# CS 7690, Advanced Image Processing Project2 Anisotropic Diffusion Xiang Hao

### Implement the PDE of the anisotropic diffusion

For one dimension: The PDE of the anisotropic diffusion is  $I_t = d(c(x, t) I_x)/dx$ 

 $\begin{array}{ll} d(c(x,t)I_x/dx = (c_r * I_r - c_l * I_l)/\Delta x & .....(1) \\ c_r * I_r - c_l * I_l = c_r * (I(x + \Delta x) - I(x))/\Delta x - c_l * (I(x) - I(x - \Delta x))/\Delta x & .....(2) \\ From (1) and (2), we can get: \\ d(c(x,t)I_x/dx = (1/\Delta x)^2 * (c_r * (I(x + \Delta x) - I(x)) - c_l * (I(x) - I(x - \Delta x))) \\ \end{array}$ 

Since  $I_t = (I(t+\Delta t) - I(t))/\Delta t$ , so  $I(t+\Delta t) = I(t) + \Delta t((1/\Delta x)^2 * (c_r * (I(x + \Delta x) - I(x)) - c_1 * (I(x) - I(x-\Delta x))))$  .....(a)

From the above equation, we know that, we can compute I(t) by an iteration. At t = 0, I(0) = I, I is the original image. At  $t = 0 + \Delta t$ , we compute I(t +  $\Delta t$ ) by using the above equation.

We do the above step over and over again, the number of the iteration is decided by the user. At each step, before we compute  $I(t + \Delta t)$ , we need to compute c and the gradient at each point.

For two dimension: The equation (a) becomes:  $I(t+\Delta t) = I(t) + \Delta t((1/\Delta x)^2 * (c_x_r * (I(x + \Delta x) - I(x)) - c_x_l * (I(x) - I(x-\Delta x))) + (1/\Delta y)^2 * (c_y_r * (I(y + \Delta y) - I(y)) - c_y_l * (I(y) - I(y-\Delta y))))$ 

We compute I(t) in the same way as the was we compute I(t) in one dimension.

### Description of the conductivity function

Usually, the conductivity function is exp-( $|I_x|/K$ )<sup>2</sup> or  $1/(1 + |I_x|^2/K)$ .

If we treat I\_x as a variable, K as a constant:

Both of the functions are monotone decreasing, which means:

If a region of a image has a higher gradient, it will suffer a smaller diffusion. On there other hand, if the region has a lower gradient, it will suffer a higher diffusion.

So, use these conductivity function will preserve the features with high gradients.

In a different view point, if we treat K as a variable, the diffusion degree of each pixel will increase as the increase of K.

Here I choose the function  $1/(1 + |I_x|^2/K)$  since it is much faster than the first one.

# Application

# The diffusion image:

| Kappa = 20 | Iteration1   | Iteration5  | Iteration10  | Iteration15  | Iteration20  |
|------------|--|---|--|--|--|
| Noisy MRI  |  |   |  |  |  |
| MRI        |  |   |  |  |  |
| Text       | De when you see<br>De hipsters on the<br>Histreet yell "HIP.<br>STER!!" and they'll<br>turn around and<br>then yet real and<br>that they acknow-<br>ledged that they<br>are hipsters | By when you see<br>By hipsters on the<br>Street yell "HIP.<br>STER!!" and they'll<br>turn around and<br>then get real and<br>that they acharow-<br>ledged that they<br>are hipsters | By the you see<br>By hipsters on the<br>STERIL" and the III<br>turn around and<br>then get real and<br>that they acknow-<br>ledged that they<br>are hipsters | By when you see<br>By hipsters on the<br>street yell "HIP-<br>STER!!" and they'll<br>turn around they'll<br>then get real and<br>that they acknow-<br>ledged that they<br>are hipsters | By when you see<br>by hipsfirs on the<br>street yell "HIP<br>STER!!" and they'll<br>turn around and<br>then get real and<br>that they acknow-<br>ledged that they<br>are hips tors |

From the above pictures, we can see in the anisotropic diffusion, the edges are preserved and the flat area and noises are blurred.

