Tuna Can Sensitivity Analysis – Runs Design

The design of the computational runs needed to perform the sensitivity analysis on the Tuna Can Case has been carried out according to Pavelic and Saxena (1969). They propose a factorial design approach which is very useful when the results of a computation are affected by a large number of variables and, then, studying the effect of each variable at a time would require too many, expensive and time consuming, runs. According to the factorial design approach a fixed number of levels for each variable is chosen and computations are run at all possible combinations of those levels.

In our case the attention is focused on the sensitivity of computations on four main variables: pool fire diameter, D_{PF}, wind speed, v_W, container position in the fire, P[1], and regression rate, RR. For D_{PF}, v_W and RR, we chose two levels of variation, denoted by subscripts min and max, while P was determined both on an absolute and relative (to D_{PF}) basis, i.e. subscripts min, min_rel, max and max_rel respectively. Then we have:

- Pool fire diameter: $D_{PF,\,min}$ = 0.5 m and $D_{PF,\,max}$ = 1 m;
- Wind speed: $v_{W,\,min}$ = 0 m/s and $v_{W,\,max}$ = 4 m/s;
- Position: P_{min} = (0, 1), P_{min_rel} = (0, D_{PF}), P_{max} = (R_{PF}, 0.5) and P_{max_rel} = (R_{PF}, 0.5 D_{PF}).
- Fuel evaporation rate, RR: RR_{min} = 1.6 mm/min and RR_{max} = 6.4 mm/min

The total number of runs required to investigate the effects of all the variables simultaneously would be equal to $\prod_i n_{l_i}^{\mathrm{var}_i}$, where n_{l_i} is the number of levels chosen for each set i of variables, var_i. In our case n_{l_i} equals 2 for D_{PF}, v_W and M_F ($\mathrm{var}_i = 2$) and 4 for P ($\mathrm{var}_i = 1$). Then, the required tests would be $2^3 \cdot 4^1 = 32$. However, being the maximum diameter, D_{PF}, equal to 1 m, the levels (P_{min}, P_{min_rel}) and (P_{max}, P_{max_rel}) are exactly the same, thus reducing the total number of runs to 28.

To reduce the number of runs further, the original runs has been divided into 3 metrics, thus allowing to analyze the effects of 3 variables, varying between two levels, at a time.

A first subsystem is chosen by keeping the regression rate constant (RR = 1.6 mm/min). Then the effect of D_{PF}, P (relative basis) and v_W can been investigated using the metric showed in Table 1:

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[1] To reduce the number of variables and, then, the number of required runs, the position of the container in the fire can be described by a point P defined by a radius, R, and a height, H. The position can be varied only along the direction identified by P_{min}= (R_{min}, H_{min}) and P_{max} = (R_{max}, H_{max}).

Test No.	D_{PF}	v_W	P
13	0.5	0	$P_{min,rel}$
14	0.5	0	P_{max_rel}
3	1	0	P_{min_rel}
9	1	0	$P_{max,rel}$
15	0.5	4	P_{min_rel}
16	0.5	4	$P_{max,rell}$
6	1	4	$P_{min,rel}$
12	1	4	P_{max_rel}

Table 1 – Metric I for the investigation of the effect of D_{PF}, P (relative basis) and v_W on the results. RR = 1.6 mm/min.

Then, keeping the diameter constant (D_{PF} = 0.5 m), the effect of P (relative basis), v_W and RR can be addressed with the metric showed in Table 2:

Test No.	v_W	P	RR
13	0	$P_{min,rel}$	1.6
14	0	$P_{max,rel}$	1.6
15	4	$P_{min,rel}$	1.6
16	4	Pmax,rel	1.6
2	0	$P_{min,rel}$	6.4
8	0	$P_{max,rel}$	6.4
5	4	$P_{min,rel}$	6.4
11	4	$P_{max,rel}$	6.4

Table 2 - Metric II for the investigation of the effect of P (relative basis), v_W and RR on the results. D_{PF} = 0.5 m.

Finally, keeping constant the diameter (D_{PF} = 0.5 m) and the regression rate (RR = 6.4 mm/min), it's possible to study the effect of P (absolute and relative basis) and v_W on the results (Table 3):

Test No.	v_W	P
1	0	P_{min}
2	0	$P_{min,rel}$
7	0	P_{max}
8	0	$P_{max,rel}$
4	4	P_{min}
5	4	$P_{min,rel}$
10	4	P_{max}
11	4	$P_{max,rel}$

Table 3 - Metric III for the investigation of the effect of P (absolute and relative basis) and v_W on the results. D_{PF} = 0.5 m and RR = 6.4 mm/min.

Given this partition, the total number of runs required reduces to 16. A complete summary of the runs is reported in Table 4.

