{R}_i \mathbf{x}\|^2 \right)$$

with solution

$$\hat{\mathbf{h}}_\lambda = [\mathbf{A}^T\mathbf{A} + \sum_{i=1}^{k} \lambda_i^2 \mathbf{R}_i^T \mathbf{R}_i]^{-1} \mathbf{A}^T \mathbf{y}.$$

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Dual Spatial Constraints

For two spatial constraints:

$$\hat{\mathbf{h}}_\lambda = (\mathbf{A}^T\mathbf{A} + \lambda_1^2 \mathbf{R}_1^T \mathbf{R}_1 + \lambda_2^2 \mathbf{R}_2^T \mathbf{R}_2)^{-1} \mathbf{A}^T \mathbf{y}.$$

Note: two regularization factors required

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Joint Time-Space Constraints

Redefine y, h, A:

$$\bar{\mathbf{y}} = \overline{\mathbf{A}}\bar{\mathbf{h}} + \bar{\mathbf{e}}$$

$$\overline{\mathbf{A}} = \begin{pmatrix} \mathbf{A} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A} & \cdots & \mathbf{0} \\ & & & \cdots & \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A} \end{pmatrix}.$$

And write a new minimization equation:

$$\hat{\mathbf{h}} = \arg\min_{\bar{\mathbf{x}}} \left(\|\overline{\mathbf{A}}\bar{\mathbf{x}} - \bar{\mathbf{y}}\|^2 + \sum_{i=1}^{k_s} \lambda_i^2 \|\overline{\mathbf{R}}_i \bar{\mathbf{x}}\|^2 + \sum_{i=1}^{k_t} \eta_i^2 \|\overline{\mathbf{T}}_i \bar{\mathbf{x}}\|^2 \right).$$

Joint Time-Space Constraints

General solution:

$$\hat{\mathbf{h}} = (\overline{\mathbf{A}}^T\overline{\mathbf{A}} + \sum_{i=1}^{k_s} \lambda_i^2 \overline{\mathbf{R}}_i^T \overline{\mathbf{R}}_i + \sum_{i=1}^{k_t} \eta_i^2 \overline{\mathbf{T}}_i^T \overline{\mathbf{T}}_i)^{-1} \overline{\mathbf{A}}^T \bar{\mathbf{y}}$$

For a single space and time constraint:

$$\begin{aligned} \hat{\mathbf{h}} &= (\overline{\mathbf{A}}^T\overline{\mathbf{A}} + \lambda^2 \overline{\mathbf{R}}^T\overline{\mathbf{R}} + \eta^2 \overline{\mathbf{T}}^T\overline{\mathbf{T}})^{-1} \overline{\mathbf{A}}^T \bar{\mathbf{y}} \\ &= [\mathbf{I}_L \otimes (\mathbf{A}^T\mathbf{A}) + \lambda^2 \mathbf{I}_L \otimes \mathbf{R}^T\mathbf{R} + \eta^2 (\mathbf{T}^T\mathbf{T}) \otimes \mathbf{I}_N]^{-1} \cdot \\ &\quad \cdot (\mathbf{I}_L \otimes \mathbf{A}^T)\bar{\mathbf{y}}. \end{aligned}$$

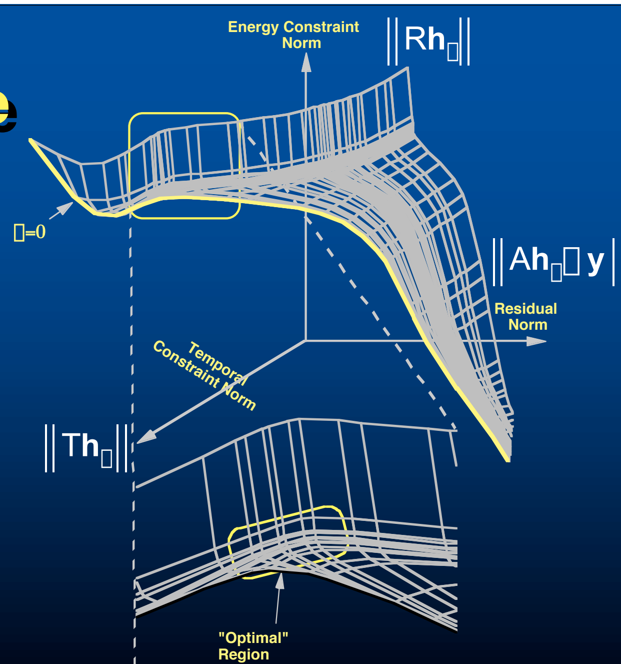
Note: two regularization factors and implicit temporal factor

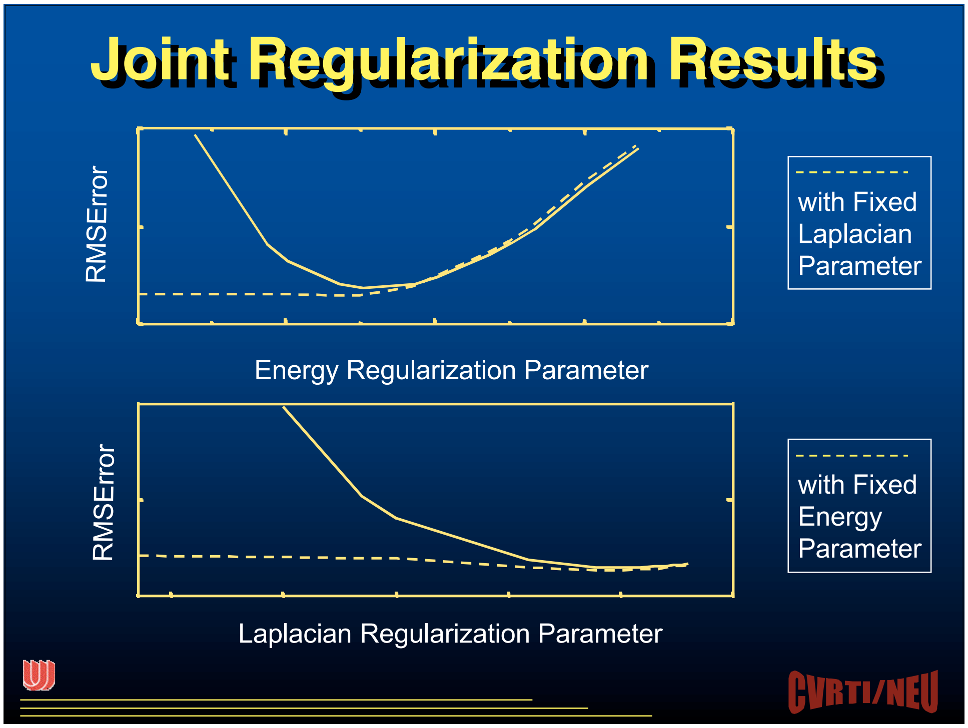
Determining Weights

- Based on *a posteriori* information
- *Ad hoc* schemes
  - CRESO: composite residual and smooth operator
  - BNC: bounded norm constraint
  - AIC: Akaike information criterion
  - L-curve: residual norm vs. solution seminorm

L-Surface

- Natural extension of single constraint approach
- “Knee” point becomes a region




