

# A Queuing-Theoretic Framework for Modeling and Analysis of Mobility in WSNs

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## ABSTRACT

In this paper, we present a complete framework for modeling and analysis of Mobility in Wireless Sensor Networks using OQNs with  $GI/G/1$  nodes and single-class customers. We formalize and present three variations - gated queues, intermittent links and intermittent servers. We suitably modify and use the Queuing Network Analyzer (QNA) to study performance measures including: throughput, average waiting time (end-to-end delay), and packet loss probability. The results are verified by simulation in OMNeT++.

## General Terms

Performance, Theory

## Keywords

Unreliable servers, squared coefficient of variation, Poisson process, Rayleigh and exponential distributions

## 1. INTRODUCTION

Open queuing networks (OQNs) have been widely used as efficient tools to analyze the performance measures in computer and communication systems ([2], [6], [8], [9]). For many classes of OQNs, elegant and efficient solution methods exist. Well known closed product-form solutions are available for simplified networks such as Jackson and BCMP networks under a number of restrictions ([14],[5]). These networks, however, do not always apply in practice.

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In wireless sensor networks with mobile nodes – we refer them as Mobile Wireless Sensor Networks (MWSNs) – the link between any two nodes will be either available or unavailable due to node mobility. Node unavailability is caused by poor node signal strengths due to low battery, weather conditions or both, or when the node is out of radio-range. In [11], the authors have developed a random walk based mobility model for a MWSN and have derived the probability distributions of link availability between any two nodes. In [4], the authors had used a queuing network for delay analysis of wireless ad hoc networks in which static nodes are distributed uniformly and independently over a torus of unit area. In this network, the transition probabilities for forwarding packets from one node to another node are considered as functions of communication area of nodes of ad hoc networks.

The mobility of nodes in a MWSN can be captured in terms of gated queues, intermittent links or intermittent servers of the queuing networks under investigation with immobile nodes. Hence, in this paper we analyze three types of OQNs with  $GI/G/1$  immobile nodes and single-class customers - one with gated nodes, second with intermittent links and third with intermittent servers. We use appropriate distributions for the time durations for gate open/close, link up/down, and server up/down. We also suitably modify the Queuing Network Analyzer (QNA), a method proposed by Kuhn [10], and later expanded by Whitt [16] to approximate performance measures of large OQNs with general inter-arrival and general service distributions; to study the performance measures of networks under investigation with general distributions for the time duration of gate open and close, link up and down, and server up and down. The performance measures include throughput, average waiting time (end-to-end delay), customer loss probability and path availability.

The paper is structured as follows. Section II introduces a general OQN and its performance measures. In section III, the QNA method is discussed. In sections IV, V and VI, we modify the QNA method for OQNs with gated nodes, with

interrupted links and with interrupted servers, respectively. In section VII, we briefly outline the validation of our results and present our conclusions.

## 2. GENERAL OQN

A queuing network is a natural extension of a collection of interactive queuing systems, referred to as nodes. Consider an OQN with  $M$  single-server and infinite buffer nodes. Let  $\mu_i$  denote the mean service rate and  $\lambda_{0i}$  denote the mean external arrival rate at node  $i$ . Let  $p_{ij}$ ,  $i, j = 1, 2, \dots, M$ , denote the transition probability by which customers finishing service at node  $i$  will join node  $j$ . The matrix  $\mathbf{P} = (p_{ij})$  is the sub-stochastic matrix such that with probability  $1 - \sum_{j=1}^M p_{ij}$  customers finishing service at node  $i$  will leave the network. The average arrival rate  $\lambda_j$  and the departure rate  $\lambda_{j0}$  of customers at node  $j$  are given as follows for  $j = 1, 2, \dots, M$ ,

$$\lambda_j = \lambda_{0j} + \sum_{i=1}^M p_{ij} \lambda_i; \quad \lambda_{j0} = \left(1 - \sum_{i=1}^M p_{ji}\right) \lambda_j. \quad (1)$$

For an acyclic network (where customers are not allowed to revisit the nodes) with Poisson external arrivals and exponential service times, the total arrivals at node  $i$  follow a Poisson process with rate  $\lambda_i$  given by (1). For cyclic networks, due to dependency amongst the arrival streams, the total arrivals at node  $i$  do not follow Poisson process though the external arrivals and service times are Poisson and exponential, respectively, in all nodes. The Jackson and BCMP networks are examples of cyclic networks [14]. For these networks, the steady-state joint probability distribution of number of customers in all the nodes of the network admits product-form solution and hence leading to closed-form solutions for performance measures like average wait times for customers in the network.

