Snakes: Active Contours

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http://www.cab.u-szeged.hu/~kato/variational/

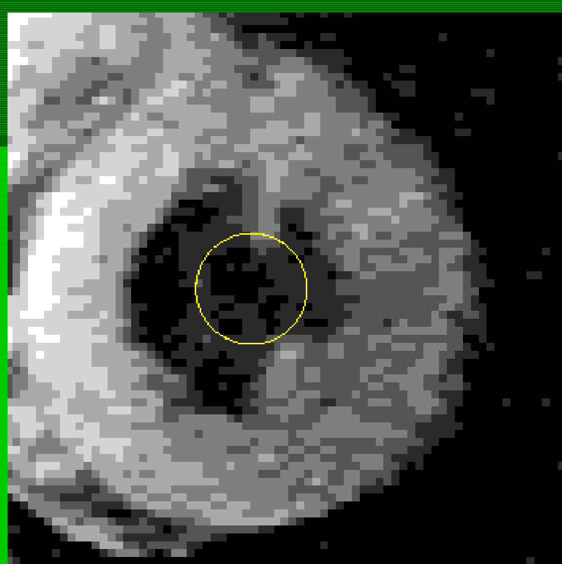
Introduction

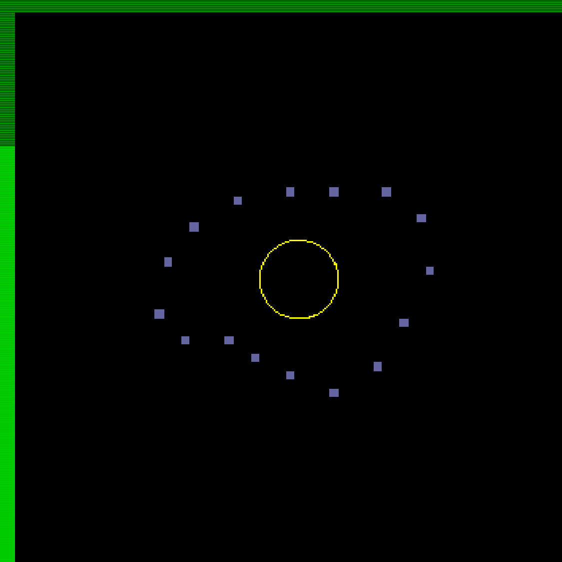
- Proposed by
  - Michael Kass
  - Andrew Witkin
    - http://www.ri.cmu.edu/people/witkin_andrew.html
  - **Demetri Terzopoulos**
    - http://mrl.nyu.edu/~dt/

Snakes: Active Contour Models. *International Journal of Computer Vision, Vol. 1, pp 321-331, 1988.*

What is a snake?

- An energy minimizing ***spline*** guided by external constraint forces and pulled by image forces toward features:
  - Edge detection
  - Subjective contours
  - Motion tracking
  - Stereo matching
  - ….





Images taken from the GVF website: http://iacl.ece.jhu.edu/projects/gvf/

Snake behavior

- A snake falls into the closest ***local*** energy minimum.
  - The local minima of the snake energy comprise the set of alternative solutions
  - A higher level knowledge is needed to choose the ***„correct one”*** from these solutions
    - High-level reasoning
    - User interaction
  - These high-level methods can ***interact*** with the contour model by pushing it toward an appropriate local minimum

Snake behavior

- They rely on other mechanisms to place them ***near*** the desired contour.
  - The existence of such an initializer is application dependent.
  - Even in the case of manual initialization, snakes are quite powerful in refining the user's input.
- Basically, snakes are trying to match a deformable model to an image by means of energy minimization.