Joint Regularization Results
RMSError
with Fixed Laplacian Parameter
Energy Regularization Parameter
RMSError
with Fixed Energy Parameter
Laplacian Regularization Parameter
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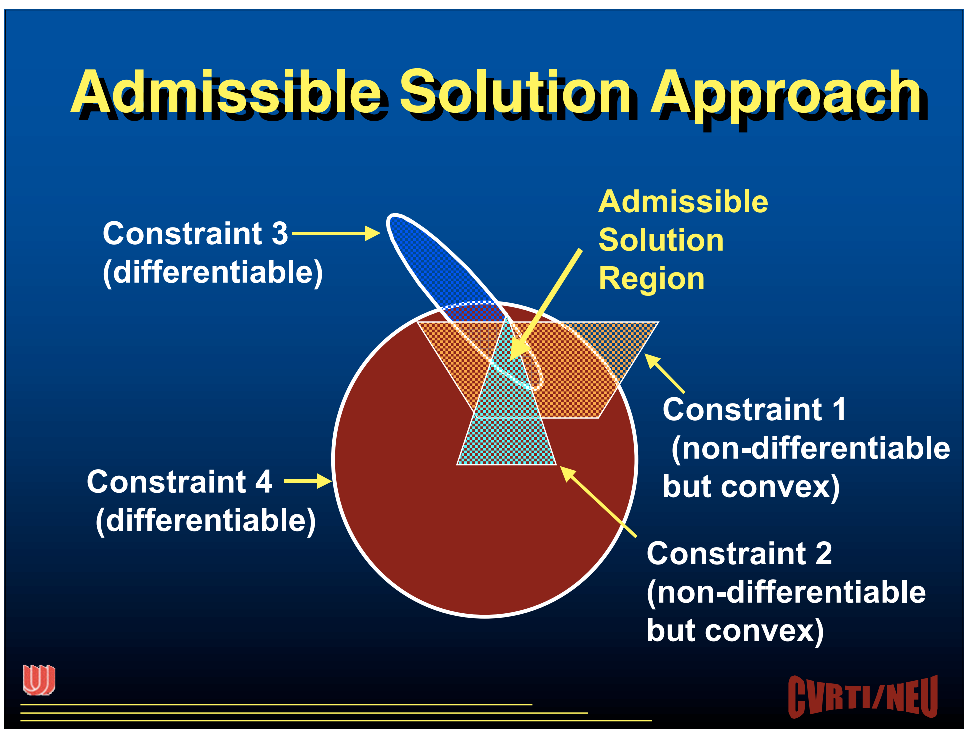
Admissible Solution Approach
Constraint 3
(differentiable)
Admissible Solution Region
Constraint 1
(non-differentiable but convex)
Constraint 4
(differentiable)
Constraint 2
(non-differentiable but convex)
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Single Constraint

Define $\phi(\mathbf{x})$ s.t.

$$\phi(\mathbf{x}) : \mathcal{R}^N \to \mathcal{R}$$

with the constraint such that

$$\phi(\mathbf{x}) - \epsilon < 0.$$

that satisfies the convex condition

$$\phi(\alpha\mathbf{x} + (1-\alpha)\mathbf{y}) \le \alpha\phi(\mathbf{x}) + (1-\alpha)\phi(\mathbf{y}) \qquad \forall\alpha \in [0,1].$$

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Multiple Constraints

Define multiple constraints $\phi_i(\mathbf{x})$

$$(\phi_i(\mathbf{x}) - \epsilon_i) \in \mathcal{H}, \text{ for } i = 1, 2, \ldots, m.$$

so that the set of these

$$\{\mathbf{x} : \phi(\mathbf{x}) < 0\}$$

represents the intersection of all constraints. When they satisfy the joint condition

$$\phi(\mathbf{x}) \le 0$$

Then the resulting **x** is the *admissible solution*

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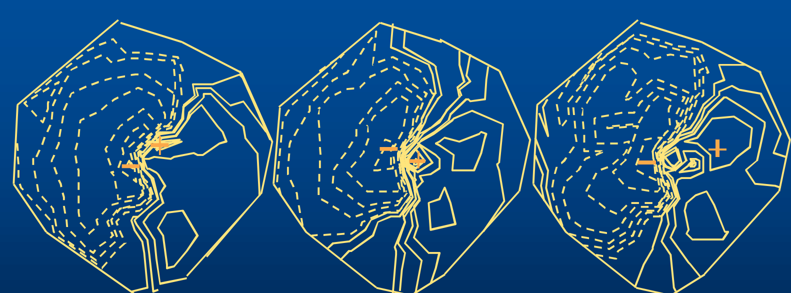
Examples of Constraints

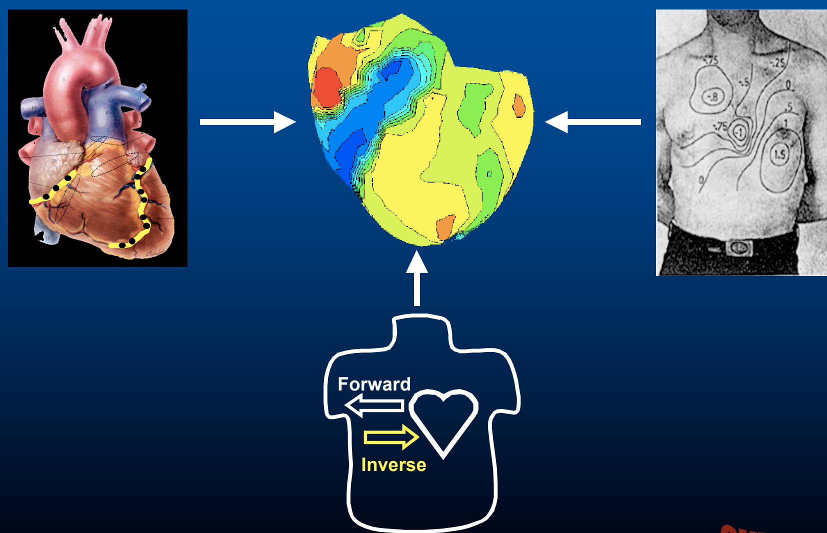
- Residual constraint $\phi(\mathbf{x}) = \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2$
- Regularization constraints $\phi(\mathbf{x}) = \|\mathbf{R}\mathbf{x}\|_2^2$
- Tikhonov constraints $\phi_\lambda(\mathbf{x}) = \left\| \begin{pmatrix} \mathbf{A} \\ \sqrt{\lambda}\mathbf{R} \end{pmatrix} \mathbf{x} - \begin{pmatrix} \mathbf{b} \\ \mathbf{0} \end{pmatrix} \right\|_2^2$
- Spatiotemporal constraints
- Weighted constraints
- Novel constraints

Admissible Solution Results

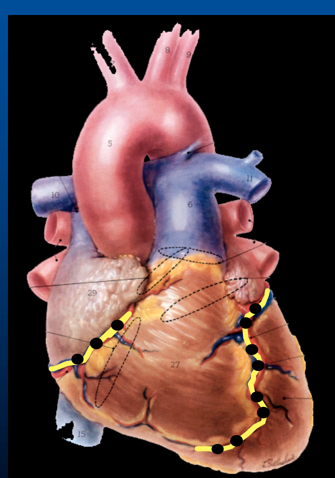


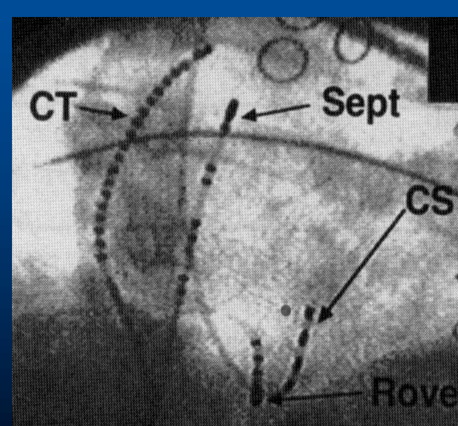
Original Regularized Admissible Solution