For networks with  $GI/G/1$  nodes, finding closed-form solutions for performance measures is more subtle. To overcome this difficulty, the QNA method is used to find approximate solutions for the performance measures of OQN with  $GI/G/1$  nodes. We discuss this method in more detail in section 3.

For the queuing networks under investigation in sections 4, 5 and 6, one can analyze the performance measures like *throughput*, *customer loss probability*, and *average waiting time*. These measures are defined as follows.

The throughput  $\mathcal{T}$  of the network is given by

$$\mathcal{T} = \sum_{j=1}^M \lambda_{j0}, \quad (2)$$

where  $\lambda_{j0}$  is given by (1).

The customer loss probability  $P_L$  is given by

$$P_L = \frac{\gamma - \mathcal{T}}{\gamma}, \quad \text{with } \gamma = \sum_{j=1}^M \lambda_{0j}. \quad (3)$$

where  $\gamma$  is the total inflow to the network. The average waiting time  $W_s$  of a customer in the network (end-to-end delay) is given by

$$W_s = \sum_{j=1}^M W_{sj}, \quad \text{with } W_{sj} = W_{qj} + \frac{1}{\mu_j}, \quad (4)$$

where  $W_{sj}$ ,  $W_{qj}$  and  $\mu_j$  are the average waiting time, average queuing time and average service rate at node  $j$ ,  $j = 1, 2, \dots, M$ . In this paper, we find analytical formulas for  $W_s$  of the proposed queuing-theoretic framework.

## 3. QNA METHOD

The QNA is an approximation technique and a software package developed at Bell Laboratories to calculate approximate congestion measures of a network of queues [16]. It is a powerful tool to analyze general queuing networks. The most important feature of the QNA is that the external arrival processes need not be Poisson, and the service-time distributions need not be exponential. The QNA can provide a fast approximate solution for large networks.

The QNA has been used extensively in many theoretical and practical applications and the results have been compared with simulation results and/or the results of other techniques ([15], [7], [13]). The low relative error percentage makes QNA one of the most important tools for analyzing general networks.

The input to QNA comprises of the number of nodes  $M$ , mean arrival rate  $\lambda_{0i}$  and squared coefficient of variation (SCV)  $c_{0i}^2$  for external arrivals at node  $i$ ,  $i = 1, 2, \dots, M$ , mean service rate  $\mu_i$  and SCV  $c_{si}^2$  for service time at node  $i$ ,  $i = 1, 2, \dots, M$ , and the transition (routing) probabilities  $p_{ij}$ ,  $i, j = 1, 2, \dots, M$ .

The total arrival rate at node  $i$  is given by (1). The traffic intensities or utilization of node  $i$  is given by  $\rho_i = \lambda_i / \mu_i$ . The arrival rate from node  $i$  to node  $j$  is given by  $\lambda_{ij} = p_{ij} \lambda_i$ . The proportion of arrivals to node  $j$  from node  $i$  is given by  $q_{ij} = \lambda_{ij} / \lambda_j$ ,  $i = 0, 1, \dots, M$ . The SCV of the total arrivals at node  $j$  is calculated as follows [16]:

$$c_{aj}^2 = a_j + \sum_{i=1}^M b_{ij} c_{ai}^2, \quad (5)$$

where  $a_j$  and  $b_{ij}$  are derived after considering merging and splitting of traffic streams and are given as follows:

$$a_j = 1 + w_j \{ (q_{0j} c_{0j}^2 - 1) + \sum_{i=1}^m q_{ij} [(1 - p_{ij}) + (p_{ij} \rho_i^2 x_i)] \} \quad (6)$$

and  $b_{ij} = w_j p_{ij} q_{ij} (1 - \rho_i^2)$ ,  $x_i = \max_{1 \leq i \leq M} (c_{si}^2, 0.2)$ ,  $w_j = [1 + 4(1 - \rho_i)^2 (v_j - 1)]^{-1}$ , and  $v_j = [\sum_{i=0}^m q_{ij}]^{-1}$ . The SCV  $c_{aj}^2$  can also be calculated as follows [16, eqn. (41)]:

$$c_{aj}^2 = 1 - w_j + w_j \sum_{i=1}^M p_{ij} c_{ij}^2, \quad (7)$$

where  $c_{ij}^2$  is the SCV of the traffic flow from node  $i$  to node  $j$  and is given by

$$c_{ij}^2 = q_{ij} [1 + (1 - \rho_i^2)(c_{ai}^2 - 1) + \rho_i^2 (c_{si}^2 - 1)] + 1 - q_{ij}. \quad (8)$$

The expressions in (6) and (8) are obtained by setting  $m = 1$  (number of servers in each node) and  $v_{ij} = 0$  in equations (25) and (41) of [16], respectively. The approximate formula for the average waiting time of a customer at node  $j$  is then given by [16]

$$W_{qj} = \frac{\lambda_j (c_{aj}^2 + c_{sj}^2) g_j}{2(1 - \rho_j)}, \quad (9)$$

where  $g_j$  is a function of  $\rho_j, c_{a_j}^2$  and  $c_{s_j}^2$ , such that  $g_j = 1$  for  $c_{a_j}^2 \geq 1$ . On substituting (9) in (4) we get the end-to-end delay  $W_s$ .

#### 4. OQN WITH GATED NODES

We consider an OQN as discussed in section 2 with the following modifications: each node has a gate which goes on and off with rates  $\alpha$  and  $\beta$  with variances  $v_{on}$  and  $v_{off}$ , respectively. When the gate is on, the customers are allowed to enter the queue, otherwise they are lost including the external arrivals as shown in Figure 1.

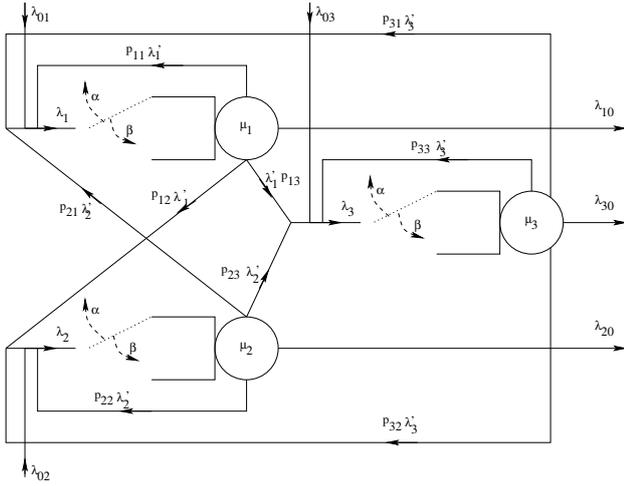


Figure 1: 3-node OQN with gated nodes

Let  $\lambda'_j$  denote the mean arrival rate of the interrupted arrival process, due to the presence of on-off gate, at node  $j$ . The total arrival rate  $\lambda_j$  at node  $j$  due to external and internal traffic flows at node  $j$  is given by  $\lambda_j = \lambda_{0j} + \sum_{i=1}^M p_{ji} \lambda'_i$ .

**THEOREM 1.** *The mean arrival rate  $\lambda'_j = p_{on} \lambda_j$  and SCV  $c_{a_j}^{\prime 2}$  of the interrupted arrival process at node  $j$ , due to presence of the on-off gate, is given by*

$$c_{a_j}^{\prime 2} = c_{a_j}^2 + k \lambda_j, \quad (10)$$

where  $p_{on}$  is the probability that the gate of node  $j$  is on and is given by

$$p_{on} = \frac{\beta}{\alpha + \beta}, \quad \text{and} \quad (11)$$

$$k = \frac{\alpha(v_{on}\alpha^2 + v_{off}\beta^2)}{(\alpha + \beta)^2}. \quad (12)$$