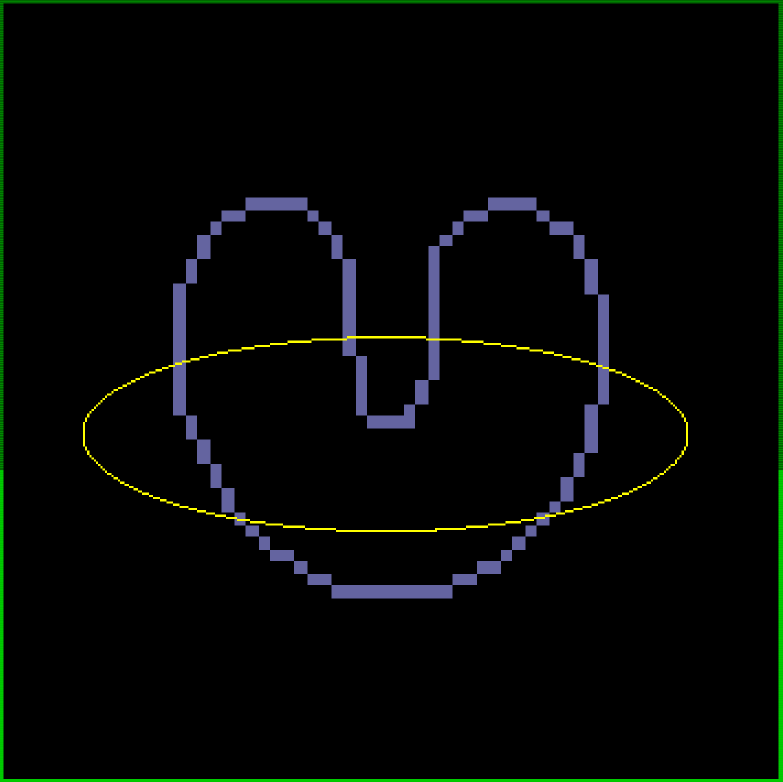


Image taken from the GVF website: http://iacl.ece.jhu.edu/projects/gvf/

Snake energy

- Parametric representation: ***v(s)=(x(s),y(s))***

$$E_{snake} = \int_0^1 E_{\text{int}}(v(s)) + E_{image}(v(s)) + E_{con}(v(s))ds$$

  - E_{int} = internal energy due to bending. Serves to impose piecewise smoothness constraint.
  - E_{image} = image forces pushing the snake toward image features (edges, etc…).
  - E_{con} = external constraints are responsible for putting the snake near the desired local minimum. It may come from:
    - Higher level interpretation
    - User interaction, etc…

Internal energy

- The snake is a ***controlled continuity*** spline
  - Regularizes the problem

$$E_{\text{int}} = (\alpha(s)\,|\,v_s(s)\,|^2 + \beta(s)\,|\,v_{ss}(s)\,|^2)/2$$

  - The first order derivative $v_s(s)$ makes the spline act like a ***membrane*** („elasticity”).
  - The second order derivative $v_{ss}(s)$ makes it act like a ***thin-plate*** („rigidity”).
  - $\alpha(s)$ and $\beta(s)$ controls the relative importance of membrane and thin-plate terms
    - Setting $\beta(s)=0$ for a point allows the snake to become second-order discontinuous and develop a corner.

Image forces

- Attracts the snake to features (data term)

$$E_{image} = w_{line}E_{line} + w_{edge}E_{edge} + w_{term}E_{term}$$

  - ***lines***: the simplest functional is the image intensity: $E_{line}=I(x,y)$
    - Depending on the sign of w_{line}, the snake will be attracted to the lightest or darkest nerby contour
  - ***edges***: one can simply set $E_{edge}= - |\nabla I(x,y)|^2$
    - attracts the snake to large intensity gradients.
  - ***terminations***: discussed later

Snake convergence

- If part of a snake finds a low-energy feature → the spline term will pull neighboring parts toward a possible continuation of the feature found.
- In fact, this places a large energy well around a good local minimum

Video taken from the website:
http://www-2.cs.cmu.edu/afs/cs/user/aw/www/gallery.html

Scale space

- Minimization by scale-continuation:
  1. Spatial smooting the edge or line functional
     - $E_{edge} = -(G_\sigma * \nabla^2 I)^2$, where G_σ is a Gaussian with σ standard deviation
     - Minima lie on zero crossings of $G_\sigma * \nabla^2 I$ (~edges)
  2. Snake comes to equilibrium on a blurry energy
  3. Slowly reduce the blurring

