Geometric Transformations and Image Warping: Mosaicing

CS 6640

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(with slides from: Jinxiang Chai, TAMU)

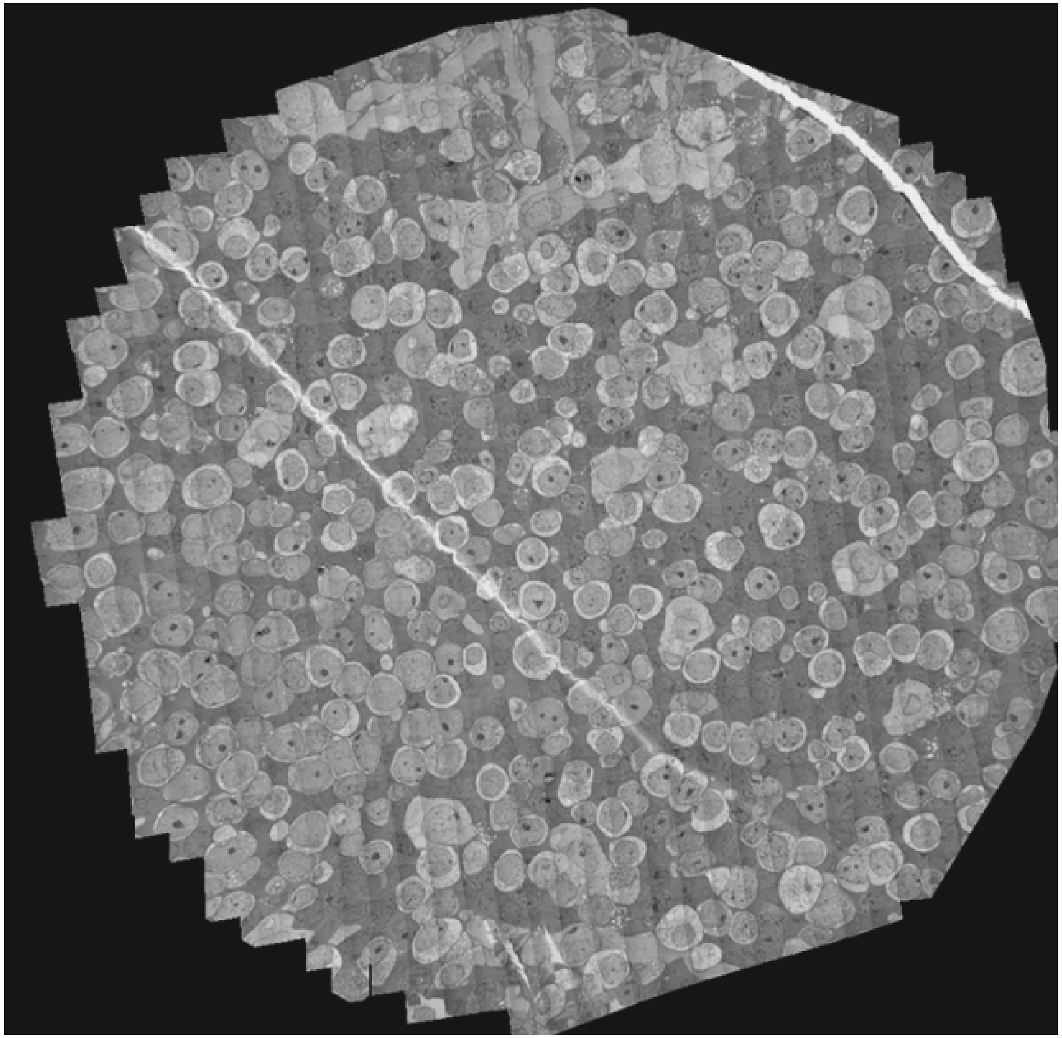
faculty.cs.tamu.edu/jchai/cpsc641_spring10/lectures/lecture8.ppt

