Surface segmentation for improved isotropic remeshing

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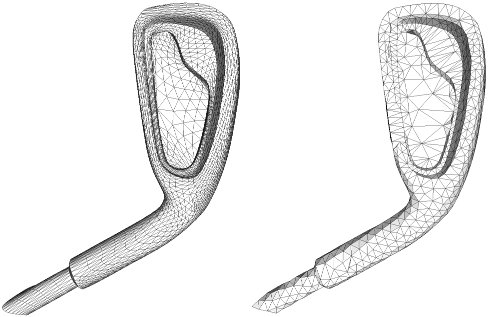
Outline

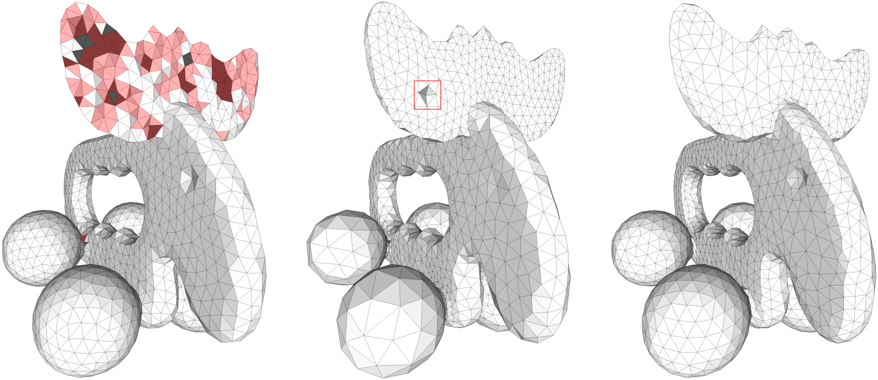
- Motivation and problem statement
- Centroidal Voronoi Tessellation
- Segmentation
- Stitching
- Results

Surface remeshing

- Surface remeshing is the process of transforming one surface mesh into another
- Reasons for remeshing
  - Decimation
  - Triangle quality improvement
- Many techniques sample the input surface S and then triangulate the points



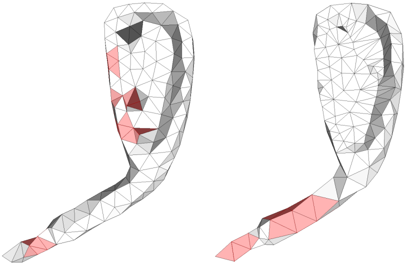
Motivation



uniform | *lfs* | κCVT

Sampling density

- Some techniques resample in parameter space [Alliez et al., 2003]
  - Global parametrization can cause distortion
  - Local parametrization: optimization is not global, stitching is required
- Direct sampling techniques require minimum sampling density



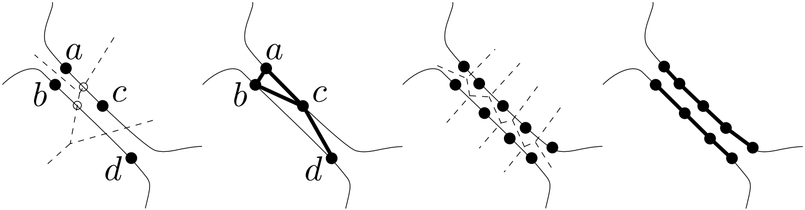
CVT remeshing

- Centroidal Voronoi Tessellation (CVT) is a method of tessellating a space
- It can be formulated as a critical point of the CVT energy function [Du et al., 1999]

$$F(X) = \sum_{i=1}^{n} \int_{\Omega_i} \rho(x) \|x - x_i\|^2 \, d\sigma \tag{1}$$

  $X = \{x_i\}$ is the set of sample points, Ω_i is the Voronoi cell of x_i with respect to the other sample points, ρ is a density function
- In the context of surface remeshing:
  - Typical density functions are $\rho = 1$ and $\rho = 1/lfs^2$ [Yan et al., 2009]
  - We use the Restricted Voronoi Diagram for the final mesh [Du et al., 2003]

- Restricted Voronoi Diagram: each Voronoi Cell is intersected with S to form Restricted Voronoi Cell (RVC)
- The dual (called Restricted Delaunay Triangulation, or RDT) is defined as usual
- Obvious problems occur when RVC is not homeomorphic to a topological disk



Sampling density

Two related theorems govern when RDT will be homeomorphic to S

Topological ball property [Edelsbrunner and Shah, 1997]