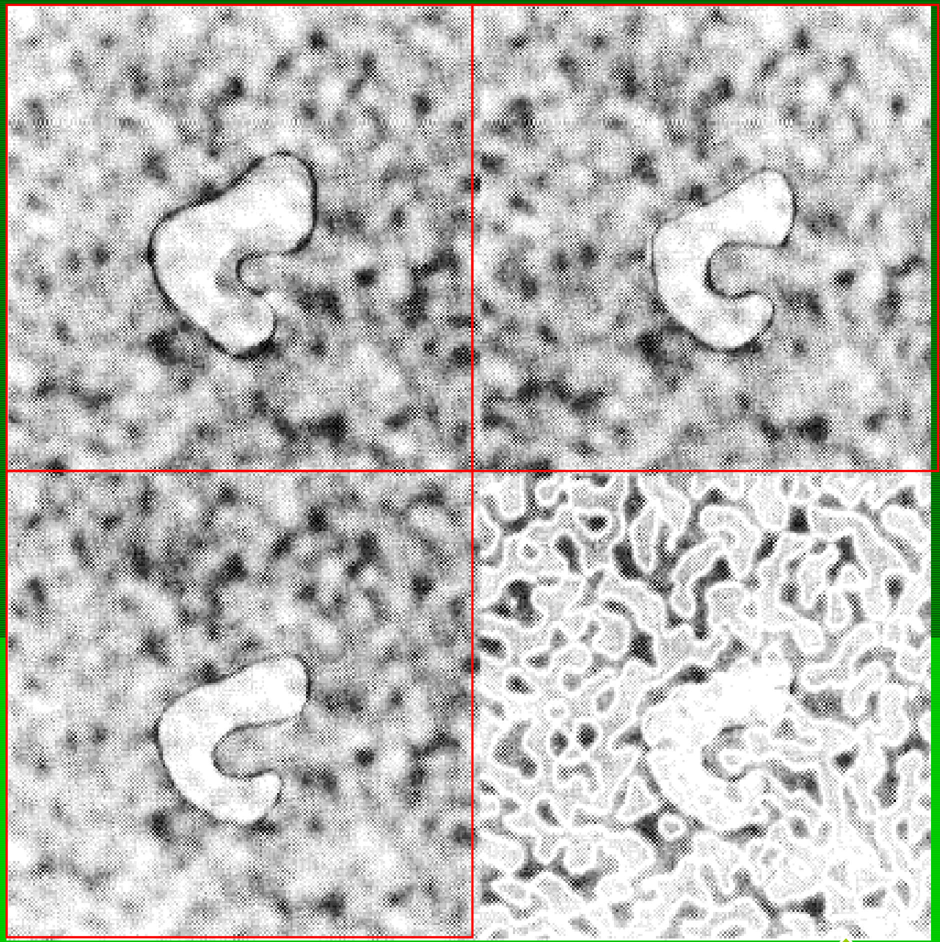


Image taken from **M. Kass & A. Witkin & D. Terzopoulos**: Snakes: Active Contour Models. *International Journal of Computer Vision, Vol. 1, pp 321-331, 1988.*

Zero crossings

Termination functional

- Attracts the snake toward termination of line segments and corners.
  - Let $C(x,y)= G_\sigma(x,y) * I(x,y))^2$ (smoothed image)
  - Let $\theta=\tan^{-1}(C_y/C_x)$ the gradient angle
    - $n=(\cos\theta, \sin\theta)$ unit vector along gradient
    - $n_\perp=(-\sin\theta, \cos\theta)$ perpendicular to gradient
  - E_{term} is defined using curvature of level lines in $C(x,y)$:

$$E_{term}=\frac{\partial\theta}{\partial n_\perp}=\frac{\partial^2 C/\partial n_\perp^2}{\partial C/\partial n}=\frac{C_{yy}C_x^2-2C_{xy}C_xC_y+C_{xx}C_y^2}{\left(C_x^2+C_y^2\right)^{3/2}}$$