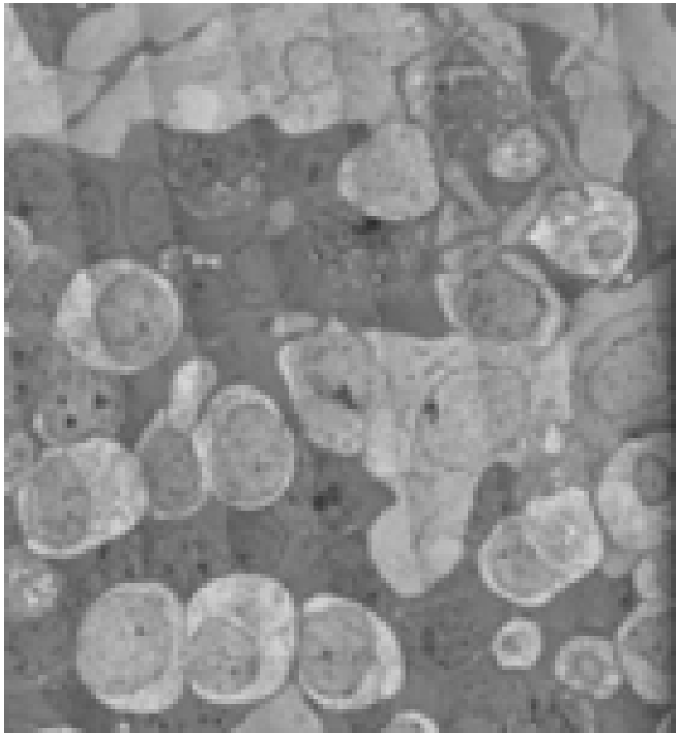
Image Mosaicing

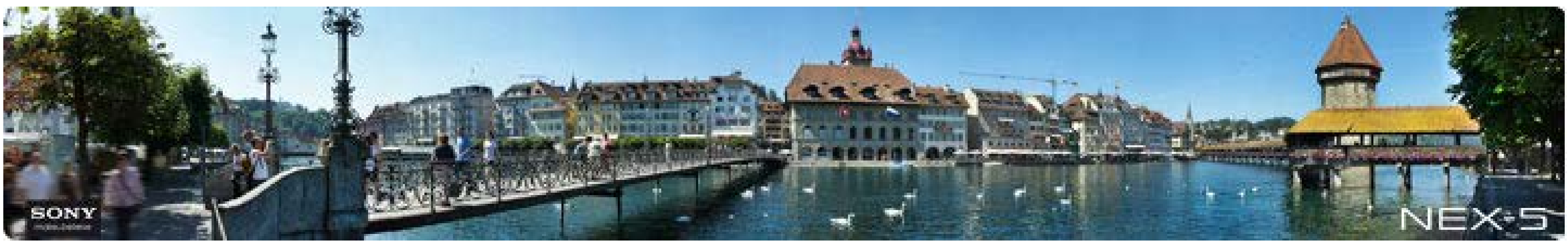
- Piecing together images to create a larger mosaic
- Doing it the old fashioned way
  - Paper pictures and tape
  - Things don’t line up
  - Translation is not enough
- Need some kind of warp
- Constraints
  - Warping/matching two regions of two different images only works when…

Applications

Microscopy (Morane Eye Inst, UofU, T. Tasdizen et al.)






SONY
NEX-5

SONY
NEX-5

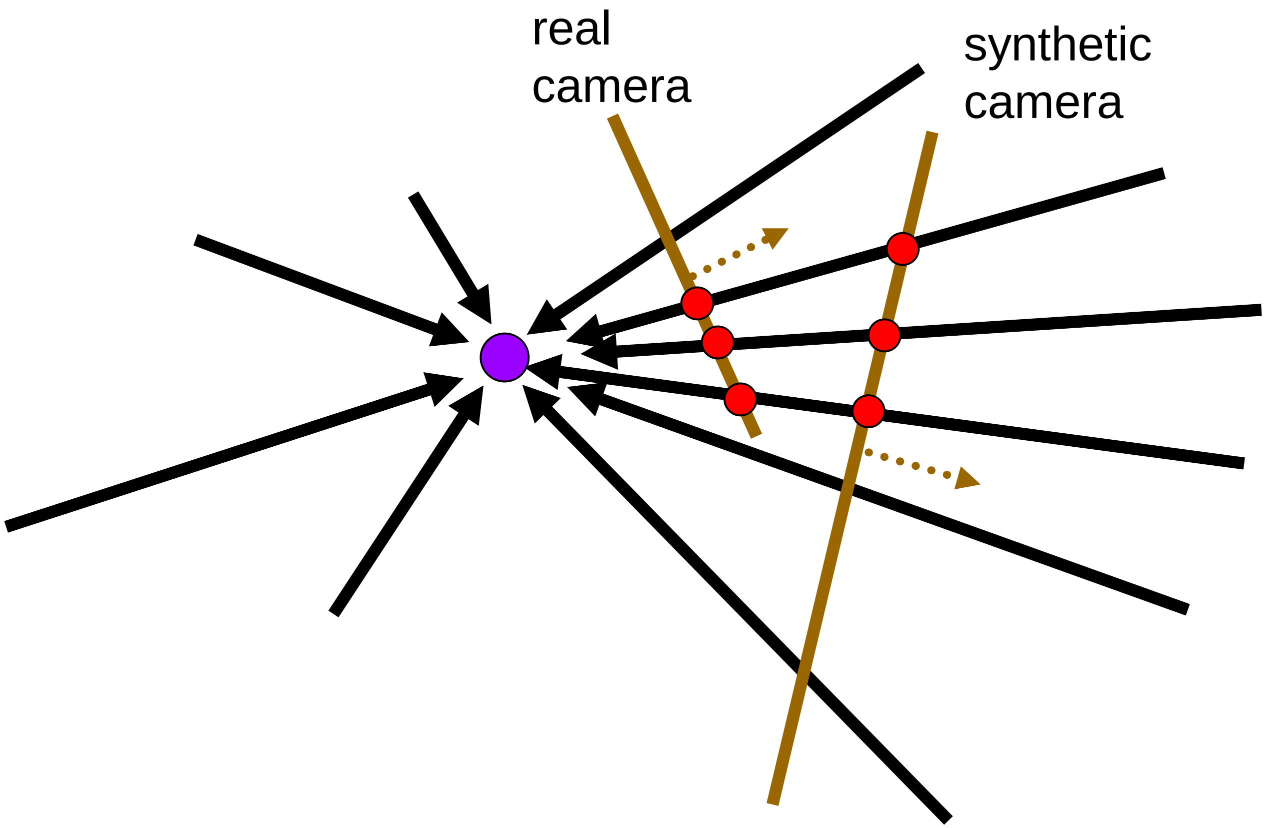
Mosaic Procedure

Basic Procedure

- Take a sequence of images from the same position
  - Rotate the camera about its optical center
- Compute transformation between second image and first
- Transform the second image to overlap with the first
- Blend the two together to create a mosaic
- If there are more images, repeat

