Computer Vision: Algorithms and Applications

Richard Szeliski

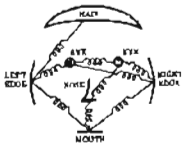
September 3, 2010 draft

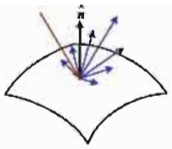
This book is dedicated to my parents,

Zdzisław and Jadwiga,

and my family,

Lyn, Anne, and Stephen.





Preface

The seeds for this book were first planted in 2001 when Steve Seitz at the University of Washington invited me to co-teach a course called "Computer Vision for Computer Graphics". At that time, computer vision techniques were increasingly being used in computer graphics to create image-based models of real-world objects, to create visual effects, and to merge real-world imagery using computational photography techniques. Our decision to focus on the applications of computer vision to fun problems such as image stitching and photo-based 3D modeling from personal photos seemed to resonate well with our students.

Since that time, a similar syllabus and project-oriented course structure has been used to teach general computer vision courses both at the University of Washington and at Stanford. (The latter was a course I co-taught with David Fleet in 2003.) Similar curricula have been adopted at a number of other universities and also incorporated into more specialized courses on computational photography. (For ideas on how to use this book in your own course, please see Table 1.1 in Section 1.4.)

This book also reflects my 20 years' experience doing computer vision research in corporate research labs, mostly at Digital Equipment Corporation's Cambridge Research Lab and at Microsoft Research. In pursuing my work, I have mostly focused on problems and solution techniques (algorithms) that have practical real-world applications and that work well in practice. Thus, this book has more emphasis on basic techniques that work under real-world conditions and less on more esoteric mathematics that has intrinsic elegance but less practical applicability.

This book is suitable for teaching a senior-level undergraduate course in computer vision to students in both computer science and electrical engineering. I prefer students to have either an image processing or a computer graphics course as a prerequisite so that they can spend less time learning general background mathematics and more time studying computer vision techniques. The book is also suitable for teaching graduate-level courses in computer vision (by delving into the more demanding application and algorithmic areas) and as a general reference to fundamental techniques and the recent research literature. To this end, I have attempted wherever possible to at least cite the newest research in each sub-field, even if the

technical details are too complex to cover in the book itself.

In teaching our courses, we have found it useful for the students to attempt a number of small implementation projects, which often build on one another, in order to get them used to working with real-world images and the challenges that these present. The students are then asked to choose an individual topic for each of their small-group, final projects. (Sometimes these projects even turn into conference papers!) The exercises at the end of each chapter contain numerous suggestions for smaller mid-term projects, as well as more open-ended problems whose solutions are still active research topics. Wherever possible, I encourage students to try their algorithms on their own personal photographs, since this better motivates them, often leads to creative variants on the problems, and better acquaints them with the variety and complexity of real-world imagery.

In formulating and solving computer vision problems, I have often found it useful to draw inspiration from three high-level approaches:

- **Scientific:** build detailed models of the image formation process and develop mathematical techniques to invert these in order to recover the quantities of interest (where necessary, making simplifying assumption to make the mathematics more tractable).
- **Statistical:** use probabilistic models to quantify the prior likelihood of your unknowns and the noisy measurement processes that produce the input images, then infer the best possible estimates of your desired quantities and analyze their resulting uncertainties. The inference algorithms used are often closely related to the optimization techniques used to invert the (scientific) image formation processes.
- **Engineering:** develop techniques that are simple to describe and implement but that are also known to work well in practice. Test these techniques to understand their limitation and failure modes, as well as their expected computational costs (run-time performance).

These three approaches build on each other and are used throughout the book.

My personal research and development philosophy (and hence the exercises in the book) have a strong emphasis on *testing* algorithms. It's too easy in computer vision to develop an algorithm that does something *plausible* on a few images rather than something *correct*. The best way to validate your algorithms is to use a three-part strategy.

First, test your algorithm on clean synthetic data, for which the exact results are known. Second, add noise to the data and evaluate how the performance degrades as a function of noise level. Finally, test the algorithm on real-world data, preferably drawn from a wide variety of sources, such as photos found on the Web. Only then can you truly know if your algorithm can deal with real-world complexity, i.e., images that do not fit some simplified model or assumptions.

In order to help students in this process, this books comes with a large amount of supplementary material, which can be found on the book's Web site http://szeliski.org/Book. This material, which is described in Appendix C, includes:

- pointers to commonly used data sets for the problems, which can be found on the Web
- pointers to software libraries, which can help students get started with basic tasks such as reading/writing images or creating and manipulating images
- slide sets corresponding to the material covered in this book
- a BibTeX bibliography of the papers cited in this book.