**Proof:** In [3], the author has analyzed switched general process (SGP), denoted by  $A$ , wherein the arrivals to the queuing system switches between two general renewal processes  $A_1$  and  $A_2$  with rates  $\lambda_1$  and  $\lambda_2$  according to a general renewal switching (on-off) periods  $V_1$  and  $V_2$  with rate  $\alpha$  and  $\beta$ , respectively. The author has derived the effective mean

arrival rate  $\lambda'$  and SCV  $C(A)$  of  $A$  as

$$\begin{aligned} \lambda' &= \frac{\lambda_1 E[V_1] + \lambda_2 E[V_2]}{E[V_1] + E[V_2]}, \\ C(A) &= \frac{\lambda_1 C(A_1) E[V_1]}{\lambda' (E[V_1] + E[V_2])} + \frac{\lambda_2 C(A_2) E[V_2]}{\lambda' (E[V_1] + E[V_2])} \\ &\quad + \frac{(\lambda_1 - \lambda_2)^2 [E[V_1]^2 Var[V_2] + E[V_2]^2 Var[V_1]]}{\lambda' (E[V_1] + E[V_2])^3}, \end{aligned}$$

where  $E[V_i]$  and  $Var[V_i]$  are the mean and variance of  $V_i$ ,  $i = 1, 2$ , respectively.

Since in our network, arrivals are not allowed to enter a node when its gate is closed, we have  $\lambda_2 = 0 = C(A_2)$ . Setting  $\lambda_1 = \lambda$ ,  $E[V_1] = 1/\alpha$ ,  $E[V_2] = 1/\beta$ ,  $Var[V_1] = v_{on}$  and  $Var[V_2] = v_{off}$ , we get

$$\lambda' = \frac{\beta}{\alpha + \beta} \lambda, \quad (13)$$

$$C(A) = C(A_1) + \lambda \frac{\alpha(v_{on}\alpha^2 + v_{off}\beta^2)}{(\alpha + \beta)^2}. \quad (14)$$

Since  $\lambda_j$  is the total arrival rate at node  $j$  of the queuing network with gated nodes, the mean arrival rate  $\lambda'_j$  and SCV  $c_{a_j}^{\prime 2}$  of the effective arrival process at this node  $j$  are obtained by replacing  $\lambda'$  by  $\lambda'_j$ ,  $\lambda$  by  $\lambda_j$ ,  $C(A)$  by  $c_{a_j}^{\prime 2}$  and  $C(A_1)$  by  $c_{a_j}^2$  in (13) and (14). Hence the theorem. Q.E.D

The utilization at node  $j$  is given by  $\rho_j = \lambda'_j / \mu_j$  and the arrival rate from node  $i$  to node  $j$  is given by  $\lambda'_{ij} = \lambda'_i p_{ij}$ . The proportion of arrivals from node  $i$  to node  $j$  is given by  $q_{ij} = \lambda'_{ij} / \lambda_j$ ,  $i \geq 0$ . Customers from node  $j$  leave the network with rate

$$\lambda_{j0} = \left(1 - \sum_{i=1}^M p_{ji}\right) \lambda'_j, \quad j = 1, 2, \dots, M. \quad (15)$$

Substituting (15) in (2) and in (3), the throughput  $\mathcal{T}$  and customer loss probability  $P_L(N)$  can be calculated, respectively.

The  $c_{a_j}^2$  in (10) is computed as follows: For  $j = 1, 2, \dots, M$ ,

$$c_{a_j}^2 = a_j + \sum_{i=1}^M b_{ij} c_{a_i}^2 + k \sum_{i=1}^M b_{ij} \lambda_i, \quad (16)$$

where  $k$  is given by (12). By replacing  $\lambda_j$  by  $\lambda'_j$  and  $c_{a_j}^2$  by  $c_{a_j}^{\prime 2}$ , we can use the QNA method (modified) presented in section 3 to compute the  $W_{qj}$ ,  $j = 1, 2, \dots, M$  of this gated OQN.

#### 5. OQN WITH INTERMITTENT LINKS

We consider an OQN as discussed in section 2 with the following modifications: the link connecting between any two nodes goes on and off with rates  $\alpha$  and  $\beta$  with variances  $v_{on}$  and  $v_{off}$ , respectively. When the link is on between nodes  $i$  and  $j$ , the customers departing node  $i$  are allowed to enter the queue of node  $j$  with probability  $p_{ij}$  as shown in Figure 2. In the model discussed in previous section, when the gate of a node is off, then the node is disconnected from the entire network since no other node can send customers to this node. This is, however, an unrealistic assumption model as for as its application to MWSNs are concerned because in MWSNs, a link between two nodes may be down, but these nodes may still be connected to other nodes in the network.