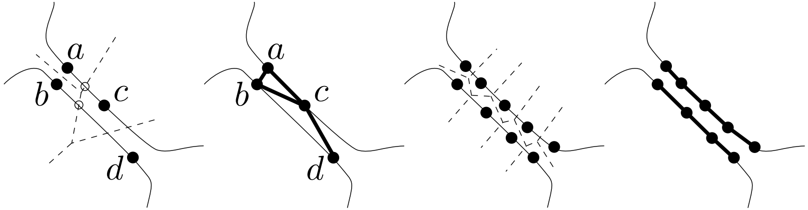
Each RVC must be a topological disk

r-sampling theorem [Amenta and Bern, 1999]

Each point p on surface S must be within $r \cdot lfs(p)$ of a seed point, where $lfs(p)$ is the local feature size at p

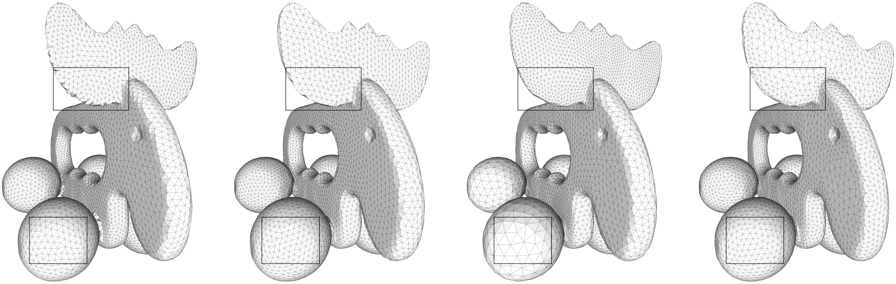
Sampling density

- Required sample density is based on local feature size (*lfs*)
- *lfs* measures curvature and thickness (see Levy short course)
- This is a disappointment: it isn't intuitive that flattish regions should still require high sampling density
- An unfortunate artifact of using euclidean distance to approximate geodesic distance



Introducing κCVT

- [Fuhrmann et al., 2010]: $\rho = \sqrt{\kappa}$
- uniform: $\rho = 1$
- *lfs*: $\rho = 1/\mathit{lfs}^2$
- κCVT: $\rho = \sqrt{\kappa}$

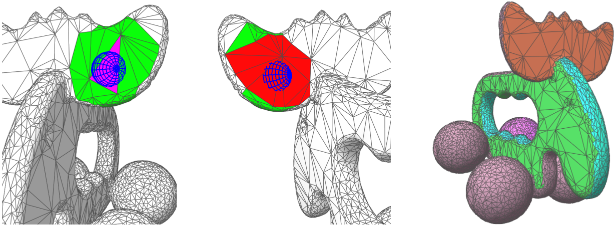


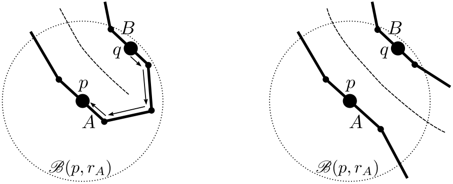
[Fuhrmann et al., 2010] uniform *lfs* κCVT

- The big question: how can we use κ as the density function while still meeting the sampling theorem?

Segmentation

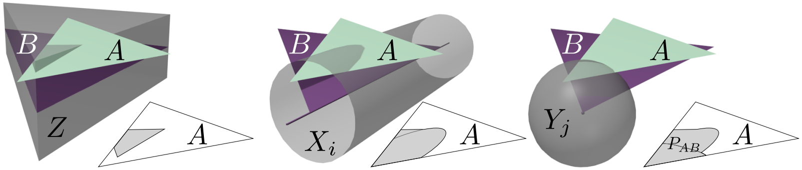
- Segment surface S such that flattish areas have high lfs
- Let A be a triangle in M_i. Heuristic: partition S into subsurfaces $M = \{M_i\}$ such that the ball $\mathscr{B}(p, r_A)$ centered at any point $p \in A$ will yield a single connected component when intersected with M_i.
  - r_A is an approximation of $lfs(A)$.





