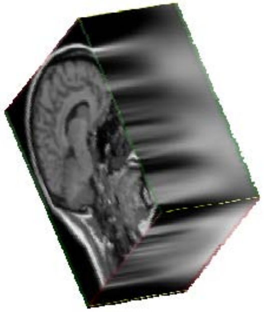
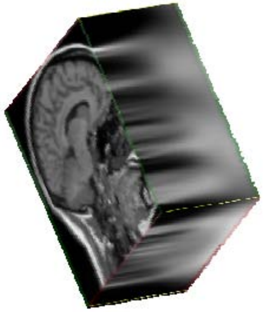


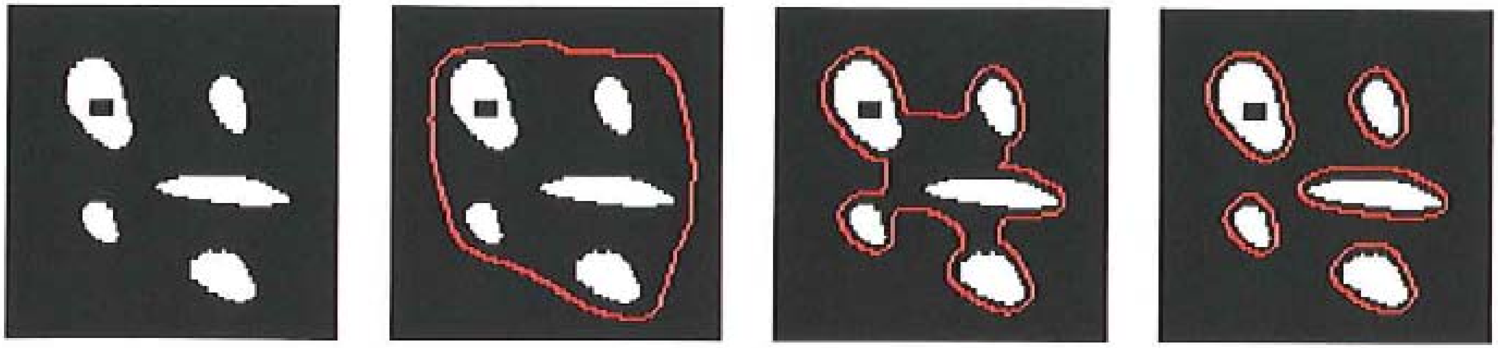
# Lecture:
# Level Set Segmentation
# Additional Notes

