CS5960: Nonparametric Methods **Due Wed. 2/18**

Homework 2: Generating Random Variables

Instructions: Be sure to electronically submit your answers in pdf format for the written part and as an R file for the coding part. You may work together and discuss the problems with your classmates, but write up your final answers entirely on your own.

1 Written Part

1. Let X and Y be a real-valued random variables with $Y = g(X)$, where $g : \mathbb{R} \to \mathbb{R}$ is a monotonic increasing function. Find and prove a formula for the quantile function of Y in terms of F_X and g.

2. Let $X \sim \text{Exp}(\lambda)$, and let $Y = \sqrt{X}$.

   (a) What is the density function f_Y?

   (b) What is the distribution function F_Y? Verify that $F_Y(0) = 0$ and $F_Y(\infty) = 1$.

   (c) What is the quantile function F_Y^{-1}?

   (d) Compute the mean, μ_Y, and variance, σ_Y^2. Hint: Use integration by parts.

2 R Simulation

3. Write a function called `BoxMuller(n)` that generates n standard normal random numbers using the Box-Muller method. For simplicity you may assume n is even.

4. Write a function that uses the Inverse Transform Method to generate n random numbers from the distribution F_Y from Problem 2 above (λ should be a parameter to the function).

5. Write a function that uses the Acceptance-Rejection Method to generate n random numbers from a Beta(α, β) distribution (α, β should be parameters to the function).

6. For each of your functions in Problems 3-5 do the following:

   (a) Generate a vector of 10,000 random numbers. Use parameters $\lambda = 1$ and $\alpha = 5$, $\beta = 2$.

   (b) Plot a histogram of the generated data with the theoretical density function superimposed for comparison.

   (c) Plot a Q-Q plot of the generated data vs. the theoretical quantiles.

   (d) Compute the mean and variance of your generated data and compare it to the theoretical values.