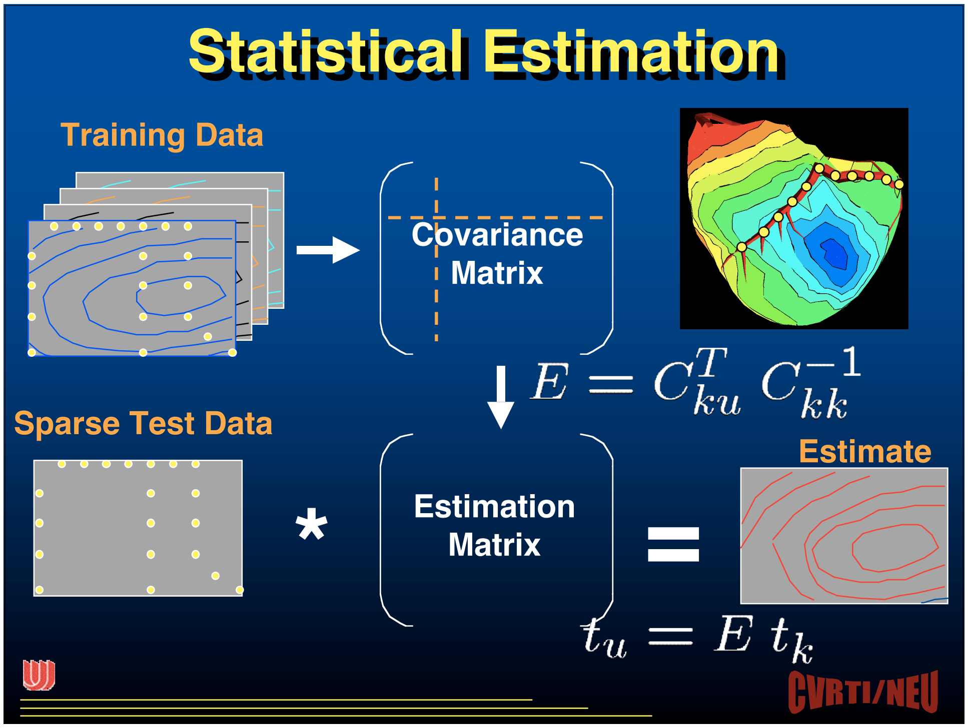
Combining Information Sources



Venous Catheter Based Mapping






Statistical Estimation
Training Data
Covariance
Matrix
$E = C_{ku}^{T}\ C_{kk}^{-1}$
Sparse Test Data
Estimation
Matrix
Estimate
*
=
$t_u = E\ t_k$