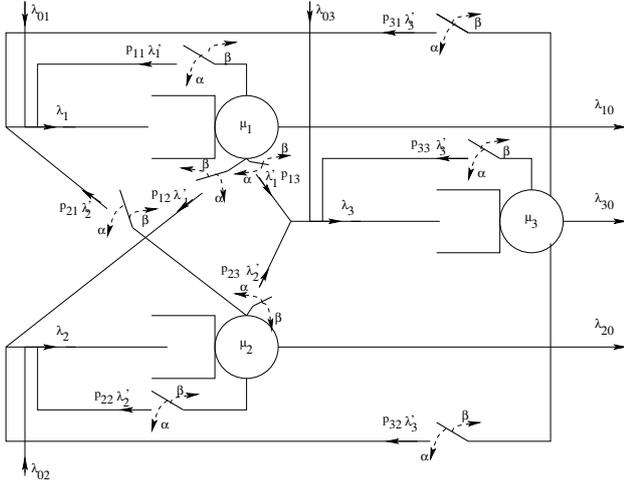


Figure 2: 3-node OQN with intermittent links

**THEOREM 2.** The mean arrival rate  $\lambda_j$  and SCV  $c_{aj}'^2$  of the interrupted arrival process, due to presence of the on-off links, at node  $j$  of the network is given by

$$\lambda_j = \lambda_{0j} + p_{on} \sum_{i=1}^M \lambda_{ij}, \quad (17)$$

$$c_{aj}'^2 = 1 - w_j \left( 1 - \sum_{i=1}^M p_{ij} c_{ij}^2 - k \sum_{i=1}^M p_{ij} \lambda_{ij} \right), \quad (18)$$

where  $p_{on}$  is the probability that the link between two nodes is on and is given by (11) and  $w_j$  is the same as the one given in (6) and  $k$  is given by (12).

**Proof:** Due to the presence of on-off links between nodes  $i$  and  $j$ , the mean arrival rate  $\lambda'_{ij}$  between these nodes is obtained by simply replacing  $\lambda$  by  $\lambda_{ij}$  and  $\lambda'$  by  $\lambda'_{ij}$  in (14). That is,

$$\lambda'_{ij} = p_{on} \lambda_{ij}. \quad (19)$$

Similarly, the SCV  $c_{ij}'^2$  of the traffic flow between node  $i$  and node  $j$  is obtained by replacing  $C(A)$  by  $c_{ij}'^2$ ,  $C(A_1)$  by  $c_{ij}^2$  and  $\lambda$  by  $\lambda_{ij}$  in (14). That is,

$$c_{ij}'^2 = c_{ij}^2 + k \lambda_{ij}, \quad (20)$$

where  $k$  is given by (12).

The total arrival rate at node  $j$  is given by  $\lambda_j = \lambda_{0j} + \sum_{i=1}^M \lambda'_{ij}$ . On substituting for  $\lambda'_{ij}$  from (19) in this equation, we get (17).

The SCV  $c_{aj}'^2$  of the total traffic flow to node  $j$  is obtained by replacing  $c_{ij}^2$  by  $c_{ij}'^2$  in (7). That is,

$$c_{aj}'^2 = 1 - w_j + w_j \sum_{i=1}^M p_{ij} c_{ij}'^2, \quad (21)$$

On substituting (20) in (21) yields (18). Hence the theorem. Q.E.D.

Using (17), (2) and (3), we can compute the  $\mathcal{T}$  and  $PL$ . Using (18) and (9) we can compute  $W_{aj}$  at node  $j$  for  $j = 1, 2, \dots, M$ .

## 6. OQN WITH INTERMITTENT SERVERS

We consider an OQN as discussed in section 2 with the following modifications: the server in each node goes on and off with rates  $\alpha$  and  $\beta$ , respectively. When the server of node  $j$  is on, the customers are served at the rate  $\mu_j$  and the server is off, the customers will wait in the queue until the server becomes on. A 3-node network with intermittent servers is shown in Figure 3.