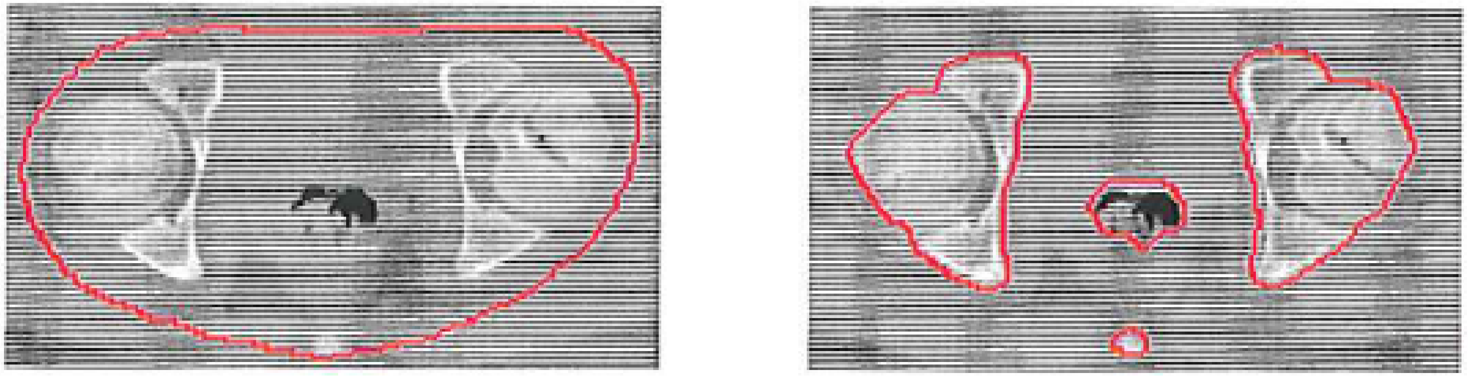
Guido Gerig

CS 7960, Spring 2010



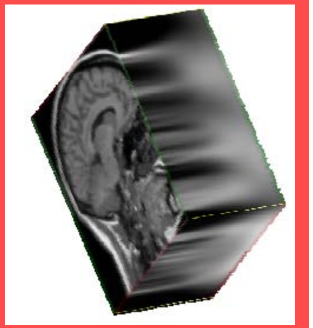
# Variable Topology

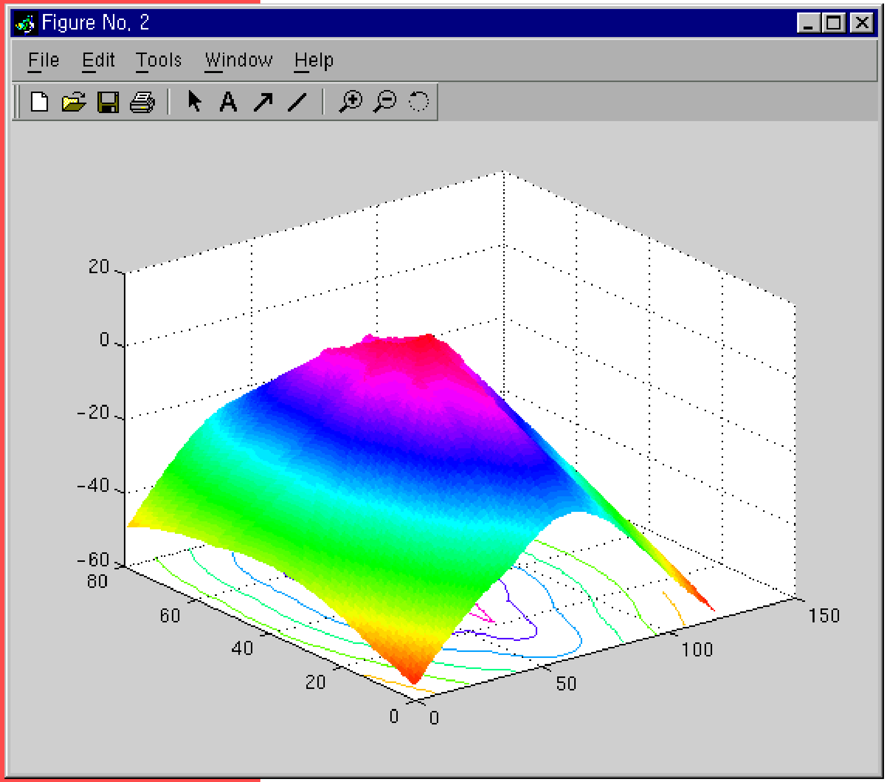


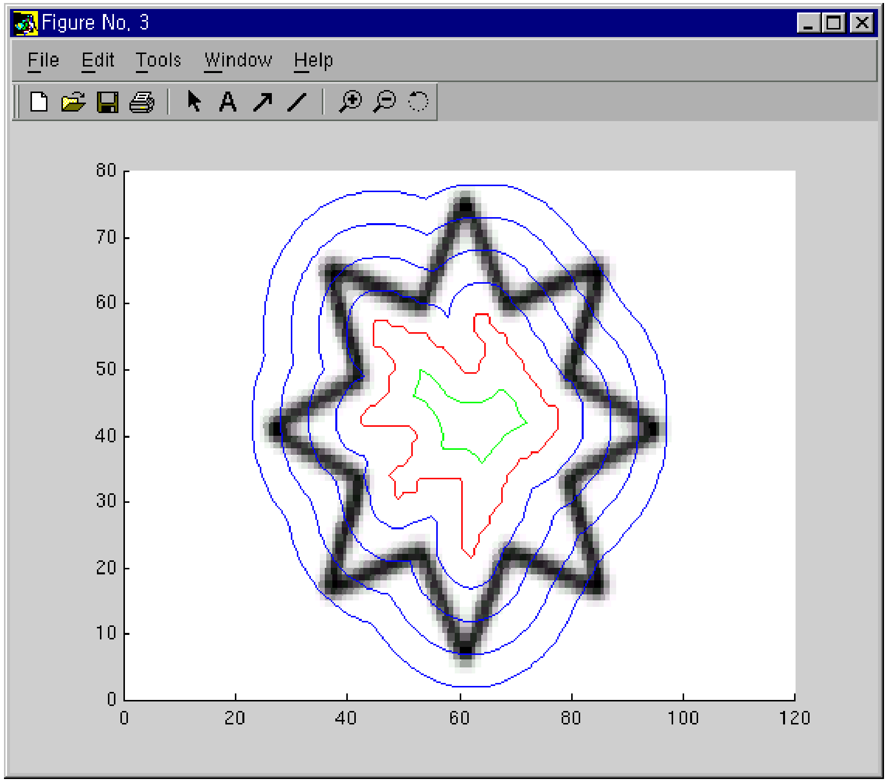


- Level set segmentation results in flexible topology

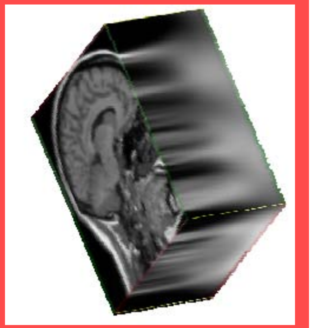
# Initialization



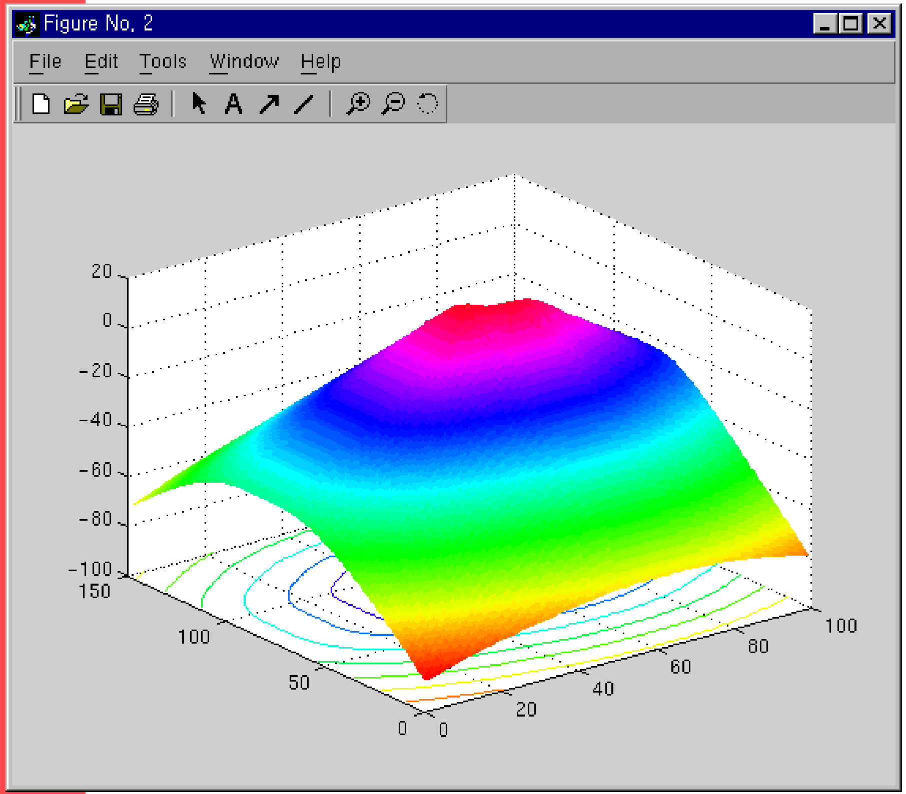


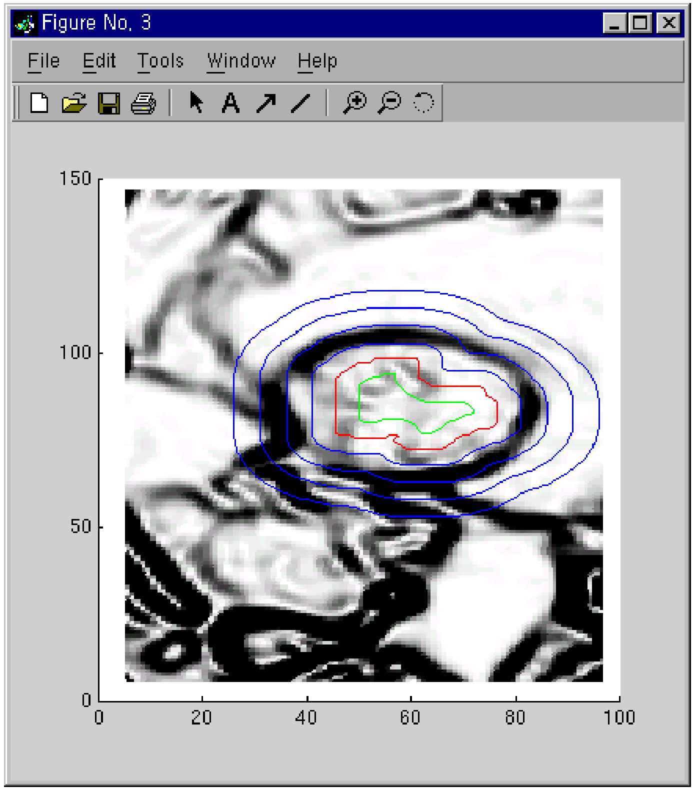




Level sets inside and outside of initialization curve

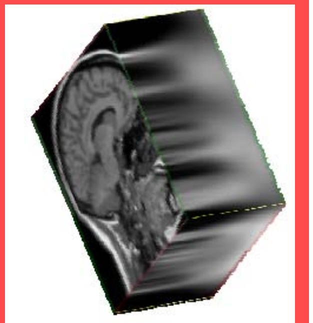


# Initialization




Level sets inside and outside of initialization curve



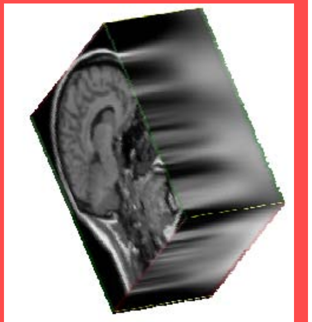
# Implicit Snakes: Caselles, Sethian, Malladi

$$\frac{\partial u}{\partial t} = |\nabla u| \operatorname{div}\left( g(I) \frac{\nabla u}{|\nabla u|} \right) + c g(I) |\nabla u|$$