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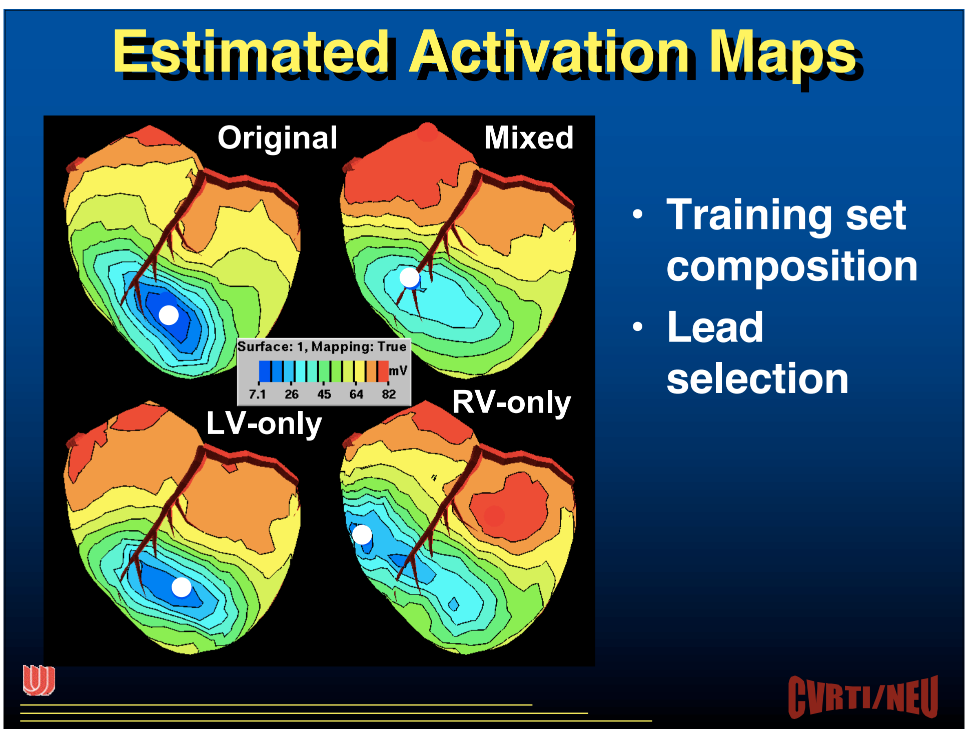
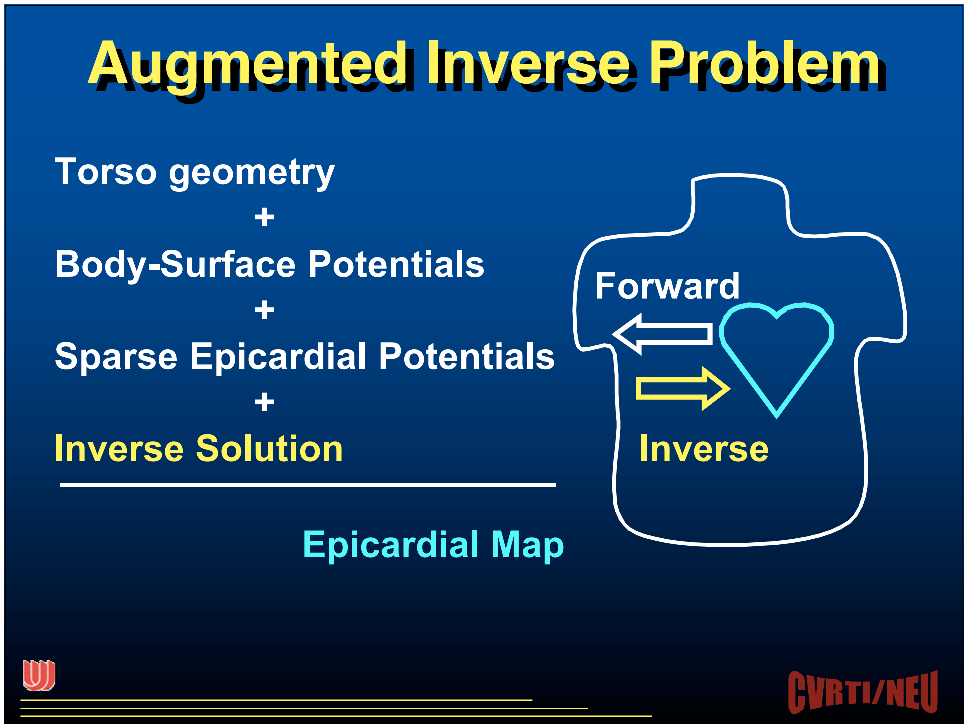
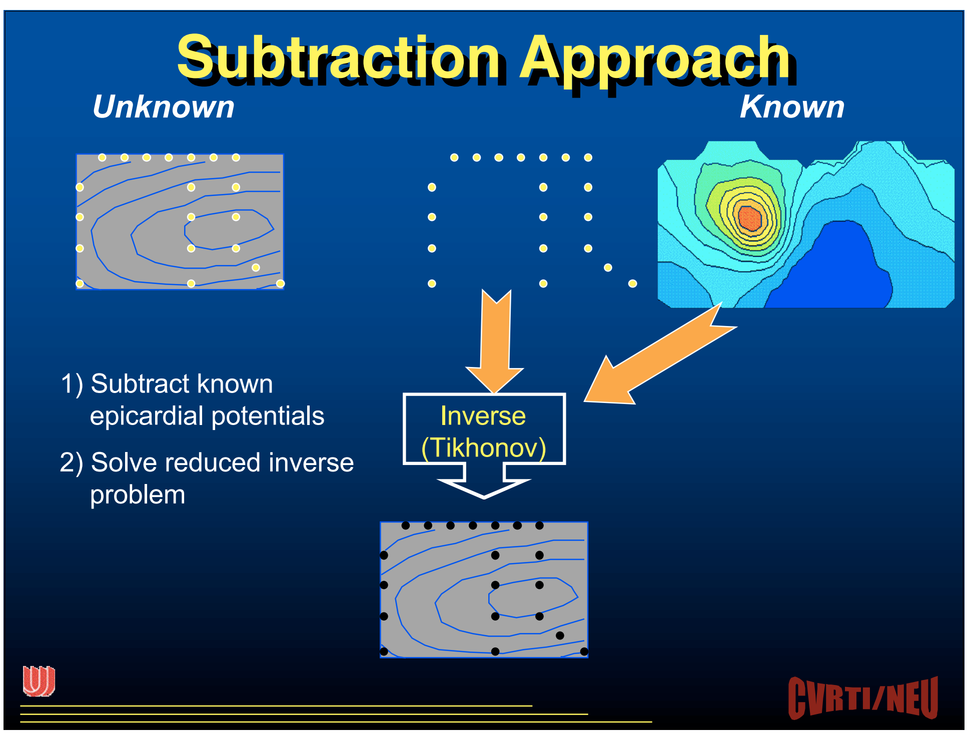
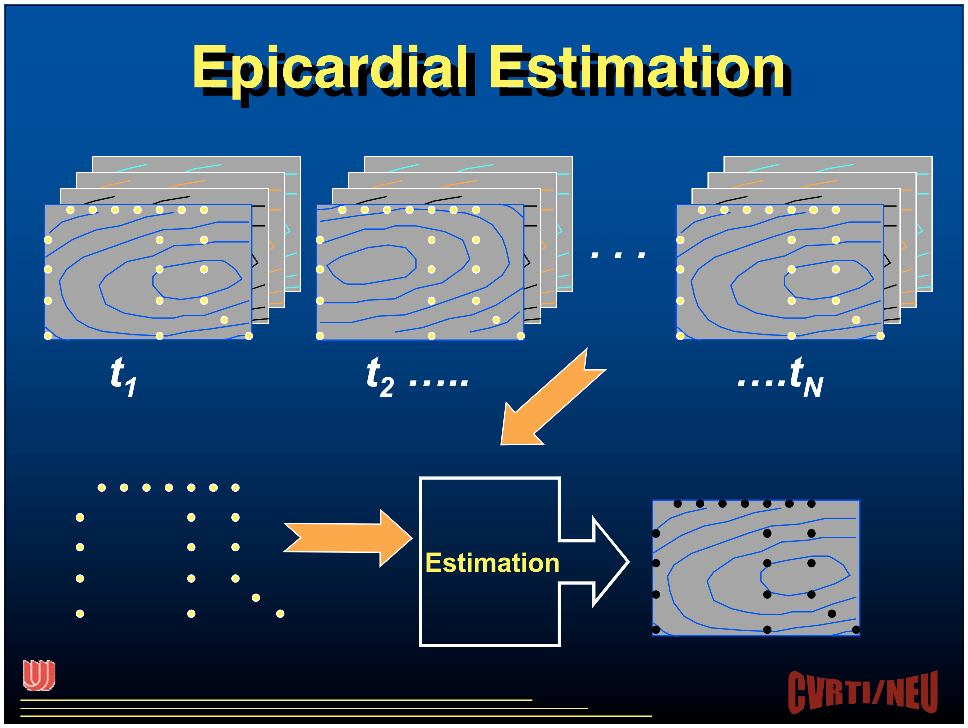
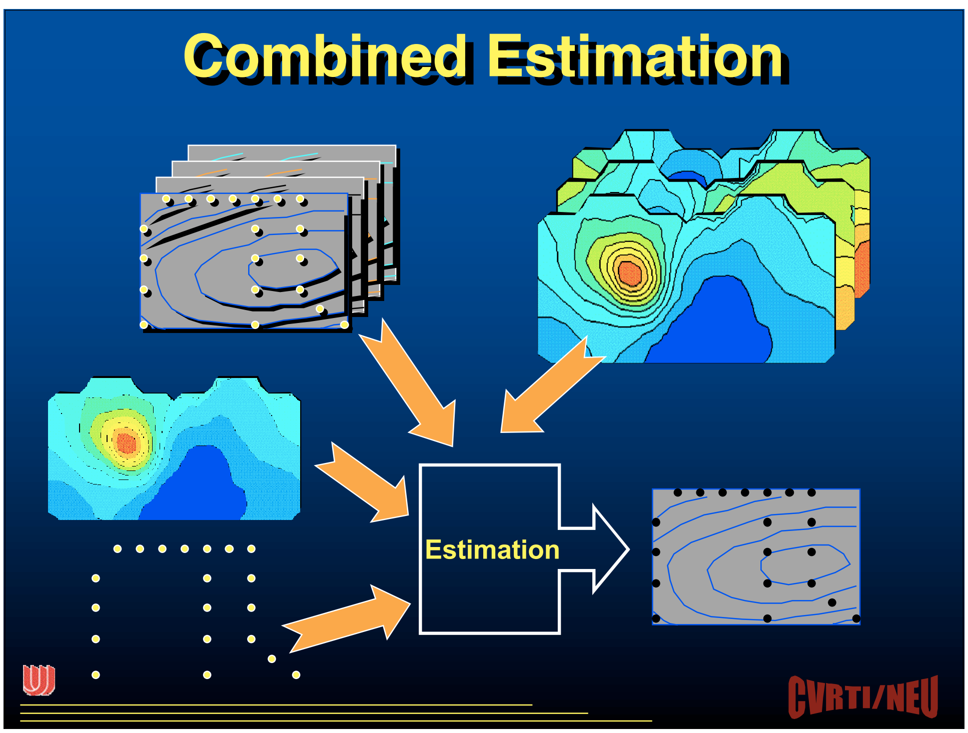
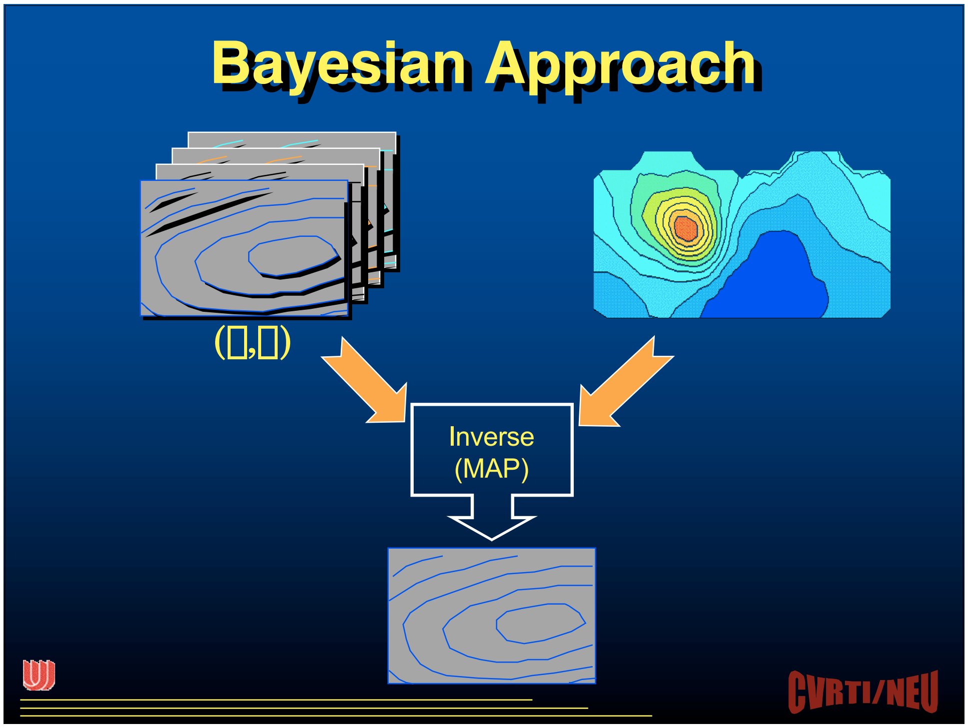