Subjective contour

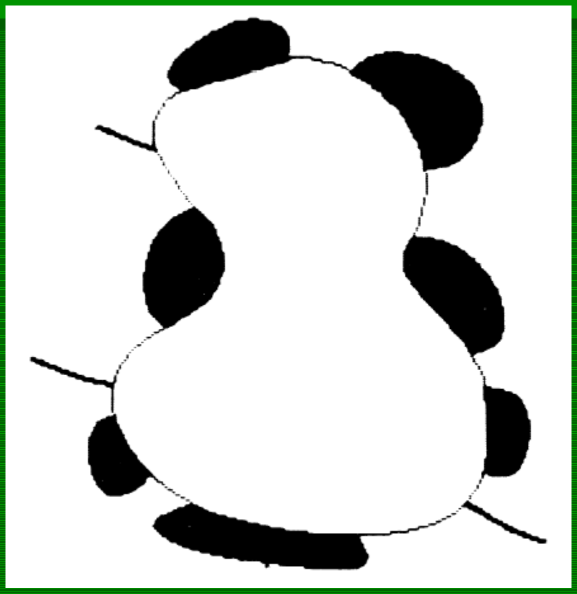


Image taken from M. Kass & A. Witkin & D. Terzopoulos: Snakes: Active Contour Models. *International Journal of Computer Vision, Vol. 1, pp 321-331, 1988.*

- Combining E_{edge} and E_{term}, we can create a snake attracted to edges and terminations
- The shape of the snake between the edges and lines in the illusion is completely determined by the spline smoothness term
- The same snake can find traditional edges in natural images

Subjective contour: hysteresys

- Snake tracking a moving subjective contour
- The snake bends until the internal spline forces overpower image forces
- Then the snake falls off the line and returns to a smoother shape