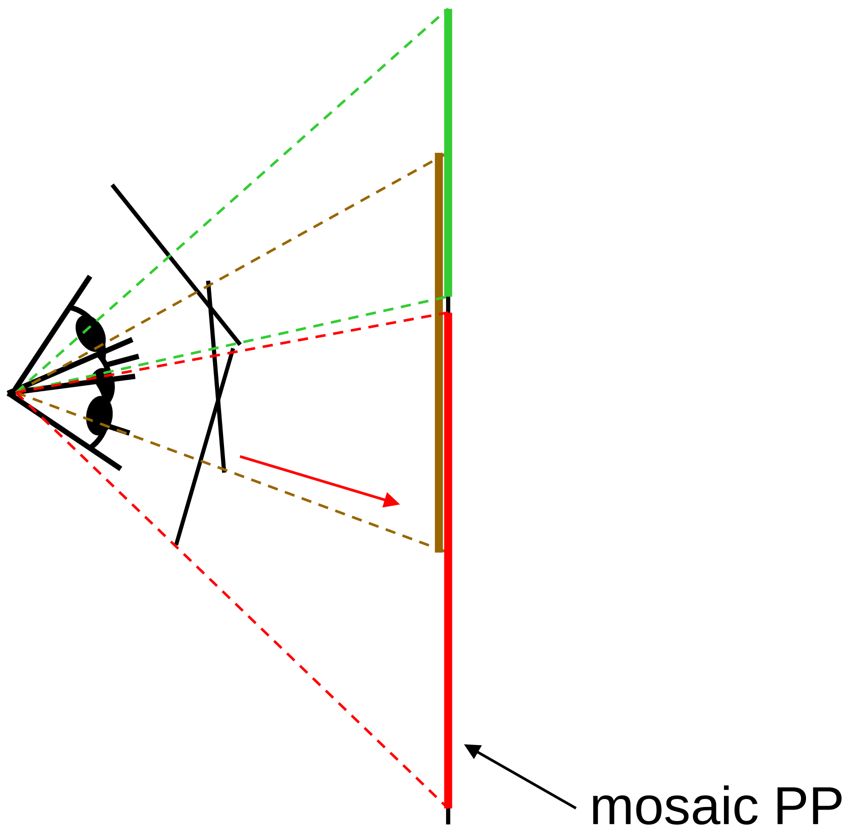
Image Mosaic

Is a pencil of rays contains all views



Can generate any synthetic camera view
as long as it has **the same center of projection**!

Image Re-projection



The mosaic has a natural interpretation in 3D

- The images are reprojected onto a common plane
- The mosaic is formed on this plane
- Mosaic is a *synthetic wide-angle camera*

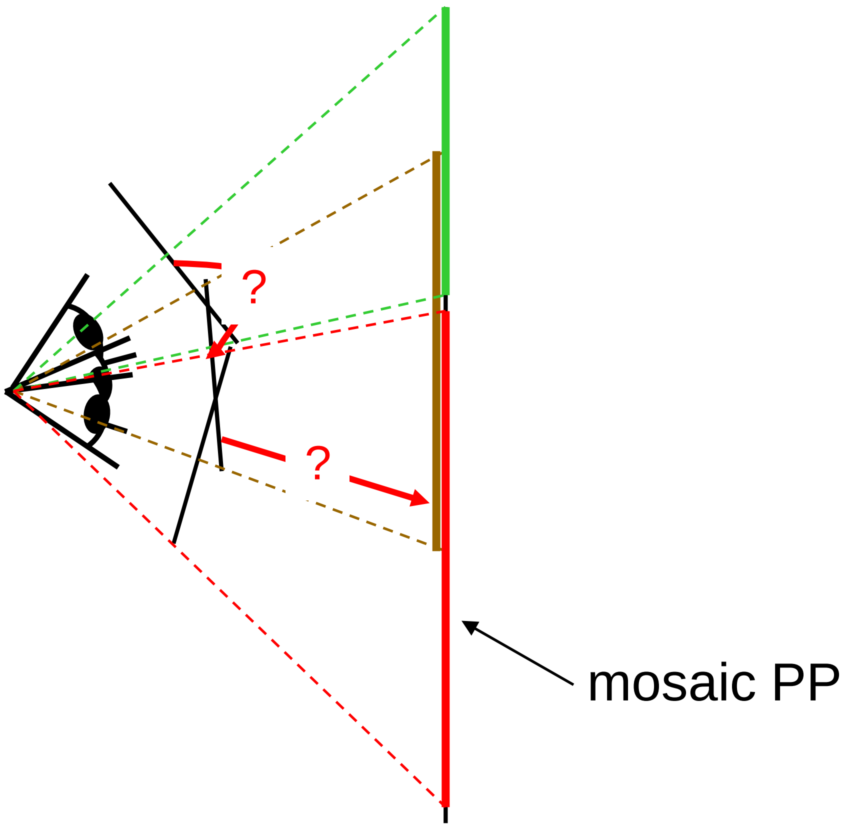
Issues in Image Mosaic

How to relate two images from the same camera center?