$$\downarrow$$

$$\frac{\partial u}{\partial t} = g(I) \underbrace{\frac{1}{N-1} \left( \Delta u - \frac{Q(\nabla u)}{|\nabla u|^2} \right)}_{\text{MCF}} + \nabla g(I) \bullet \nabla u + c g(I) |\nabla u|.$$

$$\frac{1}{N-1} \left( \Delta u - \frac{Q(\nabla u)}{|\nabla u|^2} \right)$$



# Cartesian Coordinates

$$\begin{aligned} Q(\nabla u) &= Q(u_x\ u_y) = (u_x\ u_y) \begin{pmatrix} u_{xx} & u_{xy} \\ u_{yx} & u_{yy} \end{pmatrix} \begin{pmatrix} u_x \\ u_y \end{pmatrix} \\ &= u_{xx}u_x^2 + u_{yy}u_y^2 + 2u_{xy}u_x u_y \end{aligned}$$

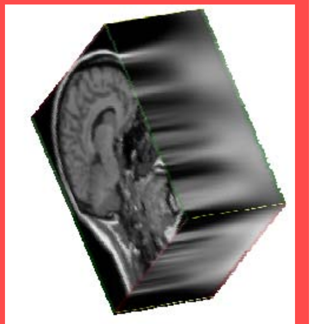
↓

2D

$$\frac{\partial u}{\partial t} = g(I)\left(\frac{u_y^2 u_{xx} + u_x^2 u_{yy} - 2u_{xy}u_x u_y}{u_x^2 + u_y^2} + c\sqrt{u_x^2 + u_y^2}\right) + g_x u_x + g_y u_y$$

3D

$$\begin{aligned} \frac{\partial u}{\partial t} &= g(I)\Big(\frac{(u_y^2+u_z^2+\varepsilon^2)u_{xx}+(u_x^2+u_z^2+\varepsilon^2)u_{yy}+(u_x^2+u_y^2+\varepsilon^2)u_{zz}-2u_{xy}u_x u_y-2u_{xz}u_x u_z-2u_{yz}u_y u_z}{2(u_x^2+u_y^2+u_z^2+\varepsilon^2)} \\ &\quad +c\sqrt{u_x^2+u_y^2+u_z^2}\Big) + g_x u_x + g_y u_y + g_z u_z \end{aligned}$$



# Implementation

$$\frac{\partial u(x,y,z)}{\partial x} \approx \frac{u(x+1,y,z)-u(x-1,y,z)}{2\Delta x}$$

$$\frac{\partial u(x,y,z)}{\partial y} \approx \frac{u(x,y+1,z)-u(x,y-1,z)}{2\Delta y}$$

$$\frac{\partial u(x,y,z)}{\partial z} \approx \frac{u(x,y,z+1)-u(x,y,z-1)}{2\Delta z}$$

$$\frac{\partial u(x,y,z)}{\partial x^2} \approx \frac{u(x+1,y,z)-2u(x,y,z)+u(x-1,y,z)}{\Delta x^2}$$

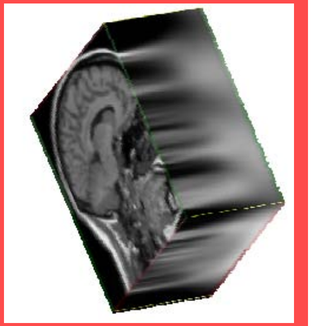
$$\frac{\partial u(x,y,z)}{\partial y^2} \approx \frac{u(x,y+1,z)-2u(x,y,z)+u(x,y-1,z)}{\Delta y^2}$$

$$\frac{\partial u(x,y,z)}{\partial z^2} \approx \frac{u(x,y,z+1)-2u(x,y,z)+u(x,y,z-1)}{\Delta z^2}$$

$$\frac{\partial u(x,y,z)}{\partial x \partial y} \approx \frac{u(x+1,y+1,z)-u(x-1,y+1,z)+u(x-1,y-1,z)-u(x+1,y-1,z)}{4\Delta x \Delta y}$$

$$\frac{\partial u(x,y,z)}{\partial x \partial z} \approx \frac{u(x+1,y,z+1)-u(x-1,y,z+1)+u(x-1,y,z-1)-u(x+1,y,z-1)}{4\Delta x \Delta z}$$

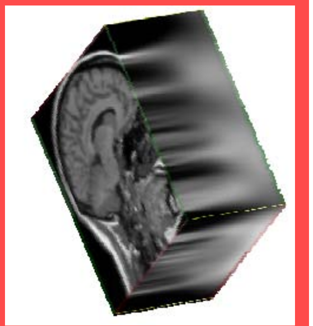
$$\frac{\partial u(x,y,z)}{\partial y \partial z} \approx \frac{u(x,y+1,z+1)-u(x,y-1,z+1)+u(x,y-1,z-1)-u(x,y+1,z-1)}{4\Delta y \Delta z}$$