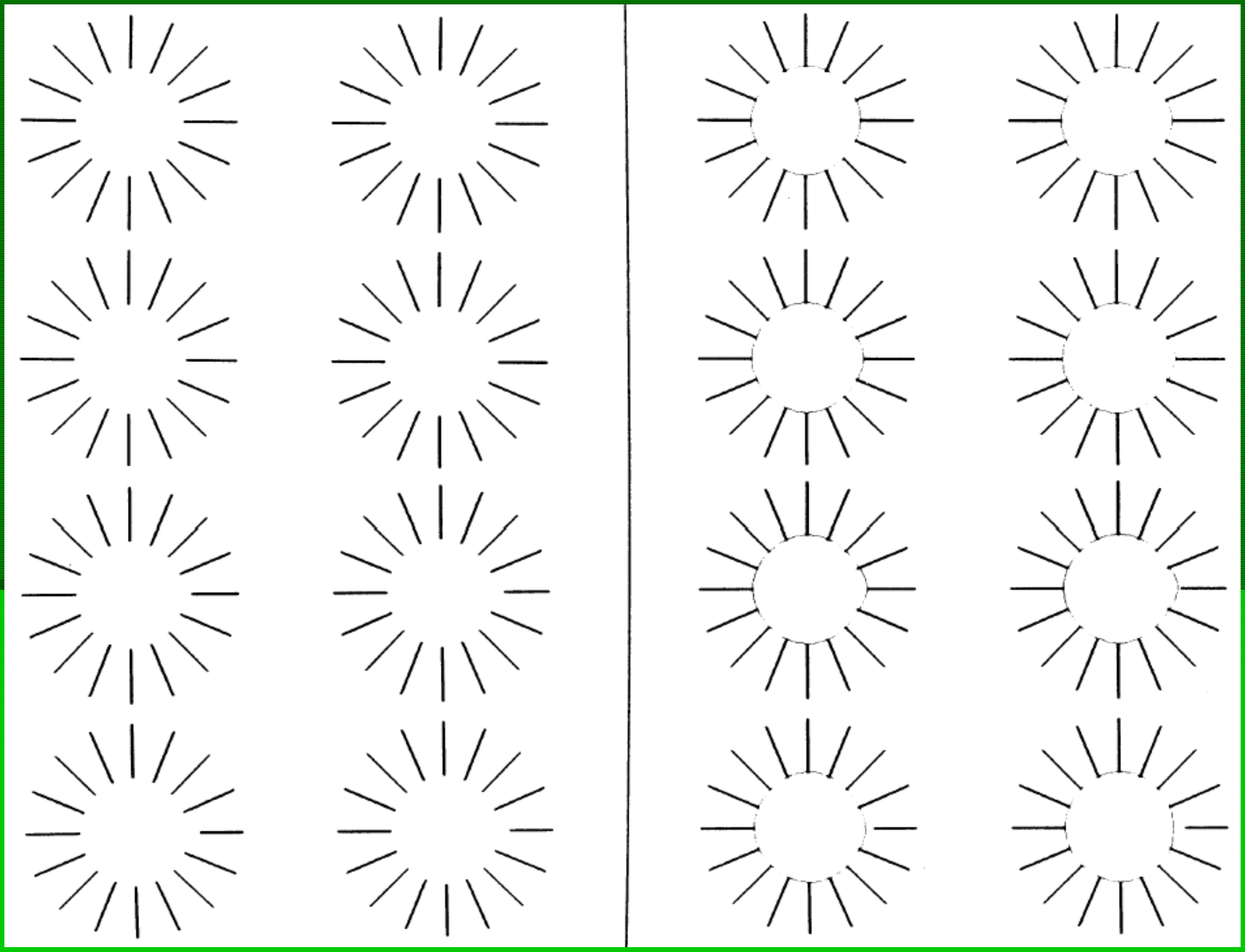


Image taken from **M. Kass & A. Witkin & D. Terzopoulos**: Snakes: Active Contour Models. *International Journal of Computer Vision, Vol. 1, pp 321-331, 1988.*

Motion tracking

- Once a snake finds a feature, it „locks on”
- If the feature begins to move, the snake will track the same local minimum
  - Fast motion could cause the snake to flip into a different minimum