- image registration

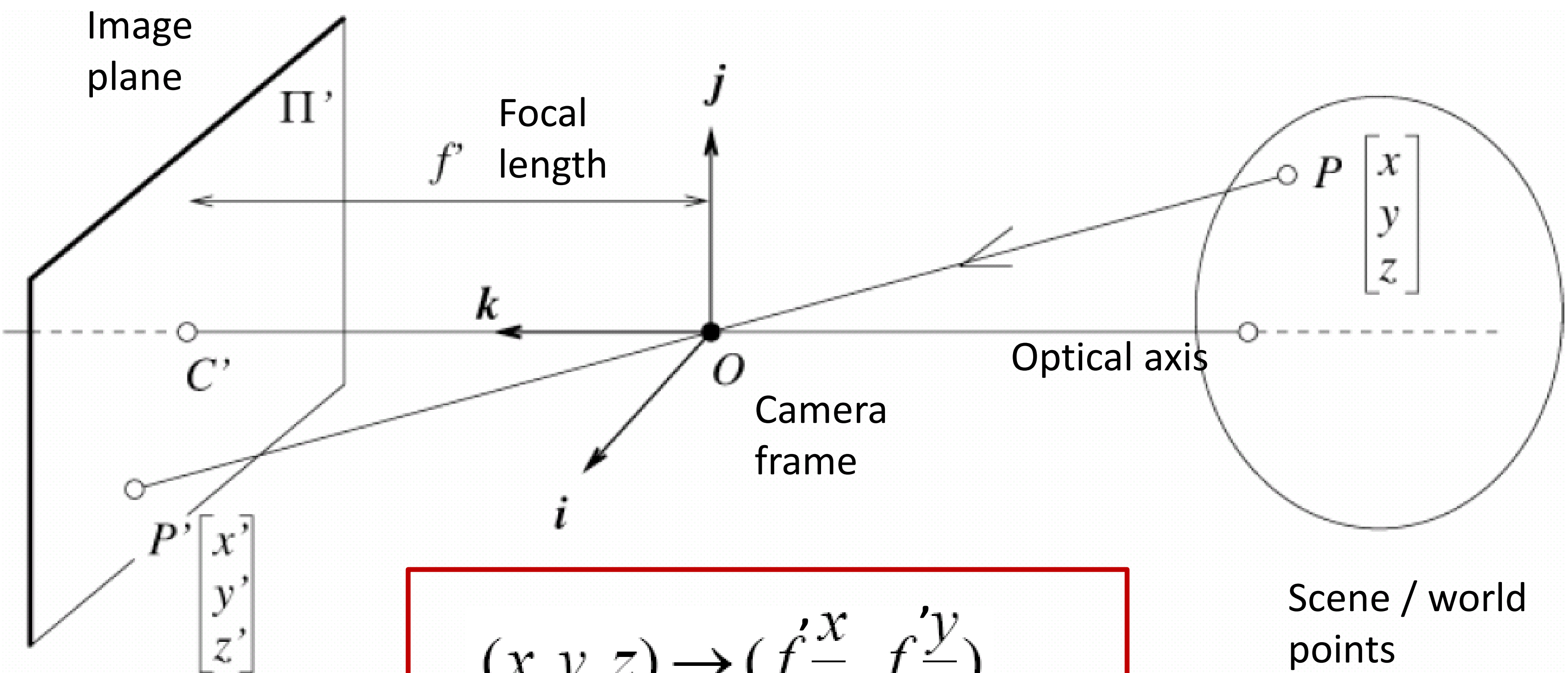
How to re-project images to a common plane?

- image warping



Perspective projection equations

- 3d world mapped to 2d projection in image plane



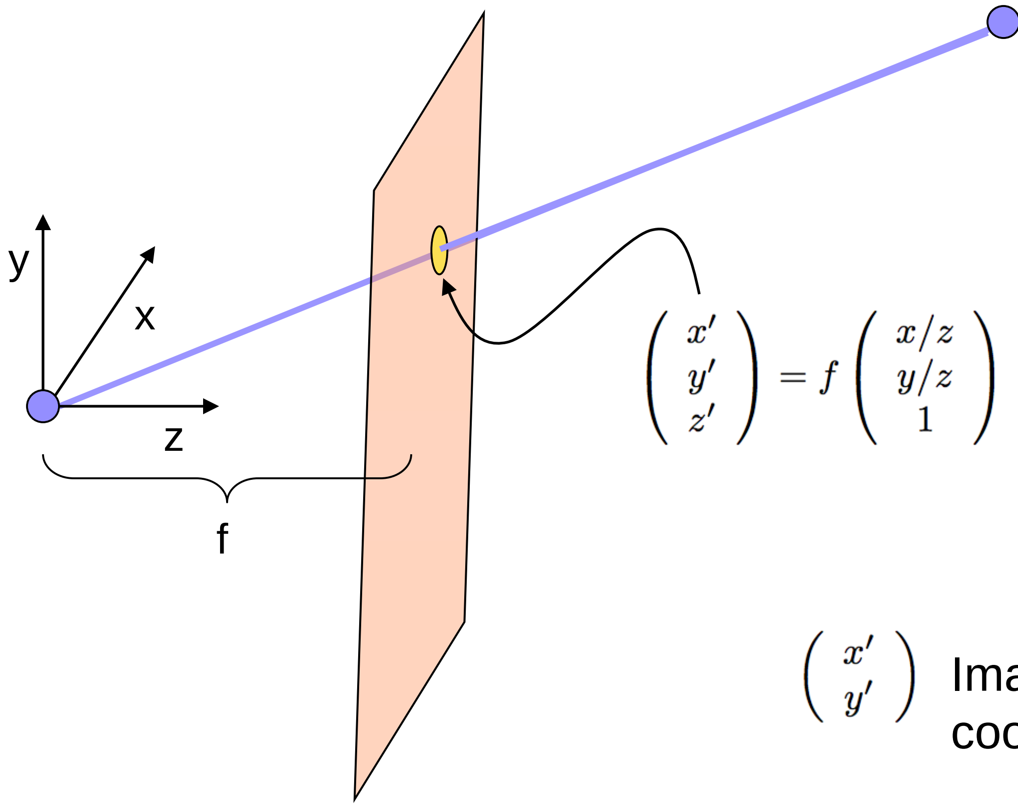
$$(x,y,z) \rightarrow (f'\frac{x}{z}, f'\frac{y}{z})$$