The latter two resources may be of more interest to instructors and researchers publishing new papers in this field, but they will probably come in handy even with regular students. Some of the software libraries contain implementations of a wide variety of computer vision algorithms, which can enable you to tackle more ambitious projects (with your instructor's consent).

Acknowledgements

I would like to gratefully acknowledge all of the people whose passion for research and inquiry as well as encouragement have helped me write this book.

Steve Zucker at McGill University first introduced me to computer vision, taught all of his students to question and debate research results and techniques, and encouraged me to pursue a graduate career in this area.

Takeo Kanade and Geoff Hinton, my Ph. D. thesis advisors at Carnegie Mellon University, taught me the fundamentals of good research, writing, and presentation. They fired up my interest in visual processing, 3D modeling, and statistical methods, while Larry Matthies introduced me to Kalman filtering and stereo matching.

Demetri Terzopoulos was my mentor at my first industrial research job and taught me the ropes of successful publishing. Yvan Leclerc and Pascal Fua, colleagues from my brief interlude at SRI International, gave me new perspectives on alternative approaches to computer vision.

During my six years of research at Digital Equipment Corporation's Cambridge Research Lab, I was fortunate to work with a great set of colleagues, including Ingrid Carlbom, Gudrun Klinker, Keith Waters, Richard Weiss, Stéphane Lavallée, and Sing Bing Kang, as well as to supervise the first of a long string of outstanding summer interns, including David Tonnesen, Sing Bing Kang, James Coughlan, and Harry Shum. This is also where I began my long-term collaboration with Daniel Scharstein, now at Middlebury College.

At Microsoft Research, I've had the outstanding fortune to work with some of the world's best researchers in computer vision and computer graphics, including Michael Cohen, Hugues Hoppe, Stephen Gortler, Steve Shafer, Matthew Turk, Harry Shum, Anandan, Phil Torr, Antonio Criminisi, Georg Petschnigg, Kentaro Toyama, Ramin Zabih, Shai Avidan, Sing Bing Kang, Matt Uyttendaele, Patrice Simard, Larry Zitnick, Richard Hartley, Simon Winder, Drew Steedly, Chris Pal, Nebojsa Jojic, Patrick Baudisch, Dani Lischinski, Matthew Brown, Simon Baker, Michael Goesele, Eric Stollnitz, David Nistér, Blaise Aguera y Arcas, Sudipta Sinha, Johannes Kopf, Neel Joshi, and Krishnan Ramnath. I was also lucky to have as interns such great students as Polina Golland, Simon Baker, Mei Han, Arno Schödl, Ron Dror, Ashley Eden, Jinxiang Chai, Rahul Swaminathan, Yanghai Tsin, Sam Hasinoff, Anat Levin, Matthew Brown, Eric Bennett, Vaibhav Vaish, Jan-Michael Frahm, James Diebel, Ce Liu, Josef Sivic, Grant Schindler, Colin Zheng, Neel Joshi, Sudipta Sinha, Zeev Farbman, Rahul Garg, Tim Cho, Yekeun Jeong, Richard Roberts, Varsha Hedau, and Dilip Krishnan.

While working at Microsoft, I've also had the opportunity to collaborate with wonderful colleagues at the University of Washington, where I hold an Affiliate Professor appointment. I'm indebted to Tony DeRose and David Salesin, who first encouraged me to get involved with the research going on at UW, my long-time collaborators Brian Curless, Steve Seitz, Maneesh Agrawala, Sameer Agarwal, and Yasu Furukawa, as well as the students I have had the privilege to supervise and interact with, including Fréderic Pighin, Yung-Yu Chuang, Doug Zongker, Colin Zheng, Aseem Agarwala, Dan Goldman, Noah Snavely, Rahul Garg, and Ryan Kaminsky. As I mentioned at the beginning of this preface, this book owes its inception to the vision course that Steve Seitz invited me to co-teach, as well as to Steve's encouragement, course notes, and editorial input.

