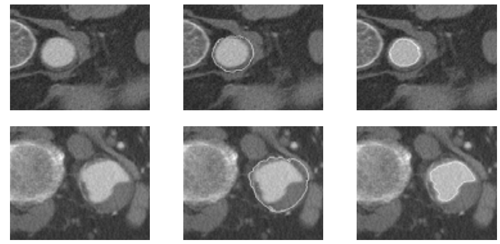
Introduction (1)

- The active contour model, or **snake**, is defined as an energy-minimizing spline.
- Active contours results from work of **Kass et.al.** in 1987.
- Active contour models may be used in image segmentation and understanding.
- The snake's energy depends on its shape and location within the image.
- Snakes can be closed or open

Introduction (2)

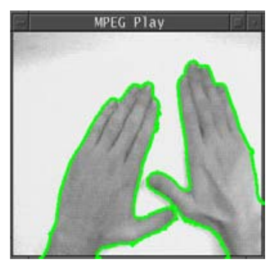


Aorta segmentation using active contours

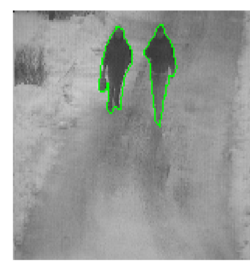
Introduction (3)

- First an initial spline (snake) is placed on the image, and then its energy is minimized.
- Local minima of this energy correspond to desired image properties.
- the snake is active, always minimizing its energy functional, therefore exhibiting dynamic behavior.
- Also suitable for analysis of dynamic data or 3D image data.

Examples (1)

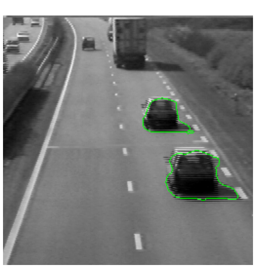


Hands

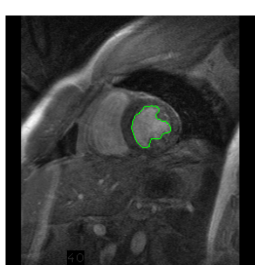


People

Examples (2)



Highway



Heart

Modeling

- The contour is defined in the (*x*, *y*) plane of an image as a parametric curve

  $\mathbf{v}(s)=(x(s), y(s))$

- Contour is said to **possess an energy** (***E**$_{snake}$*) which is defined as the sum of the three energy terms.

$$E_{snake} = E_{internal} + E_{external} + E_{constraint}$$

- The energy terms are defined so that the desired final position of the contour will have a minimum energy (E_{min})
- Therefore our problem of detecting objects reduces to an energy minimization problem.

What are these energy terms which do the trick for us??

Internal Energy (E_{int})

- Depends on the intrinsic properties of the curve.
- Sum of elastic energy and bending energy.

Elastic Energy ($E_{elastic}$):

- The curve is treated as an elastic rubber band possessing elastic potential energy.
- It discourages stretching by introducing tension.

$$E_{elastic} = \frac{1}{2}\int_s \alpha(s)\,|v_s|^2\,ds \qquad v_s = \frac{dv(s)}{ds}$$

- Weight $\alpha(s)$ allows us to control elastic energy along different parts of the contour. Considered to be constant α for many applications.
- Responsible for shrinking of the contour.

Bending Energy ($E_{bending}$):

- The snake is also considered to behave like a thin metal strip giving rise to bending energy.
- It is defined as sum of squared curvature of the contour.

$$E_{bending} = \frac{1}{2}\int_s \beta(s)\,|v_{ss}|^2\,ds$$

- **β(s)** plays a similar role to **α(s).**
- Bending energy is minimum for a circle – for a closed snake, or a line for an open one.
- Total internal energy of the snake can be defined as

$$E_{int} = E_{elastic} + E_{bending} = \int_s \frac{1}{2}(\alpha|v_s|^2 + \beta|v_{ss}|^2)ds$$

External energy of the contour (E_{ext})

- It is derived from the image.
- Define a function $E_{image}(x,y)$ so that **it takes on its smaller values at** the features of interest, such as **boundaries.**

$$E_{ext} = \int_s E_{image}(v(s))ds$$

Key rests on defining $E_{image}(x,y)$. Some examples

- $E_{image}(x,y) = -|\nabla I(x,y)|^2$
- $E_{image}(x,y) = -|\nabla(G_\sigma(x,y) * I(x,y))|^2$

Energy and force equations

- The problem at hand is to find a contour v(s) that minimize the energy functional

$$E_{snake} = \int_s \frac{1}{2}(\alpha(s)|v_s|^2 + \beta(s)|v_{ss}|^2) + E_{image}(v(s))ds$$

- Using variational calculus and by applying Euler-Lagrange differential equation we get following equation

$$\alpha v_{ss} - \beta v_{ssss} - \nabla E_{image} = 0$$

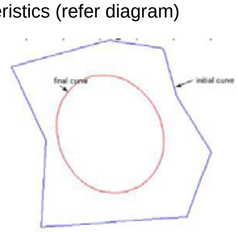
- Equation can be interpreted as a force balance equation.
- Each term corresponds to a force produced by the respective energy terms. The contour deforms under the action of these forces.