Building compatibility table and segmentation

- Find which triangles are compatible with triangle A
- Given triangle B, let $P_{A,B}$ be the set of all points on A that are within r_A of B (shaded region of 2D triangles in image)

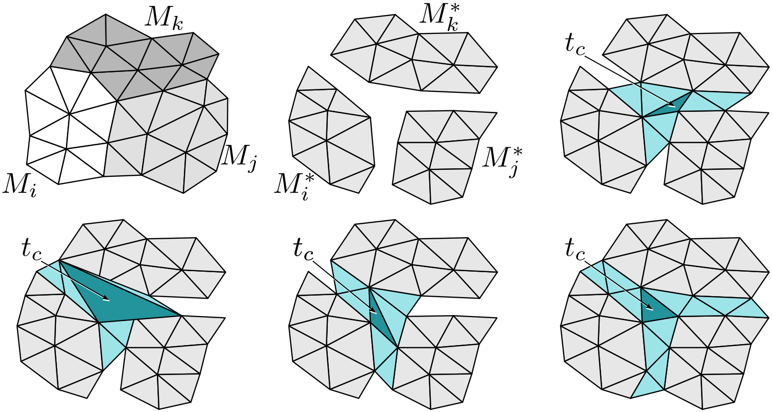


Building compatibility table and segmentation

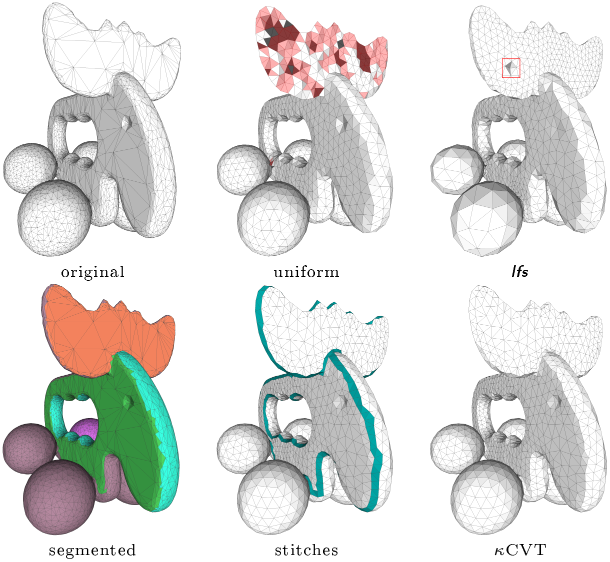
- Search out from A and use boolean set operations to build "compatibility table"
- Use region merging to find groups of compatible triangles

Subsurface remeshing and stitching

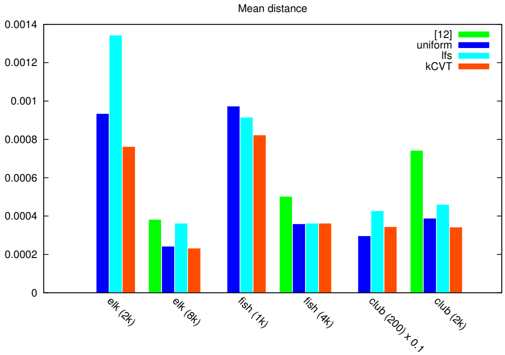
- Remesh each subsurface M_i individually using CVT with $\rho = \sqrt{\kappa}$ (hence the name of our method)
- Stitch remeshed subsurfaces $\{M_i^*\}$ back together using a search algorithm and cost function

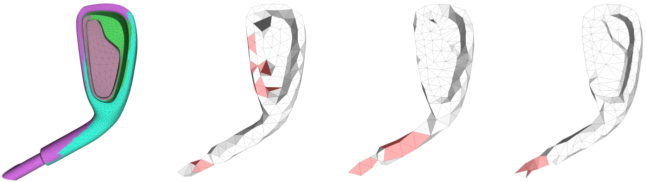


Method overview and results



Results

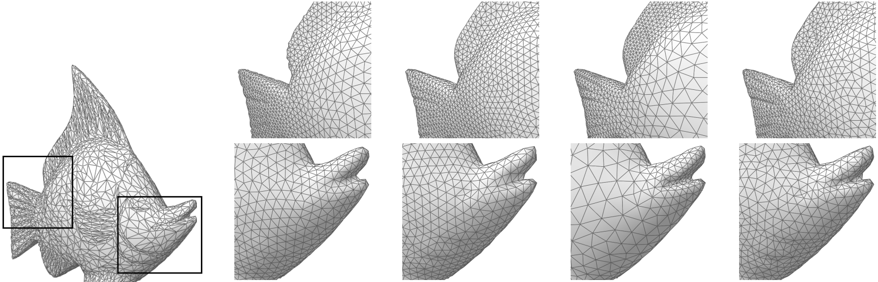




segmented uniform *lfs* κCVT

Results

Remeshing the fish model with 4000 sample points.