Scene point $\longrightarrow$ Image coordinates

Forsyth and Ponce

3D Perspective and Projection

- Camera model



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

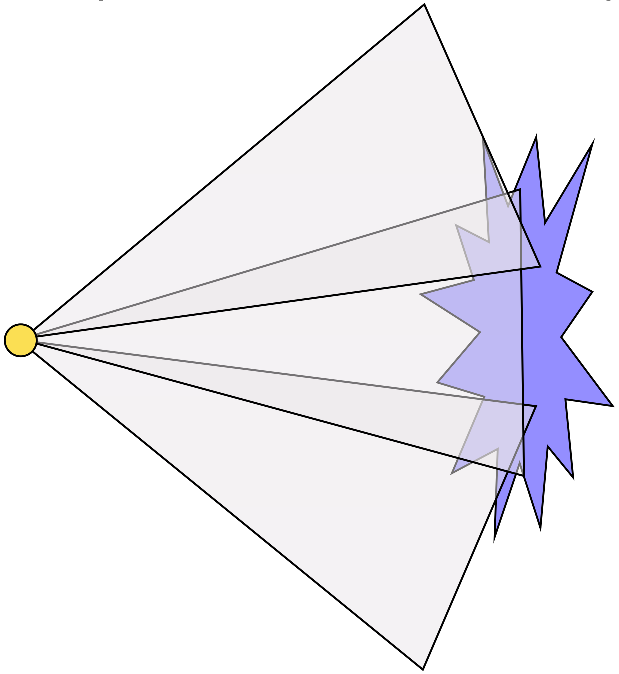
$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = f \begin{pmatrix} x/z \\ y/z \\ 1 \end{pmatrix}$$

$\begin{pmatrix} x' \\ y' \end{pmatrix}$ Image coordinates

Special Cases

- Nothing new in the scene is uncovered in one view vs another
  - No ray from the camera gets behind another

1) Pure rotations–arbitrary scene



2) Arbitrary views of planar surfaces

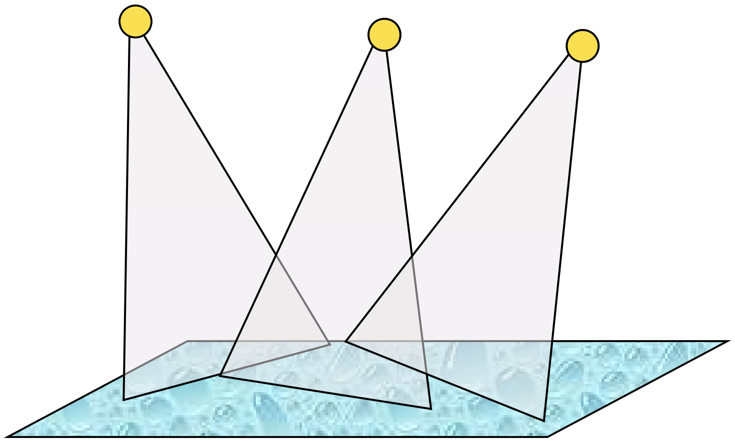
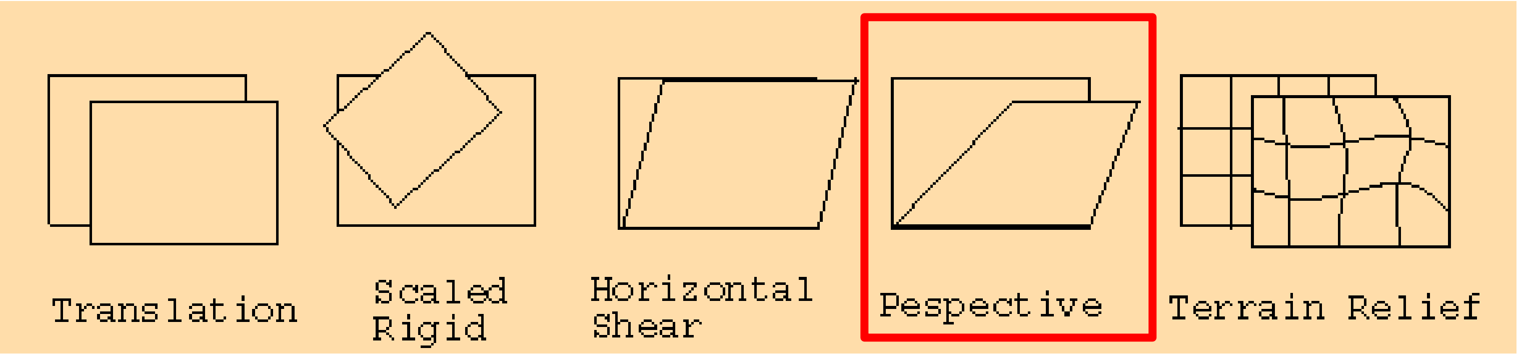