I'm also grateful to the many other computer vision researchers who have given me so many constructive suggestions about the book, including Sing Bing Kang, who was my informal book editor, Vladimir Kolmogorov, who contributed Appendix B.5.5 on linear programming techniques for MRF inference, Daniel Scharstein, Richard Hartley, Simon Baker, Noah Snavely, Bill Freeman, Svetlana Lazebnik, Matthew Turk, Jitendra Malik, Alyosha Efros, Michael Black, Brian Curless, Sameer Agarwal, Li Zhang, Deva Ramanan, Olga Veksler, Yuri Boykov, Carsten Rother, Phil Torr, Bill Triggs, Bruce Maxwell, Jana Košecká, Eero Simoncelli, Aaron Hertzmann, Antonio Torralba, Tomaso Poggio, Theo Pavlidis, Baba Vemuri, Nando de Freitas, Chuck Dyer, Song Yi, Falk Schubert, Roman Pflugfelder, Marshall Tappen, James Coughlan, Sammy Rogmans, Klaus Strobel, Shanmuganathan, Andreas Siebert, Yongjun Wu, Fred Pighin, Juan Cockburn, Ronald Mallet, Tim Soper, Georgios Evangelidis, Dwight Fowler, Itzik Bayaz, Daniel O'Connor, and Srikrishna Bhat. Shena Deuchers did a fantastic job copy-editing the book and suggesting many useful improvements and Wayne Wheeler and Simon Rees at Springer were most helpful throughout the whole book publishing process. Keith Price's Annotated Computer Vision Bibliography was invaluable in

tracking down references and finding related work.

If you have any suggestions for improving the book, please send me an e-mail, as I would like to keep the book as accurate, informative, and timely as possible.

Lastly, this book would not have been possible or worthwhile without the incredible support and encouragement of my family. I dedicate this book to my parents, Zdzisław and Jadwiga, whose love, generosity, and accomplishments have always inspired me; to my sister Basia for her lifelong friendship; and especially to Lyn, Anne, and Stephen, whose daily encouragement in all matters (including this book project) makes it all worthwhile.

Lake Wenatchee
August, 2010

Contents

Chapter 1

Introduction

Figure 1.1 The human visual system has no problem interpreting the subtle variations in translucency and shading in this photograph and correctly segmenting the object from its background.

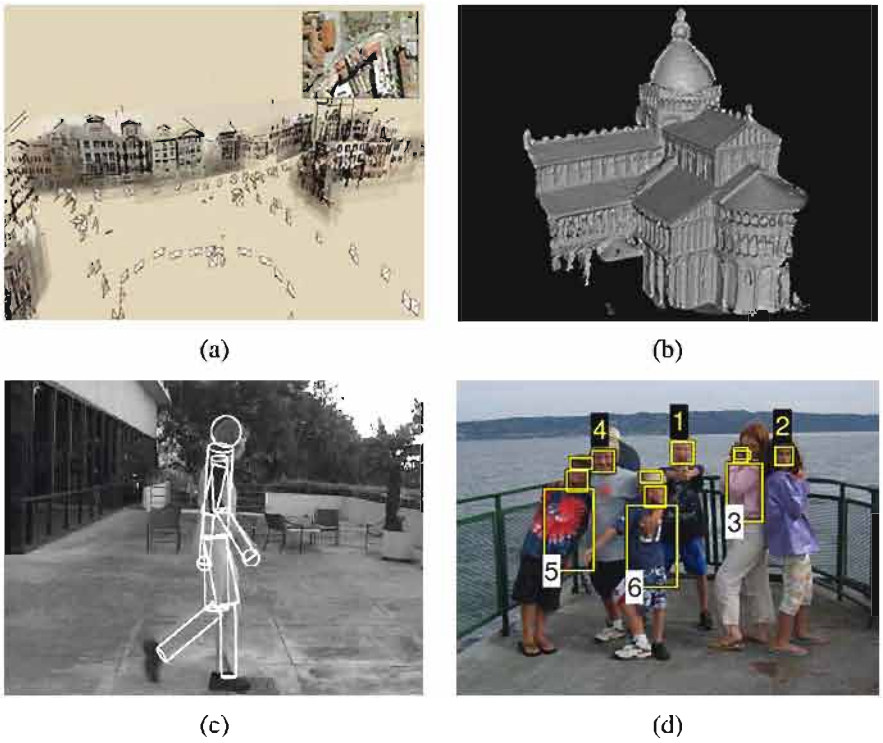


Figure 1.2 Some examples of computer vision algorithms and applications. (a) *Structure from motion* algorithms can reconstruct a sparse 3D point model of a large complex scene from hundreds of partially overlapping photographs (Snavely, Seitz, and Szeliski 2006) © 2006 ACM. (b) *Stereo matching* algorithms can build a detailed 3D model of a building façade from hundreds of differently exposed photographs taken from the Internet (Goesele, Snavely, Curless *et al.* 2007) © 2007 IEEE. (c) *Person tracking* algorithms can track a person walking in front of a cluttered background (Sidenbladh, Black, and Fleet 2000) © 2000 Springer. (d) *Face detection* algorithms, coupled with color-based clothing and hair detection algorithms, can locate and recognize the individuals in this image (Sivic, Zitnick, and Szeliski 2006) © 2006 Springer.