[Fuhrmann et al., 2010] | uniform | *l*fs | κCVT

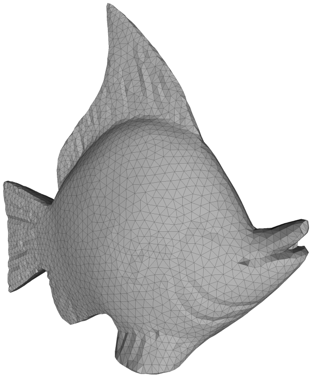
Results

- κCVT performed better than the other two CVT methods in terms of geometric error in every test but one, and showed as much as 20% improvement over the next-best method
- Topological errors were reduced to 0 in every case but one. In that case topological errors were reduced by 30% of next-best method
- Average triangle quality was similar to that of *lfs* method

Results

- So what's the catch?
  - Min triangle quality was reduced, due to stitching
  - Improvement is specific to models with flattish areas that have low local feature size
- Future work
  - Implement feature preservation
  - Perform local CVT on stitching area

Thank you.



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Sampling density theorem

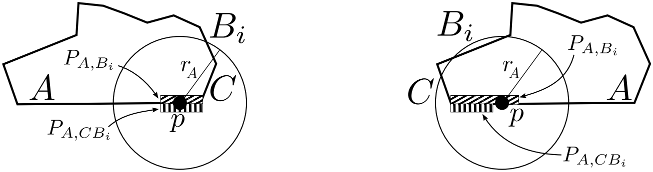
Theorem

r*-sampling theorem [Amenta and Bern, 1999] If no point* p *on surface* S *is farther than* $r \cdot \mathsf{lfs}(p)$ *from a seed point* $x \in X$ *where* r *is a constant then the Restricted Delaunay Triangulation induced by* X *is homeomorphic to* S*.*

Computation of r_A

- We define $r_p = 2 \cdot \alpha \cdot \mathit{lfs}(p)$ and $r_A = \arg\min_{v_i \in V_A} r_{v_i}$.
- All of our experiments use $\alpha = 1.1$.

Flood fill segmentation



Stitching cost function

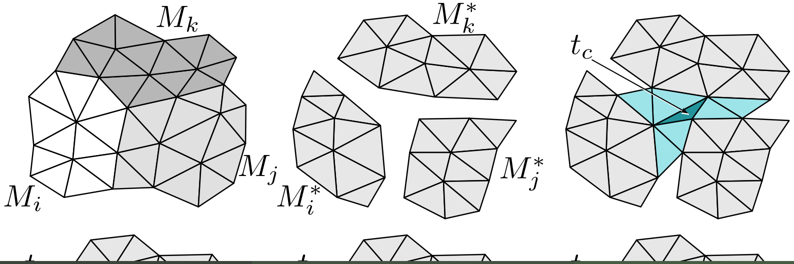
t_c is a candidate "connector"

$$cost(t_c) = \sum_{t \in T_c} area(t) \cdot Q(t)^{-\gamma}. \tag{2}$$

γ is a user-defined parameter (we used $\gamma = 0.5$) and $Q(t)$ is the triangle quality measure

$$Q_t = \frac{6}{\sqrt{3}} \frac{r_t}{h_t} \tag{3}$$

where r_t and h_t are the inradius and longest edge length of t, respectively.



Results - table

model	# seeds	method	errors	$\mathrm{H}_{\mathrm{mean}} \times 10^3$	$\mathrm{H}_{\mathrm{RMS}} \times 10^3$	Q_{min}	Q_{ave}
Elk	2000	uniform	400	0.94	1.48	0.448	0.884
		$l$fs	15	1.31	1.76	0.347	0.858
		κCVT	0	0.76	1.00	0.220	0.849
Elk	8000	[Fuhrmann et al., 2010]	0	0.38	0.63	0.058	0.902
		uniform	0	0.24	0.37	0.509	0.916
		$l$fs	0	0.36	0.49	0.451	0.893
		κCVT	0	0.23	0.34	0.259	0.885
Fish	1000	uniform	95	0.97	0.16	0.525	0.872
		$l$fs	14	0.91	0.12	0.420	0.830
		κCVT	0	0.82	0.12	0.236	0.809
Fish	4000	[Fuhrmann et al., 2010]	0	0.50	0.85	0.070	0.898
		uniform	11	0.36	0.53	0.580	0.898
		$l$fs	0	0.36	0.51	0.407	0.864
		κCVT	0	0.36	0.58	0.160	0.863
Club	200	uniform	51	2.94	4.08	0.570	0.842
		$l$fs	31	4.25	6.36	0.362	0.770
		κCVT	19	3.42	4.92	0.173	0.728
Club	2000	[Fuhrmann et al., 2010]	-	0.74	1.54	~ 0	0.832
		uniform	0	0.39	0.70	0.555	0.893
		$l$fs	0	0.46	0.85	0.314	0.834
		κCVT	0	0.34	0.62	0.082	0.855