Test No.	D_{PF}	v_W	P	RR	HF	MV	Time	Location
1	0.5	0	P_{min}	6.4	Yes	yes	5.86*	/scratch/da/alessand/Dmin_Pmin
2	0.5	0	P_{min_rel}	6.4	No	no	1.76**	/scratch/da/alessand/Dmin_Pmin_rel_new
3	1	0	P_{min_rel}	1.6	Yes	yes	?	ALC
4	0.5	4	P_{min}	6.4	Yes	no	6	/scratch/serial-old/alessand/Dmin_Pmin_Wmax
5	0.5	4	P_{min_rel}	6.4	yes	no	7.5	/scratch/serial-old/alessand/Dmin_Pmin_rel_Wmax
6	1	4	$P_{min,rel}$	1.6	no	no	< 1	ALC
7	0.5	0	P_{max}	6.4	Yes	yes	7.47*	/scratch/da/alessand/Dmin_Pmax
8	0.5	0	P_{max_rel}	6.4	Yes	no	5.11*	/scratch/da/alessand/Dmin_Pmax_rel
9	1	0	$P_{max,rel}$	1.6	Yes	no	6.34	ALC
10	0.5	4	P_{max}	6.4	Yes	no	7.4	/scratch/serial-old/alessand/Dmin_Pmax_Wmax
11	0.5	4	P_{max_rel}	6.4	No	no	5.4	/scratch/serial-old/alessand/Dmin_Pmax_rel_Wmax
12	1	4	P_{max_rel}	1.6	No	no	~	ALC
13	0.5	0	$P_{min,rel}$	1.6	No	no	~	ARCHES CLUSTER
14	0.5	0	P_{max_rel}	1.6	No	no	~	ARCHES CLUSTER
15	0.5	4	P_{min_rel}	1.6	No	no	~	ARCHES CLUSTER
16	0.5	4	$P_{max,rel}$	1.6	No	no	~	ARCHES CLUSTER

Table 4 – Factorial design for Tuna Can Sensitivity Analysis. HF = Heat Flux, MV = Movie, *After this time instabilities were observed, ** Domain extended to prevent instabilities.

To simplify writing all the possible combinations of the variables levels we can use a coding system so that the max and min conditions for each variable can be represented by +1 and -1 respectively (the max_rel and min_rel cases will be denoted by +1_rel and -1_rel). If x_1 represents the coded value of the pool fire diameter, D_{PF}, the relation between z_1 and D_{PF} is given by:

$$x_1 = \frac{D_{PF} - \langle D_{PF} \rangle}{\frac{D_{PF,\max} - D_{PF,\min}}{2}} = \frac{(D_{PF} - 0.75)}{0.25}$$

which equals +1 if $D_{PF} = D_{PF,max}$, -1 if $D_{PF} = D_{PF,min}$ and 0 if $D_{PF} = \langle D_{PF} \rangle$. We can proceed similarly for the other variables. Then x_2, x_3 and x_4 will represent the coded values of v_W, P ad RR, respectively. The metrics expressed in terms of coded values are reported in Table 5 - Table 7.

Test No.	x_1	x_2	x_3
13	-1	-1	-1_rel
14	-1	-1	+1_rel
3	+1	-1	-1_rel
9	+1	-1	+1_rel
15	-1	+1	-1_rel
16	-1	+1	+1_rel
6	+1	+1	-1_rel
12	+1	+1	+1_rel

Table 5 – Metric I in terms of coded values.

Test No.	x_2	x_3	x_4
13	-1	-1_rel	-1
14	-1	+1_rel	-1
15	+1	-1_rel	-1
16	+1	+1_rel	-1
2	-1	-1_rel	+1
8	-1	+1_rel	+1
5	+1	-1_rel	+1
11	+1	+1_rel	+1

Table 6 – Metric II in terms of coded values.

Test No.	x_2	x_3
1	-1	-1
2	-1	-1_rel
7	-1	+1
8	-1	+1_rel
4	+1	-1
5	+1	-1_rel
10	+1	+1
11	+1	+1_rel

Table 7 – Metric III in terms of coded values.

For each of the three metrics, a graphical representation can be provided. If we consider our coded variables as mutually perpendicular coordinate axes, each factorial deign can be represented by a cube (Figure 1). The eight corner points of the cube represent the eight test conditions listed in coded values in Table 5, Table 6 and Table 7. For metric III, the cubic representation is obtained considering the relative and absolute position as different variables. The center of the cube represents physically the midvalue conditions for the three variables of interest.

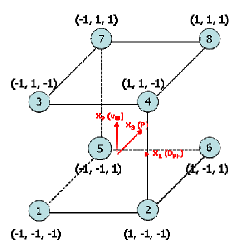

Figure 1 - Geometrical representation of factorial design.

References

Pavelic V. and Saxena U., "Statistical Experiment Design", *Chemical Engineering*, 6, 175-180, (1969)