Estimated Activation Maps
Original
Mixed
Surface: 1, Mapping: True
mV
7.1 26 45 64 82
LV-only
RV-only
• Training set composition
• Lead selection
CVRTI/NEU

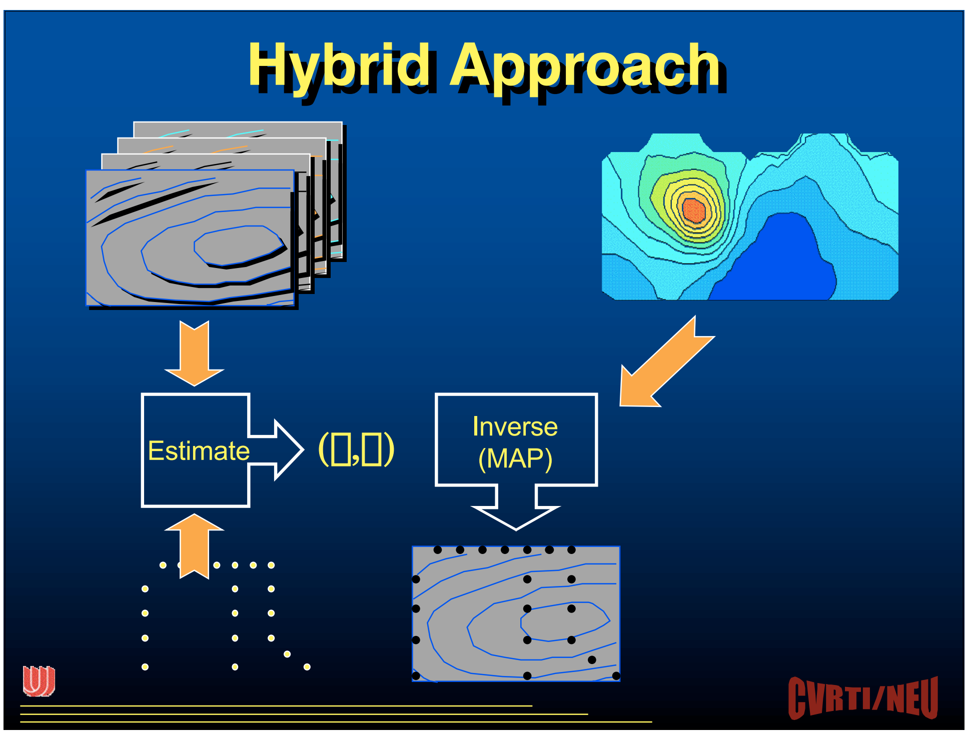

Augmented Inverse Problem
Torso geometry
+
Body-Surface Potentials
+
Sparse Epicardial Potentials
+
Inverse Solution
Epicardial Map
Forward
Inverse
CVRTI/NEU


Subtraction Approach
Unknown
Known
1) Subtract known epicardial potentials
2) Solve reduced inverse problem
Inverse (Tikhonov)
CVRTI/NEU


Epicardial Estimation
t_1
t_2 .....
....t_N
Estimation
CVRTI/NEU


Combined Estimation
Estimation
CVRTI/NEU


Bayesian Approach
(□,□)
Inverse
(MAP)
CVRTI/NEU


Hybrid Approach
Estimate
(□,□)
Inverse
(MAP)
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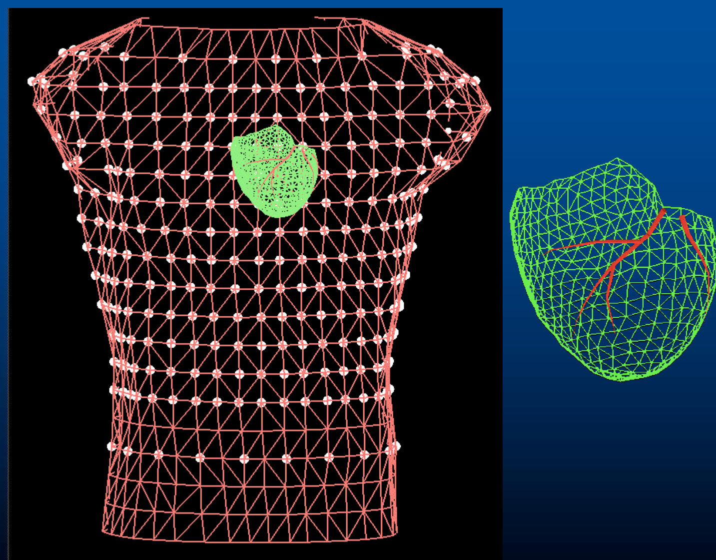
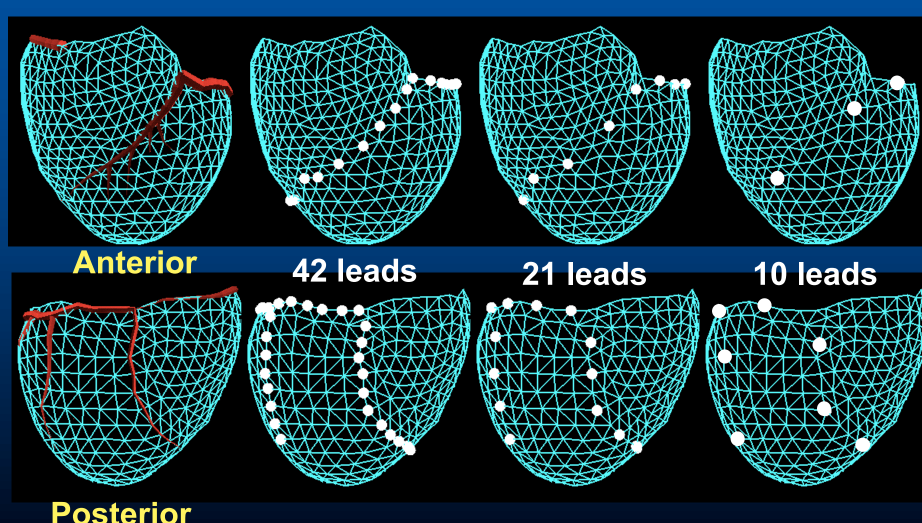
Tank/Heart Geometry