1.1 What is computer vision?

As humans, we perceive the three-dimensional structure of the world around us with apparent ease. Think of how vivid the three-dimensional percept is when you look at a vase of flowers sitting on the table next to you. You can tell the shape and translucency of each petal through the subtle patterns of light and shading that play across its surface and effortlessly segment each flower from the background of the scene (Figure 1.1). Looking at a framed group portrait, you can easily count (and name) all of the people in the picture and even guess at their emotions from their facial appearance. Perceptual psychologists have spent decades trying to understand how the visual system works and, even though they can devise optical illusions[1] to tease apart some of its principles (Figure 1.3), a complete solution to this puzzle remains elusive (Marr 1982; Palmer 1999; Livingstone 2008).

Researchers in computer vision have been developing, in parallel, mathematical techniques for recovering the three-dimensional shape and appearance of objects in imagery. We now have reliable techniques for accurately computing a partial 3D model of an environment from thousands of partially overlapping photographs (Figure 1.2a). Given a large enough set of views of a particular object or façade, we can create accurate dense 3D surface models using stereo matching (Figure 1.2b). We can track a person moving against a complex background (Figure 1.2c). We can even, with moderate success, attempt to find and name all of the people in a photograph using a combination of face, clothing, and hair detection and recognition (Figure 1.2d). However, despite all of these advances, the dream of having a computer interpret an image at the same level as a two-year old (for example, counting all of the animals in a picture) remains elusive. Why is vision so difficult? In part, it is because vision is an *inverse problem*, in which we seek to recover some unknowns given insufficient information to fully specify the solution. We must therefore resort to physics-based and probabilistic *models* to disambiguate between potential solutions. However, modeling the visual world in all of its rich complexity is far more difficult than, say, modeling the vocal tract that produces spoken sounds.

The *forward* models that we use in computer vision are usually developed in physics (radiometry, optics, and sensor design) and in computer graphics. Both of these fields model how objects move and animate, how light reflects off their surfaces, is scattered by the atmosphere, refracted through camera lenses (or human eyes), and finally projected onto a flat (or curved) image plane. While computer graphics are not yet perfect (no fully computer-animated movie with human characters has yet succeeded at crossing the *uncanny valley*[2] that separates real humans from android robots and computer-animated humans), in limited

[1] http://www.michaelbach.de/ot/sze_muelue

[2] The term *uncanny valley* was originally coined by roboticist Masahiro Mori as applied to robotics (Mori 1970). It is also commonly applied to computer-animated films such as *Final Fantasy* and *Polar Express* (Geller 2008).