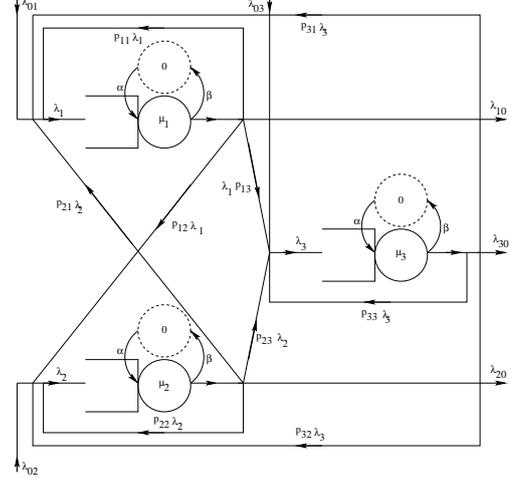


Figure 3: 3-node OQN with intermittent servers

**THEOREM 3.** For the network under investigation, the mean service rate  $\mu'_j = p_{on} \mu_j$  and the SCV of the effective service time distribution is given by

$$c_{sj}'^2 = c_{sj}^2 + k \mu_j, \quad (22)$$

where  $p_{on}$  is the probability the server at node  $j$  goes on and is given by (11) and the constant  $k$  is given by  $k = \alpha p_{on} \frac{(1+c_{off}^2)}{\beta^2}$  with  $c_{off}^2$  being the SCV of the server off distribution.

**Proof:** This model falls under the category of the unreliable server models. In [1, section 10.2.2], the authors have given the first two moments of the effective service time distribution for an unreliable server with general service and general up and down time distributions as follows<sup>1</sup>:

$$E[G] = E[B](1 + \eta E[D]) \quad (23)$$

$$E[G^2] = E[B^2](1 + \eta E[D])^2 + E[B]\eta E[D^2], \quad (24)$$

where  $G$  is the random variable denoting the effective service time of an interrupted server whose down time is given by the random variable  $D$ .  $E[B]$  and  $E[B^2]$  are the first and second moments of the processing time of this unreliable server when it is up and  $\eta$  is its expected up time. Though the second moment result is proved with the assumption that the arrivals follow Poisson process, it can be proved that it holds even for general arrival process when the arrivals are renewal processes. Dividing (24) by square of (23) yields

$$c_s'^2 = c_s^2 + \frac{\eta(1 + c_{off}^2)}{E[B]E[D]^2(1 + \eta E[D])^2}, \quad (25)$$

<sup>1</sup>Reproduced here for ease of reading and completeness.

where  $c_s'^2$ ,  $c_s^2$  and  $c_{off}^2$  are the SCVs of the effective service time, processing time and server off time distributions, respectively.

For node  $j$  of the queuing network under investigation we have  $E[B] = 1/\mu_j$ ,  $E[D] = 1/\beta$ ,  $E[G] = 1/\mu_j'$ ,  $\eta = \alpha$ ,  $c_s'^2 = c_{sj}^2$ ,  $c_s^2 = c_{sj}^2$ . Hence the theorem is proved on substituting these in (23) and (25). Hence the theorem. Q.E.D.

Since there are no changes in the arrival process and its moments, the inflow of customers is equal to the outflow as we do not lose any customers and hence  $P_L = 0$ . We can get other performance measures by simply replacing  $\mu$  by  $\mu'$ ,  $c_s^2$  by  $c_{sj}^2$  in the standard QNA.

## 7. NUMERICAL RESULTS

To verify the analytical results, we simulated these three types of queuing network models. The stopping criterion for the simulation guaranteed a maximum relative error of 5%. The relative error in the simulation was computed from the associated confidence interval, which was obtained through the usual normal distribution approximation.

We used OMNeT++ [12], a discrete-event simulation package to perform all the simulations. In the simulation, the following inputs were given depending on the type of networks. For all the three types of networks: External arrival distributions and the corresponding rates  $\lambda_{0j}$ ,  $j = 1, 2, \dots, M$ . Service time distributions and the corresponding rates  $\mu_j$ ,  $j = 1, 2, \dots, M$ . For the gated nodes network: On-off distributions of  $M$  gates and the corresponding rates  $\alpha$  and  $\beta$ . For the intermittent links network: On-off distributions of links and the corresponding rates  $\alpha$  and  $\beta$ . For the intermittent server network: On-off distributions of servers and the corresponding rates  $\alpha$  and  $\beta$ .