Test Lead Sets



Simulation Study

- 490 lead measured sock data
- Surrogate catheter potentials
  - 42 sites
  - + Gaussian noise
- Torso potentials
  - Calculated noise-free using forward model
  - + Gaussian noise

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Leave-One-Experiment Out Protocol

Training Data

LV Paced Beats

Mixed Paced Beats

Test Data

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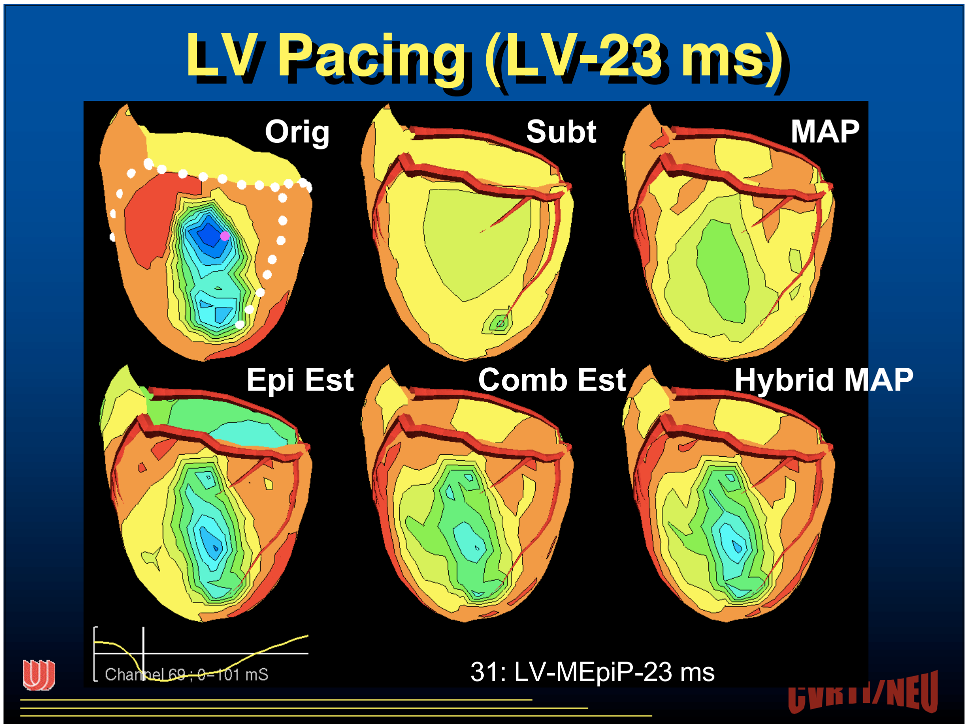
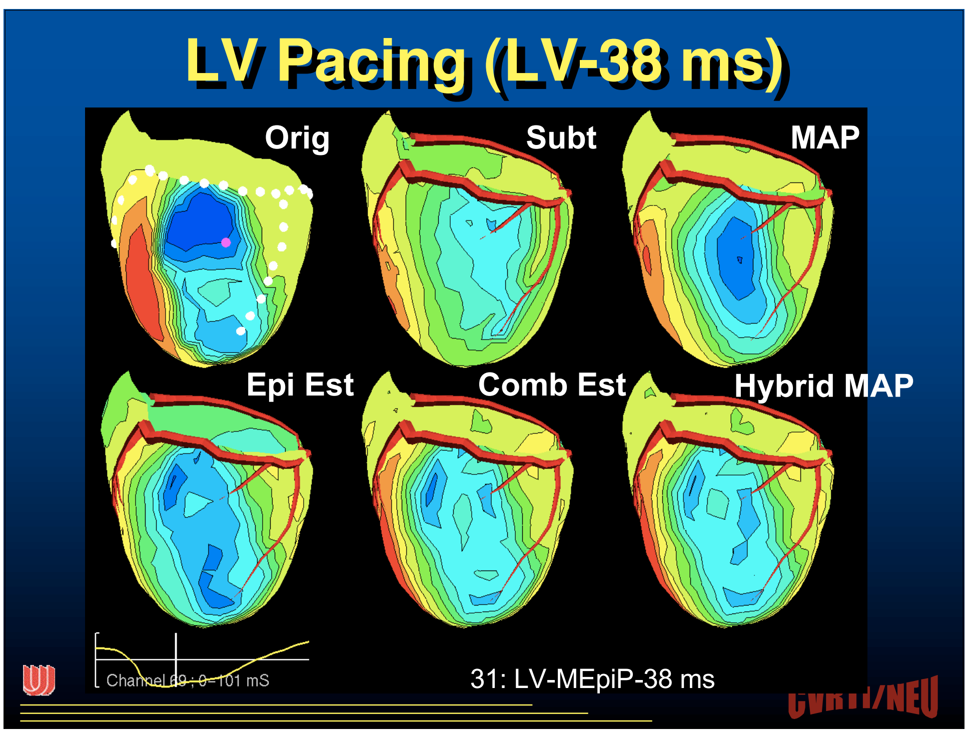
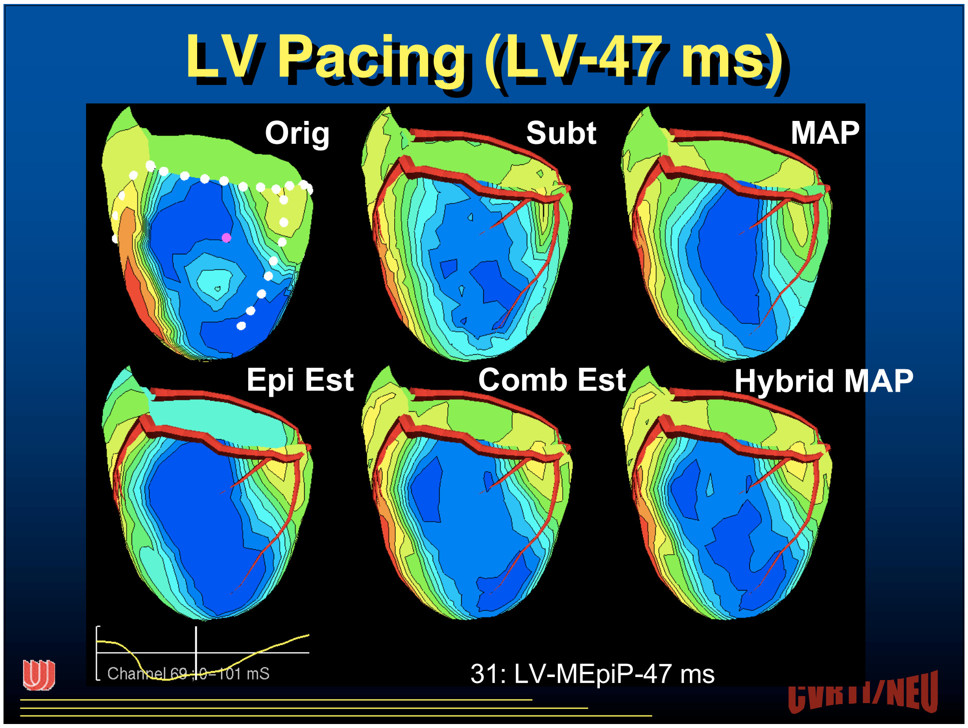
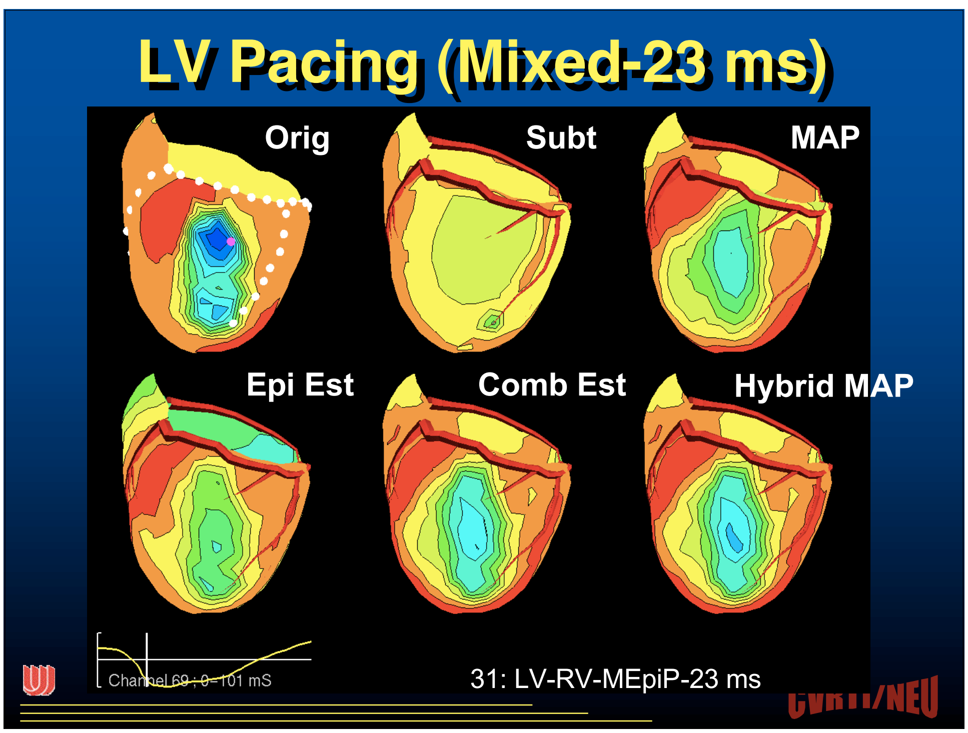
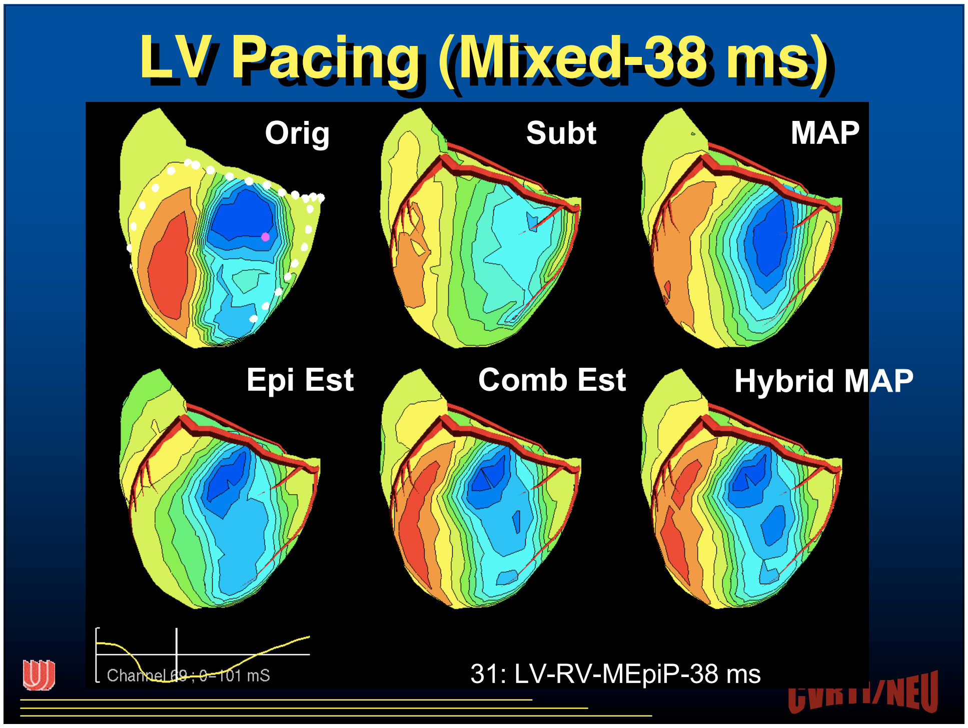