Elastic force

- Generated by elastic potential energy of the curve.

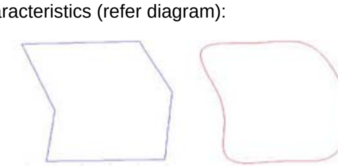
$$F_{elastic} = \alpha v_{ss}$$

- Characteristics (refer diagram)



Bending force

- Generated by the bending energy of the contour.
- Characteristics (refer diagram):



Initial curve (High bending energy)

Final curve deformed by bending force. (low bending energy)

- Thus the bending energy tries to smooth out the curve.

External force

$$F_{ext} = -\nabla E_{image}$$

- It acts in the direction so as to minimize E_{ext}

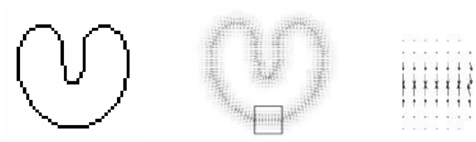


Image External force Zoomed in

Discretizing

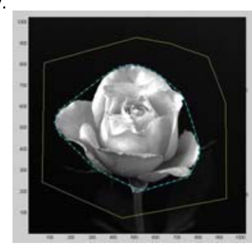
- the contour v(s) is represented by a set of control points $v_0, v_1, \ldots\ldots, v_{n-1}$
- The curve is piecewise linear obtained by joining each control point.
- Force equations applied to each control point separately.
- Each control point allowed to move freely under the. influence of the forces.
- The energy and force terms are converted to discrete form with the derivatives substituted by finite differences.

Solution and Results

Method 1:

$$\alpha v_{ss} - \beta v_{ssss} - \gamma \nabla E_{image} = 0$$

- γ is a constant to give separate control on external force.
- Solve iteratively.

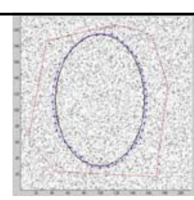


Method 2:

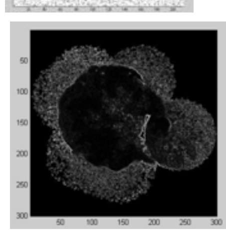
- Consider the snake to also be a function of time i.e. $v_t(s,t)$

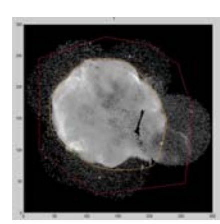
$$\alpha v_{ss}(s,t) - \beta v_{ssss}(s,t) - \nabla E_{image} = v_t(s,t) \qquad v_t(s,t) = \frac{\partial v(s,t)}{\partial t}$$

- If RHS=0 we have reached the solution.
- On every iteration update control point only if new position has a lower external energy.
- Snakes are very sensitive to false local minima which leads to wrong convergence.



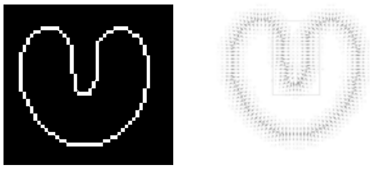
•Noisy image with many local minimas
•WGN sigma=0.1
•Threshold=15





Weakness of traditional snakes (Kass model)

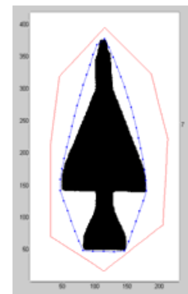
- Extremely sensitive to parameters.
- Small capture range.



- No external force acts on points which are far away from the boundary.
- Convergence is dependent on initial position.

Weakness (contd…)

- Fails to detect concave boundaries. External force cant pull control points into boundary concavity.



Gradient Vector Flow (GVF)

(A new external force for snakes)

•Detects shapes with boundary concavities.
•Large capture range.

Model for GVF snake

- The GVF field is defined to be a vector field V(x,y) = $(u(x,y), v(x,y))$
- V(x,y) is defined such that it minimizes the energy functional

$$E = \iint \mu(u_x^2 + u_y^2 + v_x^2 + v_y^2) + |\nabla f|^2 |V - \nabla f|^2 \, dxdy$$

f(x,y) is the edge map of the image.

- GVF field can be obtained by solving following equations

$$\mu\nabla^2 u - (u - f_x)(f_x^2 + f_y^2) = 0$$

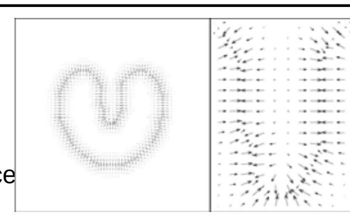
$$\mu\nabla^2 v - (v - f_y)(f_x^2 + f_y^2) = 0$$

∇^2 Is the Laplacian operator.

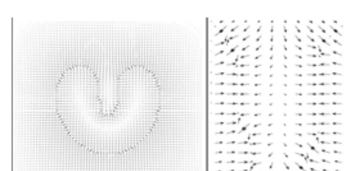
- The above equations are solved iteratively using time derivative of u and v.

Traditional external force field v/s GVF field

Traditional force



GVF force



(Diagrams courtesy "Snakes, shapes, gradient vector flow", Xu, Prince)