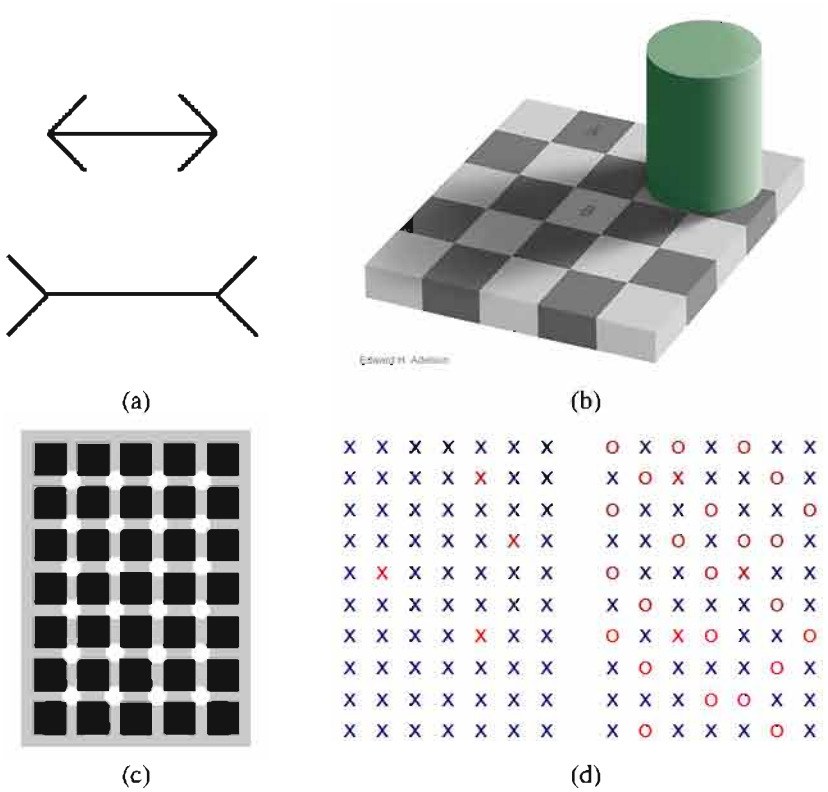


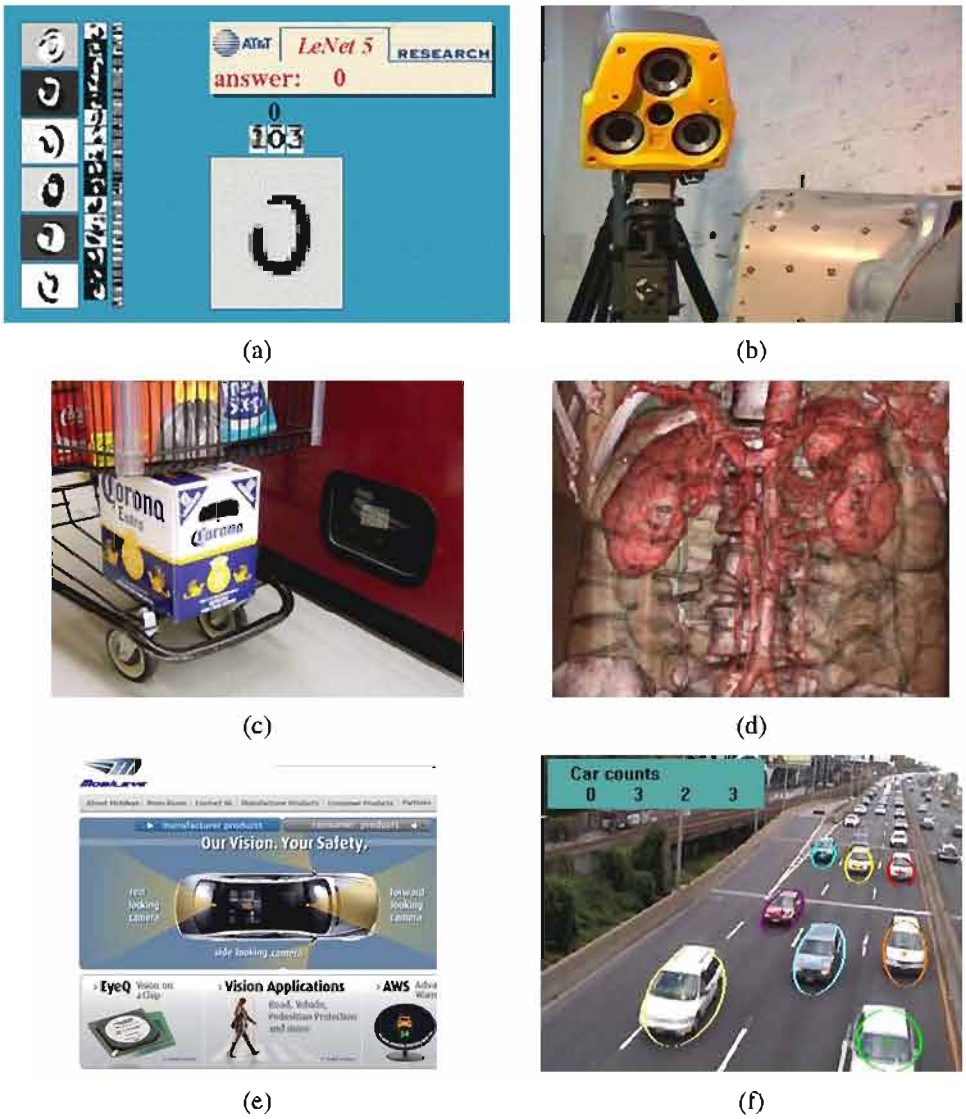
Figure 1.3 Some common optical illusions and what they might tell us about the visual system: (a) The classic Müller-Lyer illusion, where the length of the two horizontal lines appear different, probably due to the imagined perspective effects. (b) The "white" square B in the shadow and the "black" square A in the light actually have the same absolute intensity value. The percept is due to *brightness constancy*, the visual system's attempt to discount illumination when interpreting colors. Image courtesy of Ted Adelson, http://web.mit.edu/persci/people/adelson/checkershadow_illusion.html. (c) A variation of the Hermann grid illusion, courtesy of Hany Farid, http://www.cs.dartmouth.edu/~farid/illusions/hermann.html. As you move your eyes over the figure, gray spots appear at the intersections. (d) Count the red *X*s in the left half of the figure. Now count them in the right half. Is it significantly harder? The explanation has to do with a *pop-out* effect (Treisman 1985), which tells us about the operations of parallel perception and integration pathways in the brain.

domains, such as rendering a still scene composed of everyday objects or animating extinct creatures such as dinosaurs, the illusion of reality *is* perfect.