| Kappa = 20 | Iteration1  | Iteration5  | Iteration10  | Iteration15   | Iteration20   |
|------------|---|---|--|---|---|
| Noisy MRI  |   |   |  |   |   |
| MRI        |   |   |  |   |   |
| Text       | Dis when you see<br>topsing on the<br>street yell "HIP.<br>STER!!" and Haj!!!<br>four yet real and<br>that they achow<br>hat they achow<br>lodged that they<br>are hips krs | Republic you bee<br>Republic yell "HIP.<br>Street yell "HIP.<br>Street" and the<br>Her yet rail and<br>then yet rail and<br>the yet rail and<br>the they achow-<br>ladged that they<br>are hips k-s | Republic gov bee<br>Republic on the<br>streat yell "MIP.<br>Streat" and the<br>then yet real and<br>then yet real and<br>thet they about<br>lodged that they<br>are hips k-5 | Republic gov bee<br>Republic on the<br>streat gell "MIP.<br>Streat" and the<br>then yet real and<br>then yet real and<br>thet they achoever<br>lodged that they<br>are hips k-5 | Dispetter gov bee<br>provide gell "Aller<br>Streat gell "Aller<br>Streat gell "Aller<br>Storn around end<br>then get reel and<br>then get reel and<br>thet they achow-<br>lodged their they<br>are hips k-s |

From the gradient images, we can see the area with high gradients will not change two much. The area with low gradients will become darker and darker. This also explains why the anisotropic diffusion can preserve the edges.

|                   | Iteration1 | Iteration5 | Iteration10 | Iteration15 | Iteration20 |
|-------------------|------------|------------|-------------|-------------|-------------|
| Kappa = 20        |            |            |             |             |             |
| Kappa = 35        |            |            |             |             |             |
| Kappa = 50        |            |            |             |             |             |
| Kappa =300        |            |            |             |             |             |
| Kappa = 3000      |            |            |             |             |             |
| Kappa =<br>300000 |            |            |             |             |             |

## Performance under different k-values

Compare the above two results:

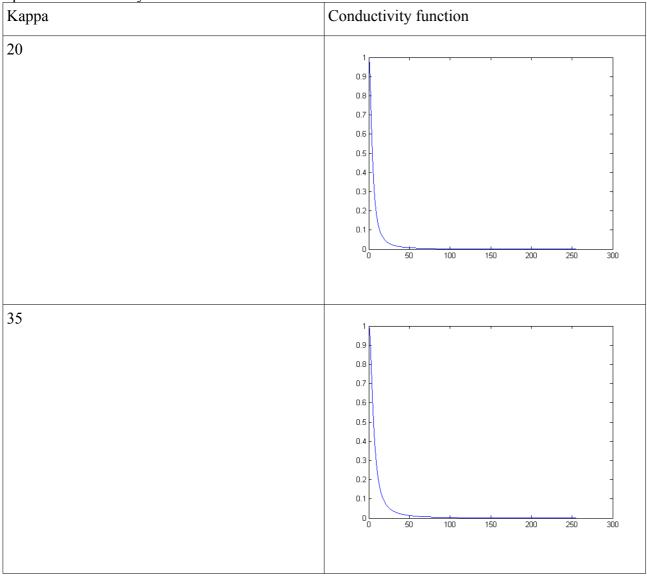
When Kappa = 30, we preserve more edges since the larger the gradient is, the less diffusion it will suffer.

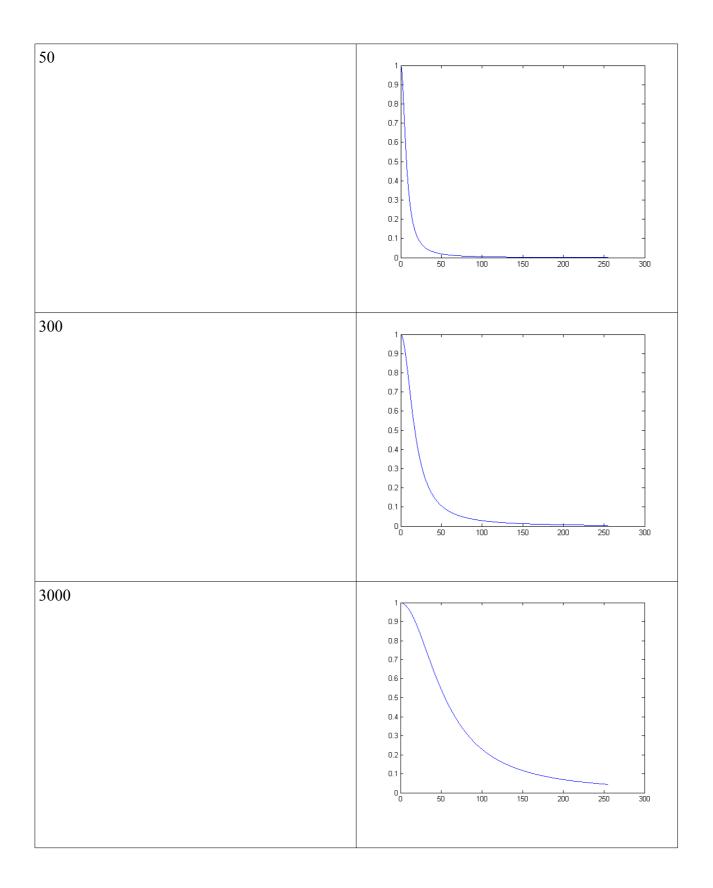
When Kappa is very large, the whole image suffer the similar diffusion, so it is similar a linear diffusion in this case.