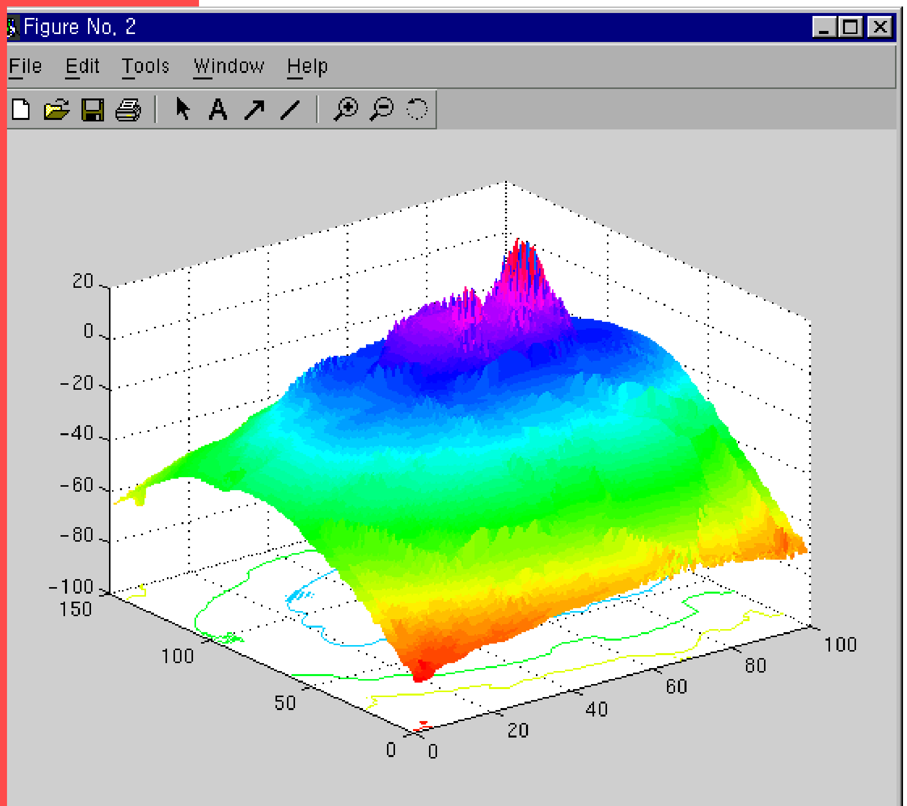
# Implementation ctd.

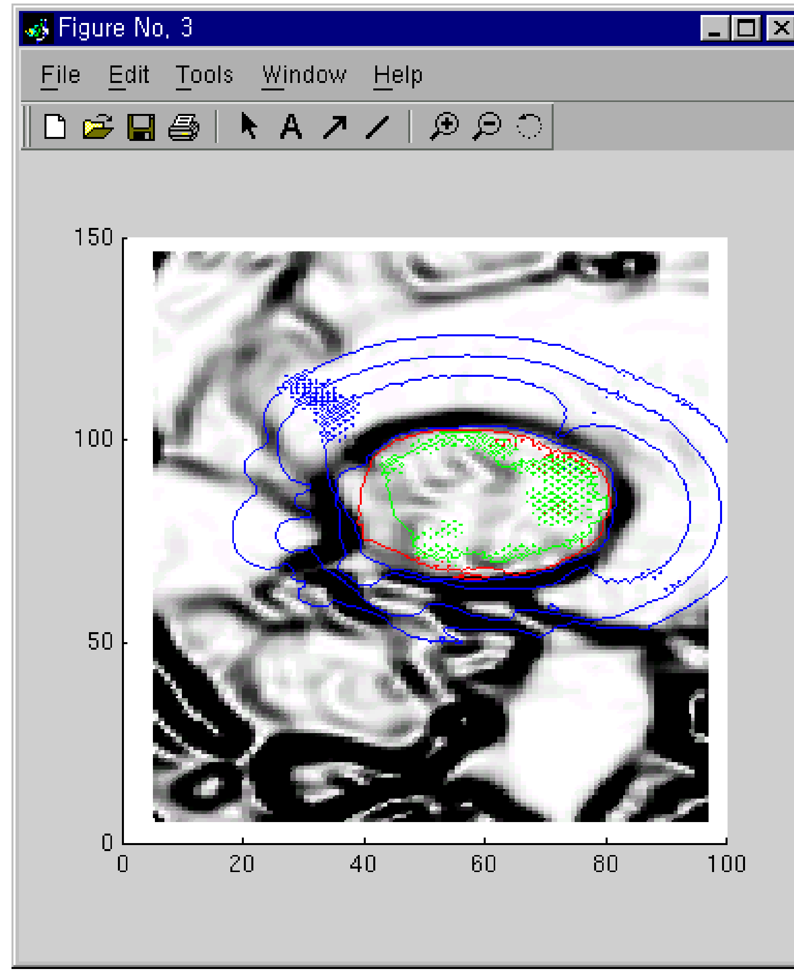
$$\frac{\partial u(\vec{x},t)}{\partial t} \approx \frac{u(\vec{x},t+\Delta t) - u(\vec{x},t)}{\Delta t}$$

$$u(\vec{x},t+\Delta t) \approx u(\vec{x},t) + \Delta t \frac{\partial u(\vec{x},t)}{\partial t}$$