In computer vision, we are trying to do the inverse, i.e., to describe the world that we see in one or more images and to reconstruct its properties, such as shape, illumination, and color distributions. It is amazing that humans and animals do this so effortlessly, while computer vision algorithms are so error prone. People who have not worked in the field often underestimate the difficulty of the problem. (Colleagues at work often ask me for software to find and name all the people in photos, so they can get on with the more "interesting" work.) This misperception that vision should be easy dates back to the early days of artificial intelligence (see Section 1.2), when it was initially believed that the *cognitive* (logic proving and planning) parts of intelligence were intrinsically more difficult than the *perceptual* components (Boden 2006).

The good news is that computer vision *is* being used today in a wide variety of real-world applications, which include:

- **Optical character recognition (OCR):** reading handwritten postal codes on letters (Figure 1.4a) and automatic number plate recognition (ANPR);
- **Machine inspection:** rapid parts inspection for quality assurance using stereo vision with specialized illumination to measure tolerances on aircraft wings or auto body parts (Figure 1.4b) or looking for defects in steel castings using X-ray vision;
- **Retail:** object recognition for automated checkout lanes (Figure 1.4c);
- **3D model building (photogrammetry):** fully automated construction of 3D models from aerial photographs used in systems such as Bing Maps;
- **Medical imaging:** registering pre-operative and intra-operative imagery (Figure 1.4d) or performing long-term studies of people's brain morphology as they age;
- **Automotive safety:** detecting unexpected obstacles such as pedestrians on the street, under conditions where active vision techniques such as radar or lidar do not work well (Figure 1.4e; see also Miller, Campbell, Huttenlocher *et al.* (2008); Montemerlo, Becker, Bhat *et al.* (2008); Urmson, Anhalt, Bagnell *et al.* (2008) for examples of fully automated driving);
- **Match move:** merging computer-generated imagery (CGI) with live action footage by tracking feature points in the source video to estimate the 3D camera motion and shape of the environment. Such techniques are widely used in Hollywood (e.g., in movies such as Jurassic Park) (Roble 1999; Roble and Zafar 2009); they also require the use of precise *matting* to insert new elements between foreground and background elements (Chuang, Agarwala, Curless *et al.* 2002).



(a) (b) (c) (d) (e) (f)

Figure 1.4 Some industrial applications of computer vision: (a) optical character recognition (OCR) http://yann.lecun.com/exdb/lenet/; (b) mechanical inspection http://www.cognitens.com/; (c) retail http://www.evoretail.com/; (d) medical imaging http://www.clarontech.com/; (e) automotive safety http://www.mobileye.com/; (f) surveillance and traffic monitoring http://www.honeywellvideo.com/, courtesy of Honeywell International Inc.

- **Motion capture (mocap):** using retro-reflective markers viewed from multiple cameras or other vision-based techniques to capture actors for computer animation;
- **Surveillance:** monitoring for intruders, analyzing highway traffic (Figure 1.4f), and monitoring pools for drowning victims;
- **Fingerprint recognition and biometrics:** for automatic access authentication as well as forensic applications.

David Lowe's Web site of industrial vision applications (http://www.cs.ubc.ca/spider/lowe/vision.html) lists many other interesting industrial applications of computer vision. While the above applications are all extremely important, they mostly pertain to fairly specialized kinds of imagery and narrow domains.