Node $j$	$1/\mu_j$	$\lambda_{0j}$	$W_s$		Error
			QNA	Simulation	
1	0.0400	2.0	0.0711	0.0682	4.0925
2	0.0400	2.0	0.1422	0.1365	4.0127
3	0.0400	2.0	0.1196	0.1137	4.9038
4	0.0400	2.0	0.2915	0.2965	1.7307
5	0.0400	2.0	0.1030	0.0979	4.9087
6	0.0400	2.0	0.2248	0.2219	1.2865
7	0.0400	2.0	0.0756	0.0723	4.3541
8	0.0400	2.0	0.1025	0.0996	2.8141
9	0.0400	2.0	0.0722	0.0690	4.3974
10	0.0400	2.0	0.0648	0.0622	3.9498

Table 1: Network with 10 gated nodes

We did not use any other information like SCV and distribution of internal arrivals. In each simulation we computed the following statistics: mean and standard deviation of inter-arrival times,  $W_{sj}$  for each node, the customer loss probability and  $W_s$  of the network.

In Tables 1 - 3, we have compared both analytical and simulation results for queuing networks with specific distributions for on-off durations of gates, links and servers and service time of servers. In all the examples, the external arrivals to the networks are assumed to follow Poisson processes.

In Table 1, we considered a network with 10 gated nodes, deterministic service, and Rayleigh on and off times with rates  $\alpha = 0.7181$ ,  $\beta = 0.0319$ . In Table 2, we considered a network with 10 nodes connected by intermittent links, exponential service, and exponential on and off times with rates  $\alpha = 10/9$ ,  $\beta = 25$ . In Table 3, we considered a net-

work with 10 intermittent server nodes, deterministic service, Rayleigh on and off times with rates  $\alpha = 0.7181$ ,  $\beta = 0.0319$ .

Node $j$	$1/\mu_j$	$\lambda_{0j}$	$W_s$		Error
			QNA	Simulation	
1	0.0400	2.0	0.0724	0.0719	0.7026
2	0.0400	2.0	0.1029	0.1026	0.3236
3	0.0400	2.0	0.0962	0.0950	1.2475
4	0.0400	2.0	0.1222	1.1207	1.2520
5	0.0400	2.0	0.0902	0.0885	1.8765
6	0.0400	2.0	0.1173	0.1159	1.2080
7	0.0400	2.0	0.0750	0.0746	0.4724
8	0.0400	2.0	0.0896	0.0881	1.6272
9	0.0400	2.0	0.0781	0.0762	2.3851
10	0.0400	2.0	0.0679	0.0675	0.5775

Table 2: 10-node Network with intermittent links

The simulation values match with the analytical values given by QNA with very less relative error. In most of the cases, the relative error (last columns in all tables) is approximately 1%.

Node $j$	$1/\mu_j$	$\lambda_{0j}$	$W_s$		Error
			QNA	Simulation	
1	0.0511	2.0	0.0504	0.0485	5.0933
2	0.1415	2.0	0.1423	0.1544	9.0869
3	0.1036	2.0	0.1043	0.1047	1.0618
4	0.0653	2.0	0.6283	0.0626	4.1172
5	0.0832	2.0	0.0833	0.0812	2.3861
6	0.4423	2.0	0.4494	0.4051	8.4119
7	0.0546	2.0	0.0544	0.0524	4.0811
8	0.0823	2.0	0.0825	0.0808	1.7634
9	0.0516	2.0	0.0514	0.0491	4.8271
10	0.0450	2.0	0.0449	0.0432	3.8929

Table 3: Network with intermittent server nodes

## 8. CONCLUSION

The research work presented in this paper provides a queueing-theoretic framework to model and analyze MWSNs by means of a mechanism that leverages the power of QNA. While QNA is an approximation technique, the results show that it results in very less error; and its use is highlighted by its applicability to model and analyze the vagaries of complex real-world networks. The paper is an attempt to analytically address this emerging need in this area; and hence we have made some realistic assumptions. Our current research is directed towards relaxing some of these assumptions, finding various other parameters - such as incorporating finite buffers, varying battery levels of a node and prevailing weather conditions; and identifying appropriate statistical distributions for such parameters.

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