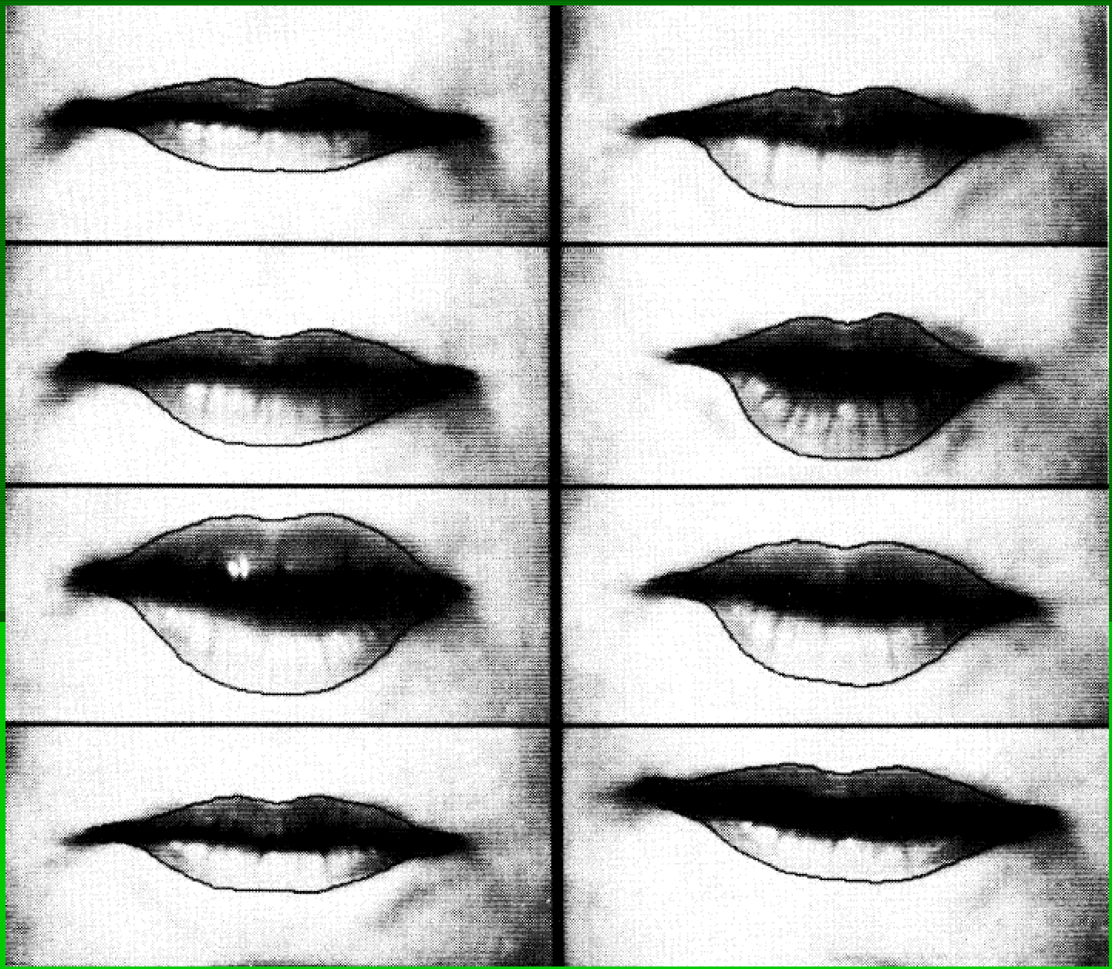


Image taken from **M. Kass & A. Witkin & D. Terzopoulos**: Snakes: Active Contour Models. *International Journal of Computer Vision, Vol. 1, pp 321-331, 1988.*

Snake energy minimization

- When ***α(s)*** and ***β(s) are constant***, we get two independent Euler-Lagrange equations.

$$\alpha x_{ss} + \beta x_{ssss} + \frac{\partial E_{ext}}{\partial x} = 0$$
$$\alpha y_{ss} + \beta y_{ssss} + \frac{\partial E_{ext}}{\partial y} = 0$$

- When ***α(s)*** and ***β(s) are not constant*** then it is simpler to use a discrete formulation:

$$E_{snake} = \sum_{i=1}^{n} E_{\text{int}}(i) + E_{ext}(i), \quad v_i = (x_i, y_i) = (x(ih), y(ih))$$

$$E_{\text{int}}(i) = \alpha_i \, | v_i - v_{i-1} |^2 / 2h^2 + \beta_i \, | v_{i-1} - 2v_i + v_{i+1} |^2 / 2h^4$$

Snake energy minimization

- Let $f_x(i) = \partial E_{ext} / \partial x_i$ where derivatives are approximated by finite differences if they cannot be computed analitically.
- The corresponding Euler equations

$$\alpha_i(v_i - v_{i-1}) - \alpha_{i+1}(v_{i+1} - v_i)$$
$$+ \beta_{i-1}(v_{i-2} - 2v_{i-1} + v_i)$$
$$- 2\beta_i(v_{i-1} - 2v_i + v_{i+1})$$
$$+ \beta_{i+1}(v_i - 2v_{i+1} + v_{i+2})$$
$$+ (f_x(i), f_y(i)) = 0$$

- In matrix form where ***A*** is a pentadiagonal banded matrix:

$$Ax + f_x(x, y) = 0$$
$$Ay + f_y(x, y) = 0$$

Snake energy minimization

$$Ax_t + f_x(x_{t-1}, y_{t-1}) = -\gamma(x_t - x_{t-1})$$
$$Ay_t + f_y(x_{t-1}, y_{t-1}) = -\gamma(y_t - y_{t-1})$$

γ is the step size

- Taking into account the derivatives requires changing ***A*** at each iteration. Speed up:
  - We assume that f_x and f_y are constant during a time step → explicit Euler method w.r.t. the external forces.
  - internal forces are specified by ***A*** → we can evaluate the time derivative at ***t*** rather than ***t-1***

Snake energy minimization

- At equilibrium, the time derivative vanishes.
- The Euler equations can be solved by matrix inversion:

$$x_t = (A + \gamma I)^{-1}(\gamma x_{t-1} - f_x(x_{t-1}, y_{t-1}))$$

$$y_t = (A + \gamma I)^{-1}(\gamma y_{t-1} - f_y(x_{t-1}, y_{t-1}))$$

- The inverse can be calculated by **LU** decomposition in **O(n)** time.