In this book, we focus more on broader *consumer-level* applications, such as fun things you can do with your own personal photographs and video. These include:

- **Stitching:** turning overlapping photos into a single seamlessly stitched panorama (Figure 1.5a), as described in Chapter 9;
- **Exposure bracketing:** merging multiple exposures taken under challenging lighting conditions (strong sunlight and shadows) into a single perfectly exposed image (Figure 1.5b), as described in Section 10.2;
- **Morphing:** turning a picture of one of your friends into another, using a seamless *morph* transition (Figure 1.5c);
- **3D modeling:** converting one or more snapshots into a 3D model of the object or person you are photographing (Figure 1.5d), as described in Section 12.6
- **Video match move and stabilization:** inserting 2D pictures or 3D models into your videos by automatically tracking nearby reference points (see Section 7.4.2)[3] or using motion estimates to remove shake from your videos (see Section 8.2.1);
- **Photo-based walkthroughs:** navigating a large collection of photographs, such as the interior of your house, by flying between different photos in 3D (see Sections 13.1.2 and 13.5.5)
- **Face detection:** for improved camera focusing as well as more relevant image searching (see Section 14.1.1);
- **Visual authentication:** automatically logging family members onto your home computer as they sit down in front of the webcam (see Section 14.2).

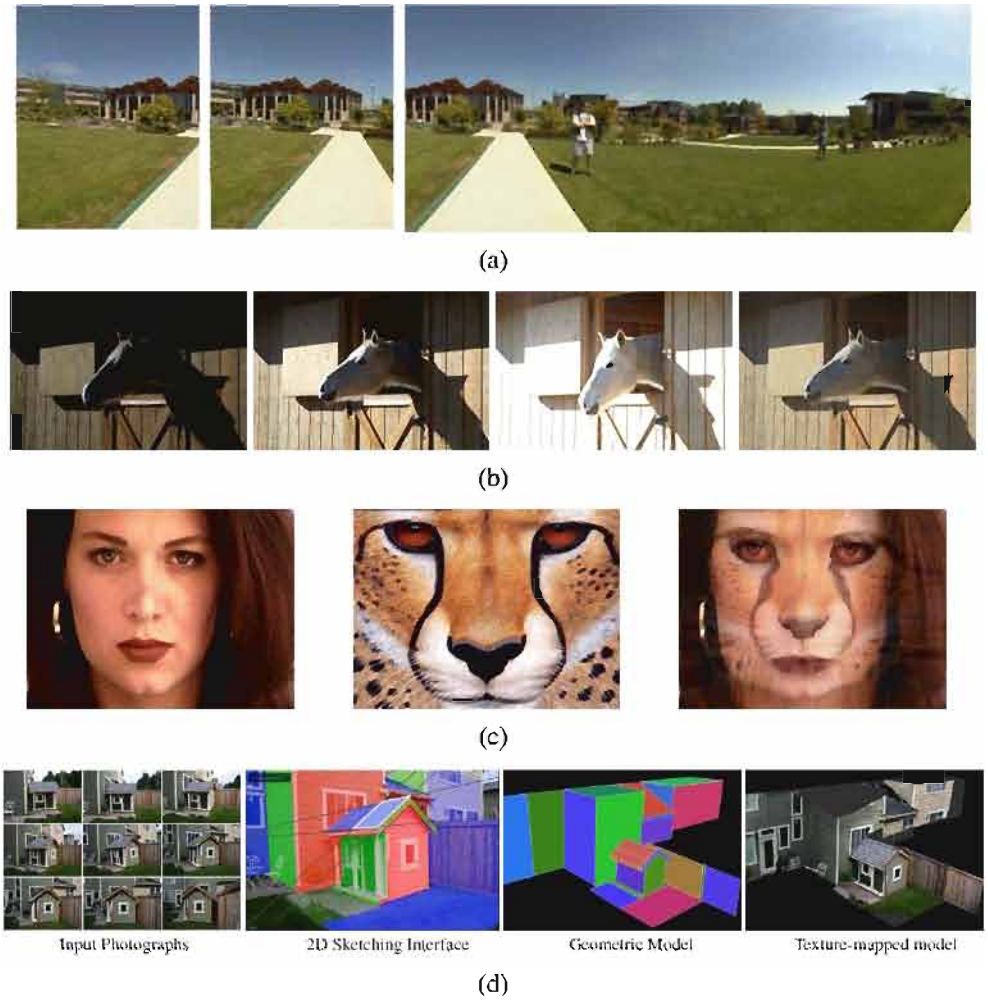


Figure 1.5 Some consumer applications of computer vision: (a) image stitching: merging different views (Szeliski and Shum 1997) © 1997 ACM; (b) exposure bracketing: merging different exposures; (c) morphing: blending between two photographs (Gomes, Darsa, Costa *et al.* 1999) © 1999 Morgan Kaufmann; (d) turning a collection of photographs into a 3D model (Sinha, Steedly, Szeliski *et al.* 2008) © 2008 ACM.