Image Homologies

- Images taken under cases 1,2 are perspectively equivalent to within a linear transformation
  - Projective relationships – equivalence is

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \equiv \begin{pmatrix} d \\ e \\ f \end{pmatrix} \Longleftrightarrow \begin{pmatrix} a/c \\ b/c \\ 1 \end{pmatrix} = \begin{pmatrix} d/f \\ e/f \\ 1 \end{pmatrix}$$

Transformations



$$\begin{pmatrix} X' \\ Y' \\ 1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

affine

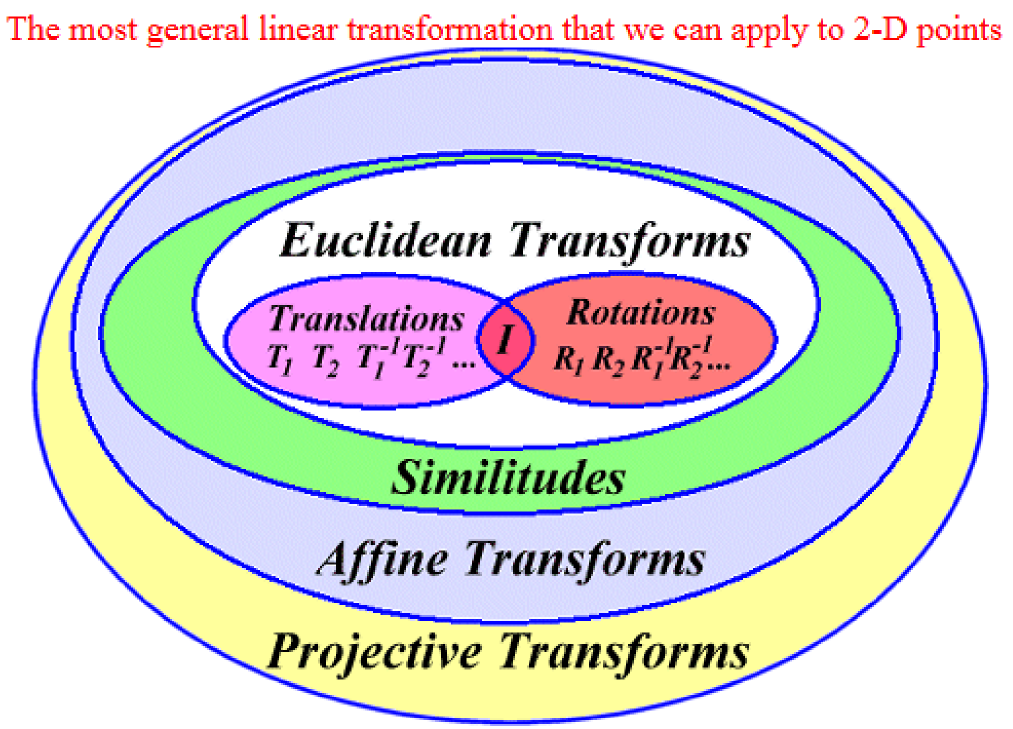
$$\boxed{\begin{pmatrix} X' \\ Y' \\ W \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}}$$

New image coordinates can be found as $x' = X'/W$, $y' = Y'/W$

x’, y’: homographies

Homography: Projective Spaces

Projective Transformations



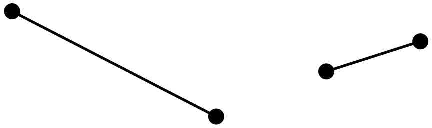
There is something different about this group of transformations. The result will not necessarily lie on our selected plane. Since our world (to this point) is 2D we need some way to deal with this.

$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

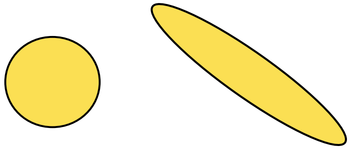
Source: http://groups.csail.mit.edu/graphics/classes/6.837/F01/Lecture07/Slide17.html

Perspective Projection Properties

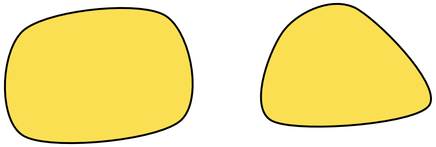
- Lines to lines (linear)