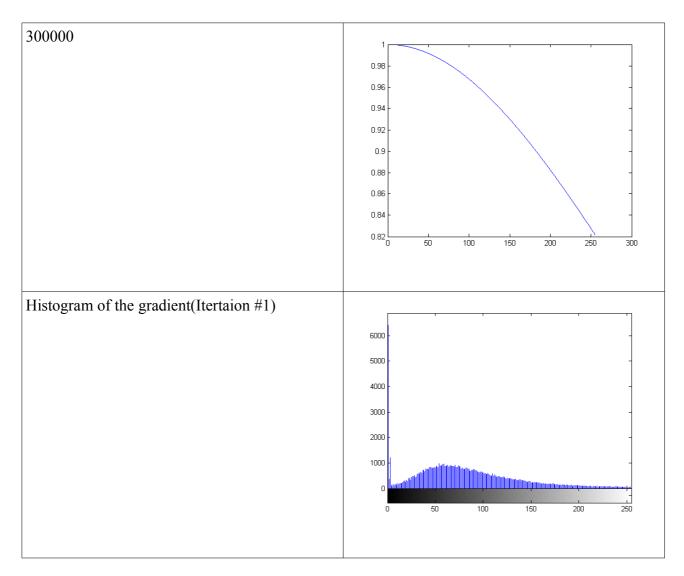
In addition, the images, especially the CSF boundary, do not change too much from kappa = 300 to kappa = 300000. Even form kappa = 50 to kappa = 300, the changes are not very significant. The reasons are:

- 1) The diffusion degree is increasing as the increases of the kappa.
- 2) As kappa is increasing. For some features(For example, the csf edges in the above pictures), before kappa goes to certain value(for the edges of the above picture, I guess the value is around 25), the features will have few diffusion, since their gradients are large enough. But, as the kappa increases larger and larger, the features' gradients are not big enough to get few diffusion, so these features will be blurred.

I plot the conductivity functions here:







From the above pictures,

1)We can see that when kappa = 300000, the range of the y axis(the amount of the diffusion) is  $0.82\sim1$ , when the gradient is less than 255. That's why the anisotropic diffusion is similar with isotropic diffusion when kappa is very large

2)From the histogram of the gradients, we know that most of the gradients are less than 100. When kappa is larger 300, the  $c(I_x > 100)$  we get from the conductivity function is become larger and larger, which means most of the image will be blurred. This also explains why the images does not change too much from kappa = 300 to kappa = 300000.

3)Base on the histogram of the gradients, we can come up a way to automatically choose the kappa parameter.

## Performance under different iteration numbers

| Kappa = 20 | Iteration1 | Iteration5 | Iteration10 | Iteration15 | Iteration20 |
|------------|------------|------------|-------------|-------------|-------------|
| Noisy MRI  |            |            |             |             |             |
| Gradient   |            |            |             |             |             |

In the Noisy MRI images, as the number of iteration increases, the boundary does not change too much, but the noise is blurred as the number of iteration increases.

In the gradient images, the boundary, which is the very bright, does not change too much either. The dark areas become darker and darker.

So, when kappa is equal to 20, as the number of iteration increases, the edges(boundaries) will be preserved and the noises and flat areas will be blurred.

However, I am wondering if the edges will also be blurred when the number of the iteration goes to infinite. So I increased the maximum of the number of the iteration from 20 to 200.

| Kappa = 20 | Iteration50 | Iteration80 | Iteration120 | Iteration150 | Iteration200 |
|------------|-------------|-------------|--------------|--------------|--------------|
| Noisy MRI  |             |             |              |              |              |
| Gradient   |             |             |              |              |              |

From the above results, obviously everything is blurred as the number of iteration goes to a very larger number.

We can also guess that as the number of iteration goes to infinite, every pixel of the image will have the same intensity.