LV Pacing (LV-23 ms)
Orig
Subt
MAP
Epi Est
Comb Est
Hybrid MAP
Channel 69 ; 0-101 mS
31: LV-MEpiP-23 ms


LV Pacing (LV-38 ms)
Orig
Subt
MAP
Epi Est
Comb Est
Hybrid MAP
Channel 69 ; 0-101 mS
31: LV-MEpiP-38 ms


LV Pacing (LV-47 ms)
Orig
Subt
MAP
Epi Est
Comb Est
Hybrid MAP
31: LV-MEpiP-47 ms


LV Pacing (Mixed-23 ms)
Orig
Subt
MAP
Epi Est
Comb Est
Hybrid MAP
31: LV-RV-MEpiP-23 ms


LV Pacing (Mixed-38 ms)
Orig
Subt
MAP
Epi Est
Comb Est
Hybrid MAP
Channel 69 ; 0–101 mS
31: LV-RV-MEpiP-38 ms
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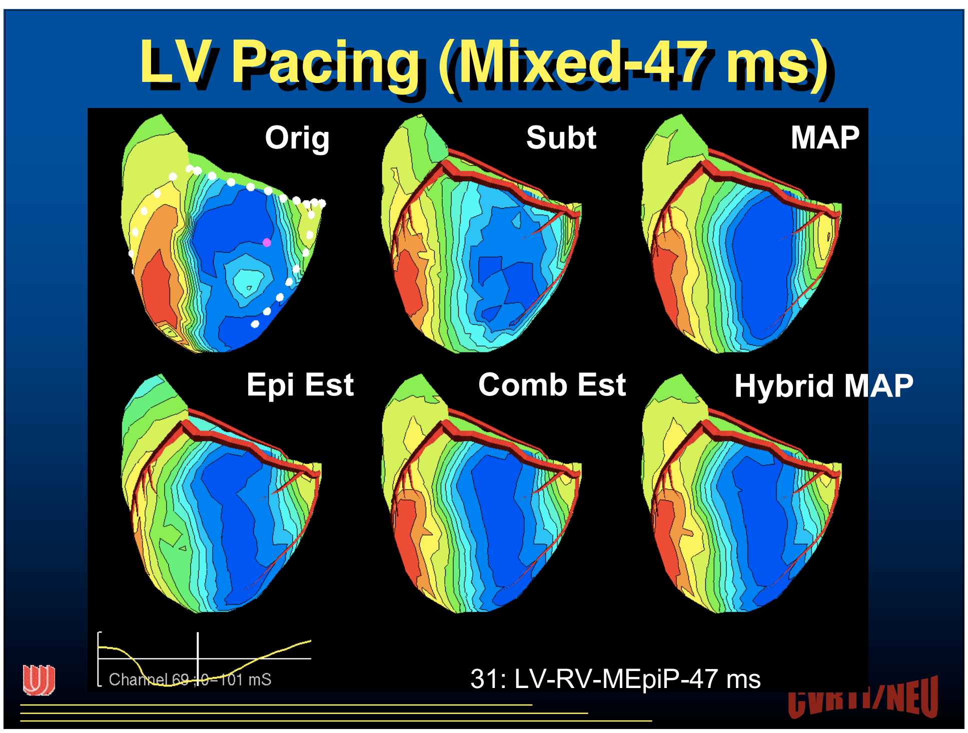
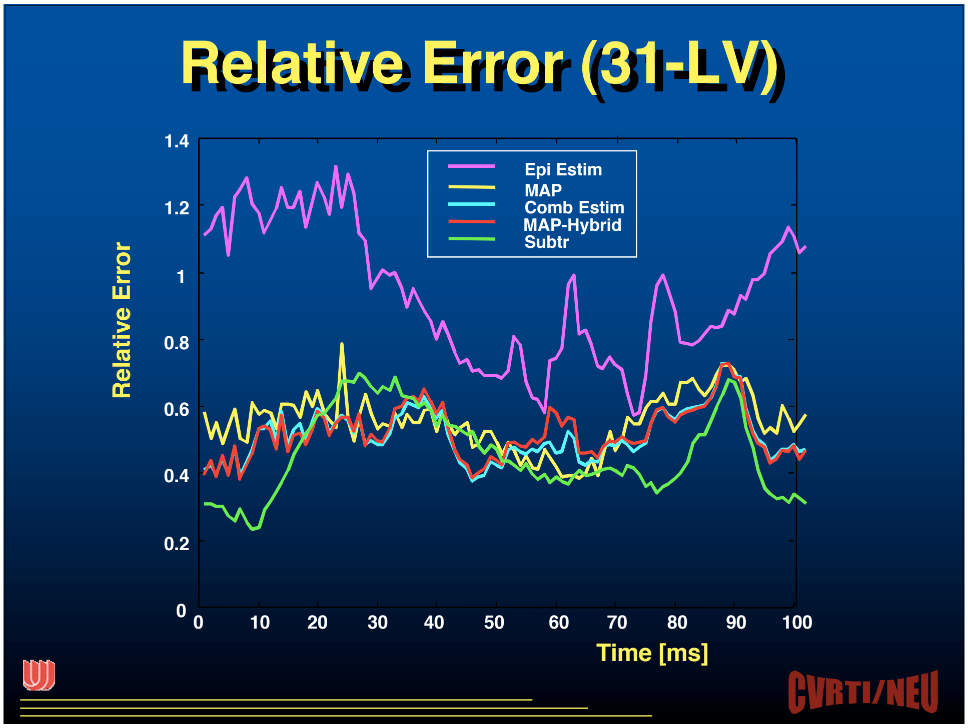
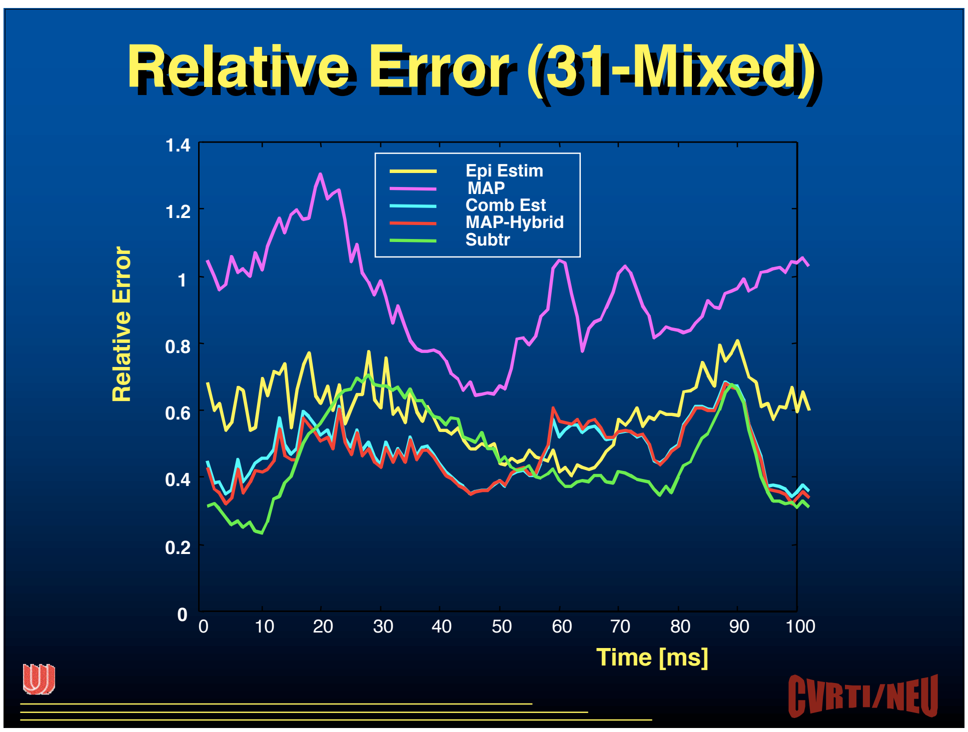

LV Pacing (Mixed-47 ms)
Orig
Subt
MAP
Epi Est
Comb Est
Hybrid MAP
Channel 69 ; 0–101 mS
31: LV-RV-MEpiP-47 ms
CVRTI/NEU


Relative Error (31-LV)
Relative Error
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
10
20
30
40
50
60
70
80
90
100
Time [ms]
Epi Estim
MAP
Comb Estim
MAP-Hybrid
Subtr
CVRTI/NEU


Relative Error (31-Mixed)
Relative Error
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
10
20
30
40
50
60
70
80
90
100
Time [ms]
Epi Estim
MAP
Comb Est
MAP-Hybrid
Subtr
CVRTI/NEU

Estimation Findings

- Estimation alone: noisy, unstable results
- Estimation + inverse: smoothing improves stability

Inverse Solution Findings

- All solutions better than simple Tikhonov
- MAP usually improved with addition of catheter measurements (Hybrid MAP)

Role of Statistics (Training)

- Generally helps
- But can add artifacts, e.g., spurious breakthroughs or wavefronts
- Torso potentials can reduce artifacts

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Acknowledgements

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  - Lucas Lorenzo
- CDSP
  - Dana Brooks
  - Ghandi Ahmad

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