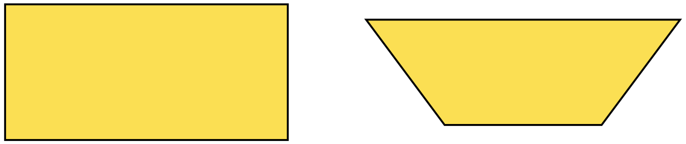
- Conic sections to conic sections



- Convex shapes to convex shapes



- Foreshortening



- See demo slides and math at:

  http://groups.csail.mit.edu/graphics/classes/6.837/F01/Lecture07/Slide22.html

Materials

- *Excellent material to derive homography matrix:*
  - *www.cs.toronto.edu/~jepson/csc2503/**tutorial2.pdf***
  - *www.cs.toronto.edu/pub/jepson/teaching/vision/2503/**tutorial2.pdf***

Degrees of Freedom

A projective transform has 8 free-parameters

$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

http://groups.csail.mit.edu/graphics/classes/6.837/F01/Lecture07/Slide20.html

Degrees of Freedom

A projective transform has 8 free-parameters

$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

which can be expressed as the following rational linear equation:

$$x' = \frac{p_{11}x + p_{12}y + p_{13}}{p_{31}x + p_{32}y + 1} \qquad y' = \frac{p_{21}x + p_{22}y + p_{23}}{p_{31}x + p_{32}y + 1}$$

http://groups.csail.mit.edu/graphics/classes/6.837/F01/Lecture07/Slide20.html

Degrees of Freedom

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rearranging terms gives a linear expression in the coefficients:

$$x' = p_{11}x + p_{12}y + p_{13} - p_{31}xx' - p_{32}yx'$$

$$y' = p_{21}x + p_{22}y + p_{23} - p_{31}xy' - p_{32}yy'$$

http://groups.csail.mit.edu/graphics/classes/6.837/F01/Lecture07/Slide20.html

Transforming Images To Make Mosaics

Linear transformation with matrix P

$$\bar{x}^* = P\bar{x} \qquad P = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & 1 \end{pmatrix} \qquad \begin{aligned} x^* &= p_{11}x + p_{12}y + p_{13} \\ y^* &= p_{21}x + p_{22}y + p_{23} \\ z^* &= p_{31}x + p_{32}y + 1 \end{aligned}$$

Perspective equivalence

$$\begin{aligned} x' &= \frac{p_{11}x + p_{12}y + p_{13}}{p_{31}x + p_{32}y + 1} \\ y' &= \frac{p_{21}x + p_{22}y + p_{23}}{p_{31}x + p_{32}y + 1} \end{aligned}$$

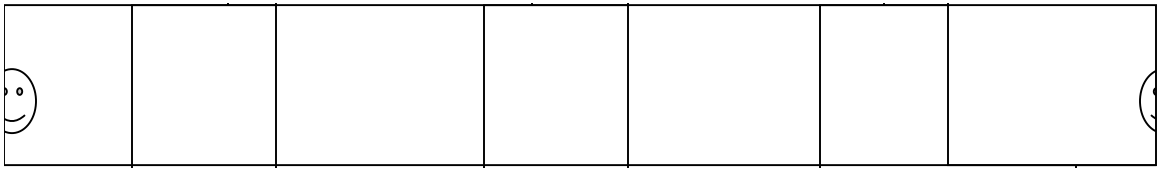
Multiply by denominator and reorganize terms

$$\begin{aligned} p_{31}xx' + p_{32}yx' - p_{11}x - p_{12}y - p_{13} &= -x' \\ p_{31}xy' + p_{32}yy' - p_{21}x - p_{22}y - p_{23} &= -y' \end{aligned}$$

Linear system, solve for P

$$\begin{pmatrix} -x_1 & -y_1 & -1 & 0 & 0 & 0 & x_1x'_1 & y_1x'_1 \\ -x_2 & -y_2 & -1 & 0 & 0 & 0 & x_2x'_2 & y_2x'_2 \\ & & & \vdots & & & & \\ -x_N & -y_N & -1 & 0 & 0 & 0 & x_Nx'_N & y_Nx'_2 \\ 0 & 0 & 0 & -x_1 & -y_1 & -1 & x_1y'_1 & y_1y'_1 \\ 0 & 0 & 0 & -x_2 & -y_2 & -1 & x_2y'_2 & y_2y'_2 \\ & & & \vdots & & & & \\ 0 & 0 & 0 & -x_N & -y_N & -1 & x_Ny'_N & y_Ny'_N \end{pmatrix} \begin{pmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{21} \\ p_{23} \\ p_{23} \\ p_{31} \\ p_{32} \end{pmatrix} = \begin{pmatrix} -x'_1 \\ -x'_2 \\ \vdots \\ -x'_N \\ -y'_1 \\ -y'_2 \\ \vdots \\ -y'_N \end{pmatrix}$$

