

## CS6210 Homework Set 4 (Fall 2010)

### 1. Fixed Points

(50pt)

For the equation  $f(x) = x^2 - 3x + 2 = 0$  each of the following functions yields an equivalent fixed point problem:

(i)  $g_1(x) = (x^2 + 2)/3$  (ii)  $g_2(x) = \sqrt{3x - 2}$  (iii)  $g_3(x) = 3 - 2/x$  (iv)  $g_4(x) = (x^2 - 2)/(2x - 3)$ .

a) Analyze the convergence properties of the simple iteration  $x_{i+1} = g_j(x_i)$  schemes ( $j = 1, 2, 3, 4$ ) for the root  $x = 2$  by considering  $|g'_j(2)|$ .

b) Confirm the analysis by implementing each scheme and verifying its convergence (or lack thereof) and appropriate convergence rate.

c) Besides convergence rates, comment on any other interesting features (e.g. geometric) you observe about the convergence.

d) Organize your answers in compact form for easy access and easy comparisons.

### 2. Roots of a Polynomial

(50pt)

a) Express Chebyshev polynomial  $T_7(x)$  as a sum of powers of  $x$ .

b) Implement bisection, Newton and secant methods and test them by finding the smallest and largest roots of  $T_7(x)$ . For each method make clear the termination criterion used and why it was used.

c) Compare, discuss, and explain your results paying attention to the following considerations:

i) Convergence rates.

ii) Evaluating all polynomials recursively.

iii) Accuracy.