### **Discussion of preservation of details**

|             | Iteration1  | Iteration5  | Iteration10  | Iteration15   | Iteration20   |
|-------------|---|---|--|---|---|
| Kappa = 20  | Deputtion you see<br>Hispoters on the<br>Historet yell "HIP.<br>STER!!" and they'll<br>turn around and<br>then get real and<br>that they acknow-<br>ledged that they<br>are hips tors | Dis when you see<br>thisstory on the<br>History gell "HIP.<br>STER!!" and they'll<br>turn around and<br>then get real and<br>that they acknow-<br>ledged that they<br>are hips tors     | By when you see<br>by hipsters on the<br>street yell "HIP.<br>STER!!" and they'll<br>turn around and<br>then yet real and<br>that they acknow-<br>ledged that they<br>are hipsters | Dis when you ice<br>this stores on the<br>History gell "HIP.<br>STER!!" and they'll<br>turn around and<br>then get real and<br>that they acknow-<br>ledged that they<br>are hips tors | Discrete years and the<br>street year and the<br>street year and the<br>steer and the<br>two actors and<br>the get real and<br>that they achors<br>ledged that they<br>are hips to s  |
| Kappa = 40  | Drewhen you see<br>Drewhen you see<br>Historet you "Hip.<br>STER!!" and they'll<br>turn around and<br>then get real and<br>that they acknow-<br>ledged that they<br>are hips ters     | De when you see<br>De hipstris on the<br>Historic yell "HIP.<br>STER!!" and they'll<br>turn around and<br>then get real and<br>that they acknow-<br>ledged that they<br>are hipstris    | By when you see<br>By hipstry on the<br>stream yell "HIP.<br>STER!!" and they'll<br>turn around and<br>then yet real and<br>that they acknow-<br>ledged that they<br>are hipstrs   | Reporter you see<br>Reporter you see<br>How and the<br>STERN and they'll<br>turn around and<br>then get real and<br>that they achorour-<br>ledged that they<br>are hipsters           | Reported you see<br>Reported to the street of |
| Kappa = 80  | Dr when you see<br>Dr hipstors on the<br>Histreet yell "HIP.<br>STERI!" and they!!!<br>turn around and<br>then get real mad<br>that they acknow-<br>ledged that they<br>are hipstors  | Det when you see<br>this transfers on the<br>street yell "HIP.<br>STER!!" and they'll<br>turn around and<br>then yet real and<br>that they acknow-<br>ledged that they<br>are hips ters | De when you see<br>De hipsters an the<br>street yell "HIP.<br>STERIS" and they'll<br>turn around and<br>then yet real and<br>that they acknow-<br>ledged that they<br>are hipsters | Det when you see<br>Det hipstors on the<br>street yell "HIP.<br>STERIS" and Hay'll<br>torn around and<br>then yet real and<br>that they acknew-<br>ledged that they<br>are hipstors   | By the gov see<br>the power on the<br>street yell "HIP.<br>STER!!" and they'll<br>then yet real and<br>that they acknow-<br>ledged that they<br>are hips kers   |
| Kappa = 150 | Low when you see<br>Historet yell "HIP.<br>STER!!" and Heg'll<br>turn around and<br>then get real and<br>that they acknow-<br>ledged that they<br>are hips tors                       | Dis when you see<br>District yell "HIP.<br>STER!!" and they'll<br>there around and<br>then get real and<br>that they acknow-<br>ledged that they<br>are hips tors                       | By when you see<br>By hipsters on the<br>street yell "HIP.<br>STER!!" and they'll<br>then yet real and<br>that they acknow-<br>ledged that they<br>are hipsters                    | By the yer ice<br>the higher on the<br>street yell "HIP.<br>STER!!" and they'll<br>two normal and<br>then yet real and<br>that they achore-<br>ledged that they<br>are high thes      | Street yell "Hip.<br>STER!!" and Hop?!!<br>STER!!" and Hop?!!<br>two acrowd and<br>then get real and<br>that they acknow.<br>ledged short they<br>are hips tors   |

As the kappa increases, the overall diffusion is increasing. We may use this property to segment the region of the each word first by using large kappa and then segment the words into characters.

In order to segment the text into words, we can also use a lot of iteration.

The conductivity function I choose here is a monotone decreasing function. This kind of functions preserve pixels with high gradient and blur pixels with low gradient. It is kind of a "high pass filter".

Similarly, we can also construct some "low pass filter" and "band pass filter" if we need. For example, by band pass filter, I mean we can set the conductivity function to be a Gaussian(mu, sigma) function. In this case, the pixels with the gradient as mu will be preserved.