Image Stitching



Stitch pairs together, blend, then crop

Image Stitching

A big image stitched from 5 small images

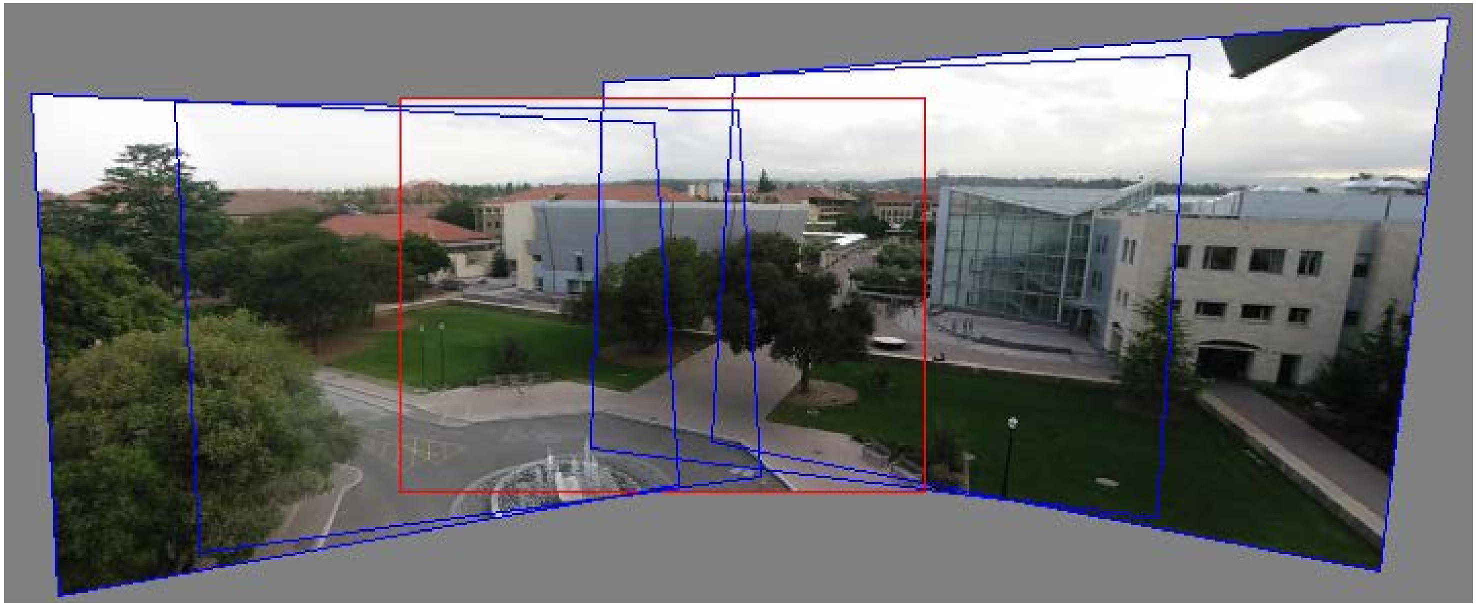


Image Mosaicing


4 Correspondences

5 Correspondences

6 Correspondences

Mosaicing Issues

- Need a canvas (adjust coordinates/origin)
- Blending at edges of images (avoid sharp transitions)
- Adjusting brightnesses
- Cascading transformations

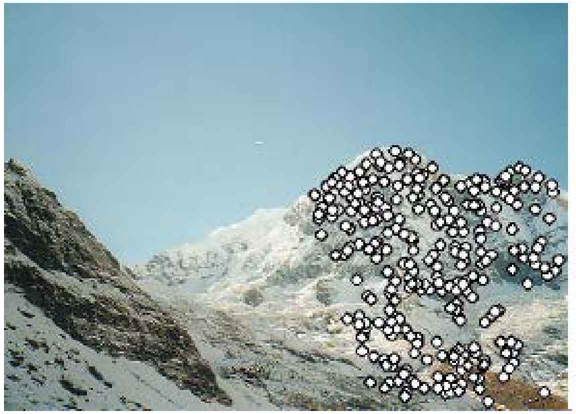
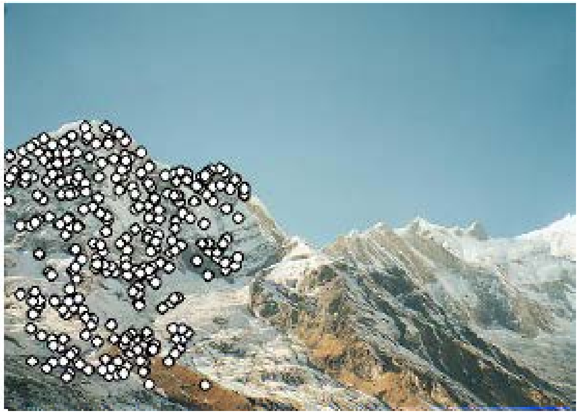
Recognizing panoramas

- A fully automatic 2D image stitcher system

Recognizing panoramas

- A fully automatic 2D image stitcher system

- Image matching with SIFT features
- For every image, find the M best images with RANSAC
- Form a graph and find connected component in the graph
- Stitching and blending.

Automatic Solutions

Intensity Based Image Mosaicing

- Transformation:

$$x_i^{'} = \frac{m_0 x_i + m_1 y_i + m_2}{m_6 x_i + m_7 y_i + 1}$$

$$y_i^{'} = \frac{m_3 x_i + m_4 y_i + m_5}{m_6 x_i + m_7 y_i + 1}$$

- **Problem**: Determining the transformation parameters $\mathbf{m_i}$ between every two adjacent images, in order to merge the set of images into a single complete image.
- **Idea**: to choose the parameters $\mathbf{m_i}$ such that the sum of squared difference between all pixels between the two images is minimized

$$E = \sum_i \left[I' \left(x_i^{'}, y_i^{'} \right) - I \left(x_i, y_i \right) \right]^2 = \sum_i e_i^2$$

- Non-linear minimization, e.g. by Levenberg Marquardt algorithm
- See: http://www.umiacs.umd.edu/~hismail/Mosaic/node2.html#SECTION00020000000000000000