# Explicit Implementation:
## FTCS (forward in time centered space)

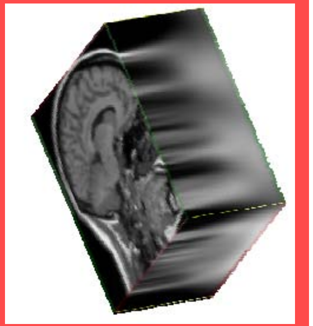


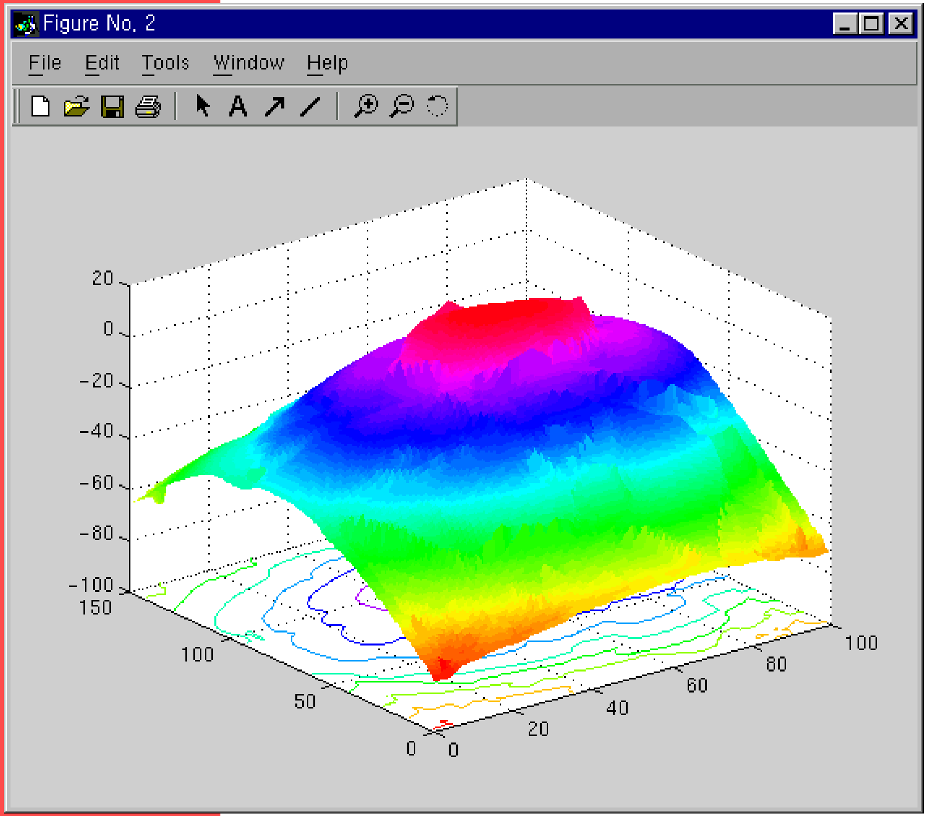




*dt = 0.6, speed = 0.7, iteration : 20*

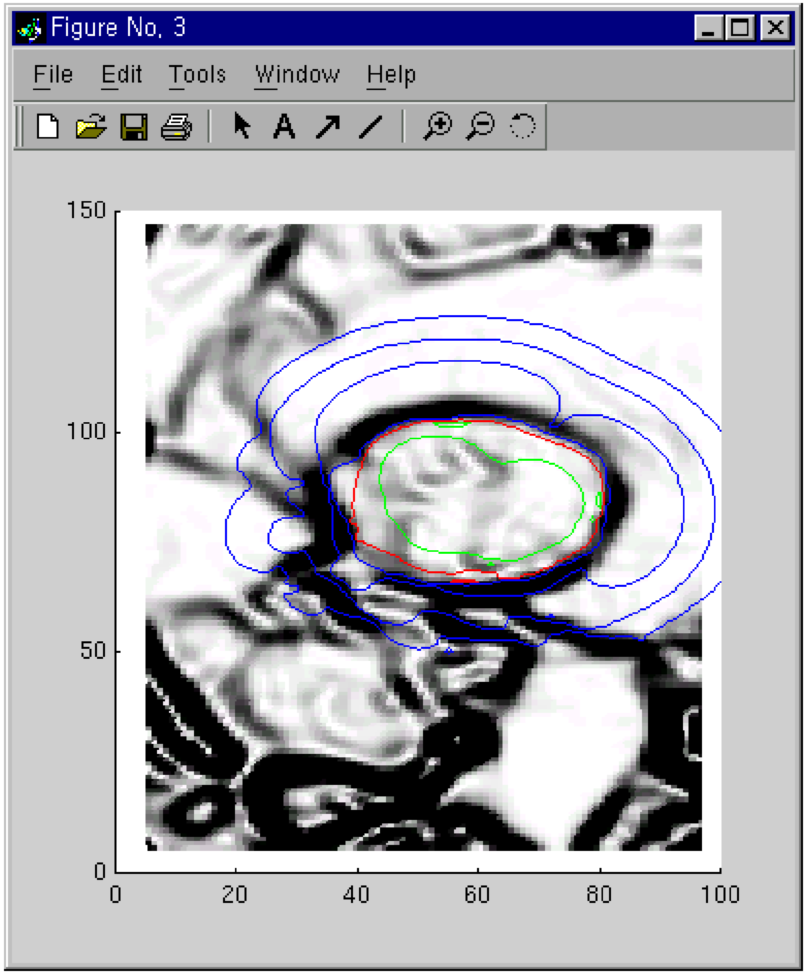
Problem: instability if time steps too large → slow

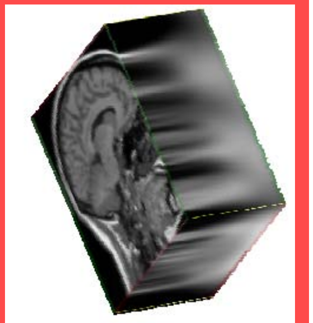
# Alternative: Implicit Implementation of PDE

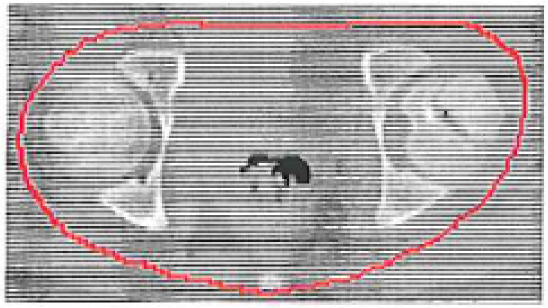





*dt = 0.6, speed = 0.7, iteration : 20*







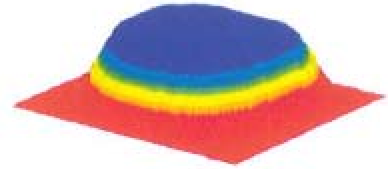


Abb.1: Initialisierung von u(x,y)

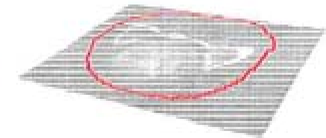


Abb.2: Initialisierung der Snake

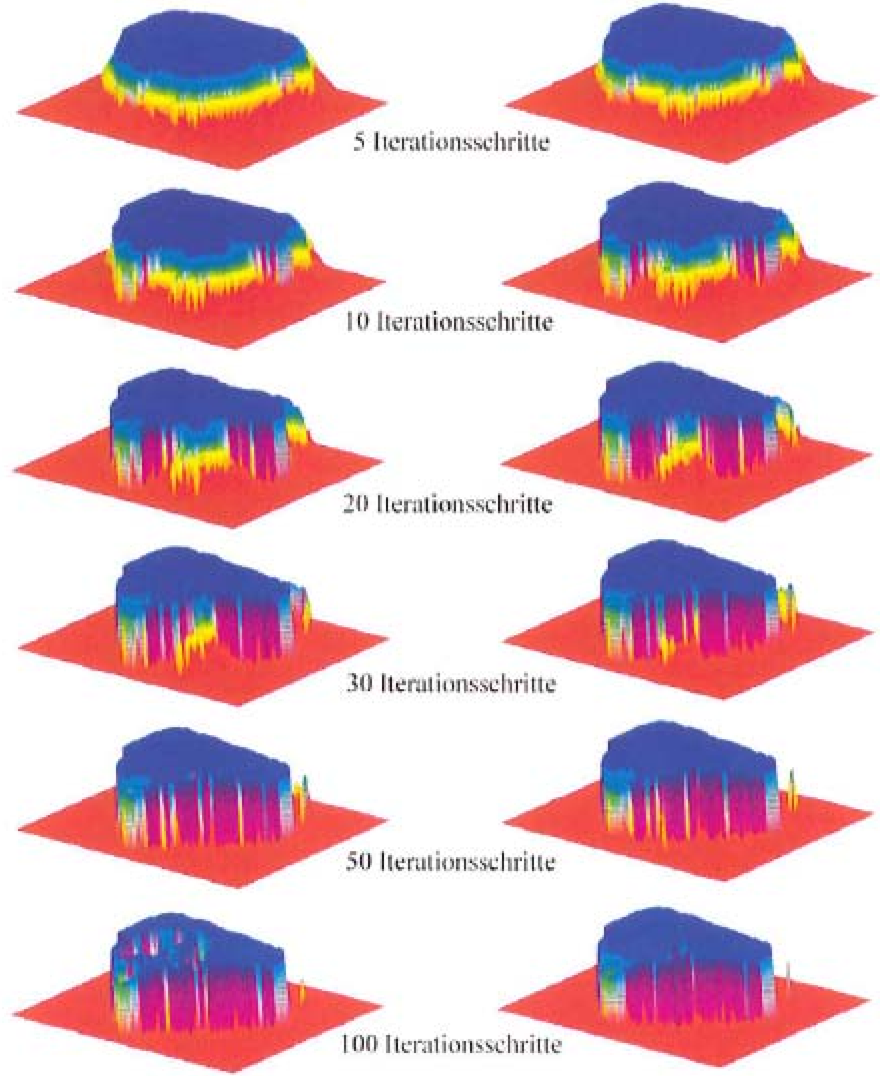
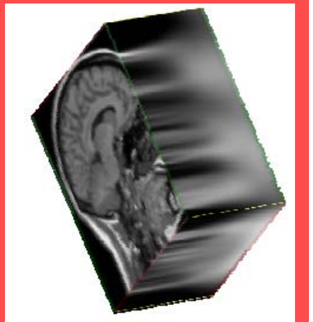
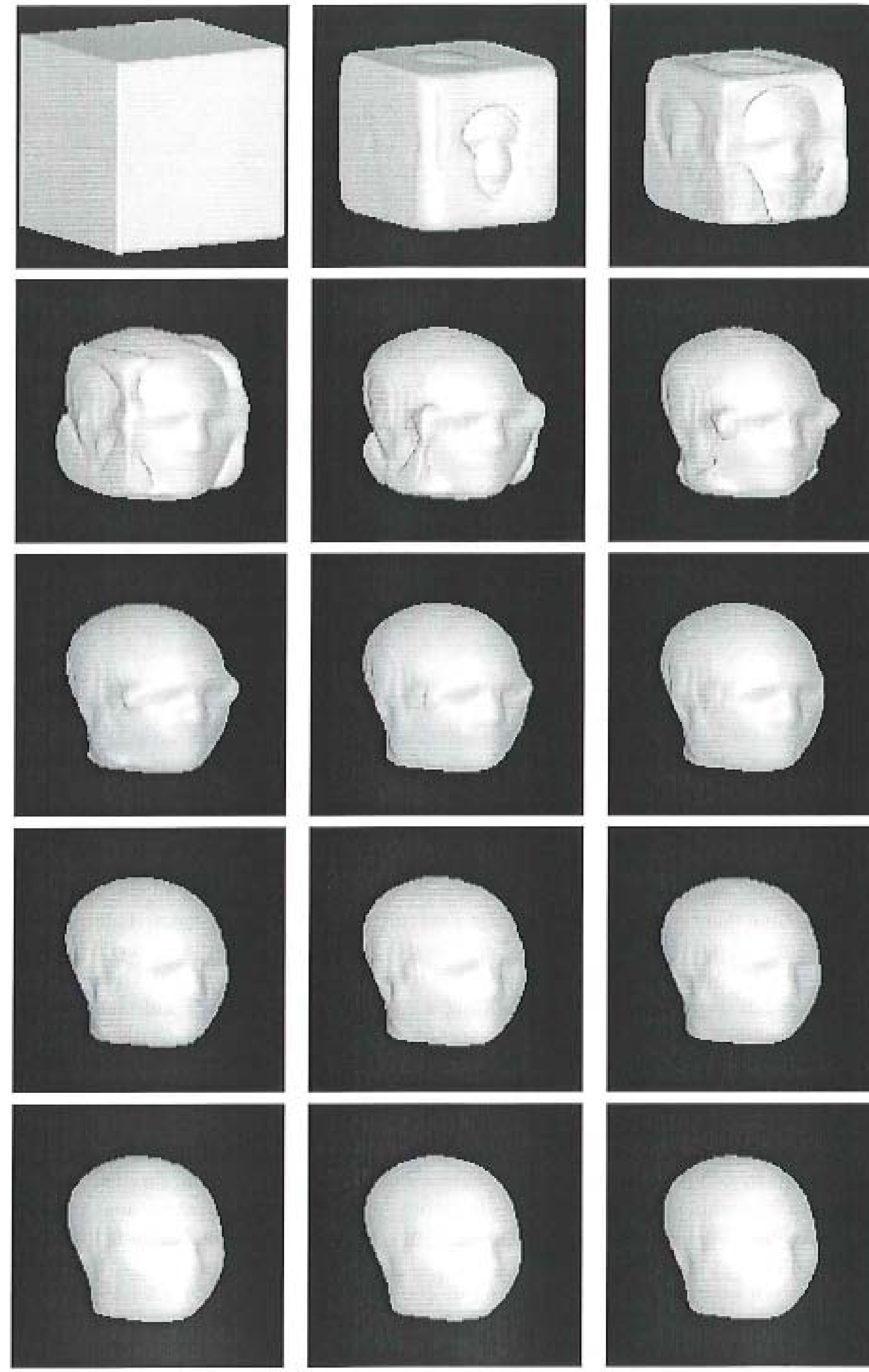
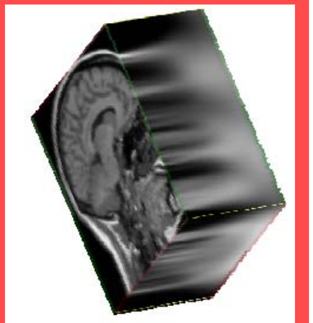



Abb.3: Turello-Version

Abb.4: Schlegel-Version







# itkSNAP: Freely available tool

http://www.itksnap.org/pmwiki/pmwiki.php?n=Main.Downloads