The great thing about these applications is that they are already familiar to most students; they are, at least, technologies that students can immediately appreciate and use with their own personal media. Since computer vision is a challenging topic, given the wide range of mathematics being covered[4] and the intrinsically difficult nature of the problems being solved, having fun and relevant problems to work on can be highly motivating and inspiring.

The other major reason why this book has a strong focus on applications is that they can be used to *formulate* and *constrain* the potentially open-ended problems endemic in vision. For example, if someone comes to me and asks for a good edge detector, my first question is usually to ask *why*? What kind of problem are they trying to solve and why do they believe that edge detection is an important component? If they are trying to locate faces, I usually point out that most successful face detectors use a combination of skin color detection (Exercise 2.8) and simple blob features Section 14.1.1; they do not rely on edge detection. If they are trying to match door and window edges in a building for the purpose of 3D reconstruction, I tell them that edges are a fine idea but it is better to tune the edge detector for long edges (see Sections 3.2.3 and 4.2) and link them together into straight lines with common vanishing points before matching (see Section 4.3).

Thus, it is better to think back from the problem at hand to suitable techniques, rather than to grab the first technique that you may have heard of. This kind of working back from problems to solutions is typical of an **engineering** approach to the study of vision and reflects my own background in the field. First, I come up with a detailed problem definition and decide on the constraints and specifications for the problem. Then, I try to find out which techniques are known to work, implement a few of these, evaluate their performance, and finally make a selection. In order for this process to work, it is important to have realistic **test data**, both synthetic, which can be used to verify correctness and analyze noise sensitivity, and real-world data typical of the way the system will finally be used.

However, this book is not just an engineering text (a source of recipes). It also takes a **scientific** approach to basic vision problems. Here, I try to come up with the best possible models of the physics of the system at hand: how the scene is created, how light interacts with the scene and atmospheric effects, and how the sensors work, including sources of noise and uncertainty. The task is then to try to invert the acquisition process to come up with the best possible description of the scene.

The book often uses a **statistical** approach to formulating and solving computer vision problems. Where appropriate, probability distributions are used to model the scene and the noisy image acquisition process. The association of prior distributions with unknowns is often

[3] For a fun student project on this topic, see the "PhotoBook" project at http://www.cc.gatech.edu/dvfx/videos/dvfx2005.html.

[4] These techniques include physics, Euclidean and projective geometry, statistics, and optimization. They make computer vision a fascinating field to study and a great way to learn techniques widely applicable in other fields.

called *Bayesian modeling* (Appendix B). It is possible to associate a risk or loss function with mis-estimating the answer (Section B.2) and to set up your inference algorithm to minimize the expected risk. (Consider a robot trying to estimate the distance to an obstacle: it is usually safer to underestimate than to overestimate.) With statistical techniques, it often helps to gather lots of training data from which to learn probabilistic models. Finally, statistical approaches enable you to use proven inference techniques to estimate the best answer (or distribution of answers) and to quantify the uncertainty in the resulting estimates.

Because so much of computer vision involves the solution of inverse problems or the estimation of unknown quantities, my book also has a heavy emphasis on **algorithms**, especially those that are known to work well in practice. For many vision problems, it is all too easy to come up with a mathematical description of the problem that either does not match realistic real-world conditions or does not lend itself to the stable estimation of the unknowns. What we need are algorithms that are both **robust** to noise and deviation from our models and reasonably **efficient** in terms of run-time resources and space. In this book, I go into these issues in detail, using Bayesian techniques, where applicable, to ensure robustness, and efficient search, minimization, and linear system solving algorithms to ensure efficiency. Most of the algorithms described in this book are at a high level, being mostly a list of steps that have to be filled in by students or by reading more detailed descriptions elsewhere. In fact, many of the algorithms are sketched out in the exercises.

Now that I've described the goals of this book and the frameworks that I use, I devote the rest of this chapter to two additional topics. Section 1.2 is a brief synopsis of the history of computer vision. It can easily be skipped by those who want to get to "the meat" of the new material in this book and do not care as much about who invented what when.

The second is an overview of the book's contents, Section 1.3, which is useful reading for everyone who intends to make a study of this topic (or to jump in partway, since it describes chapter inter-dependencies). This outline is also useful for instructors looking to structure one or more courses around this topic, as it provides sample curricula based on the book's contents.

1.2 A brief history

In this section, I provide a brief personal synopsis of the main developments in computer vision over the last 30 years (Figure 1.6); at least, those that I find personally interesting and which appear to have stood the test of time. Readers not interested in the provenance of various ideas and the evolution of this field should skip ahead to the book overview in Section 1.3.

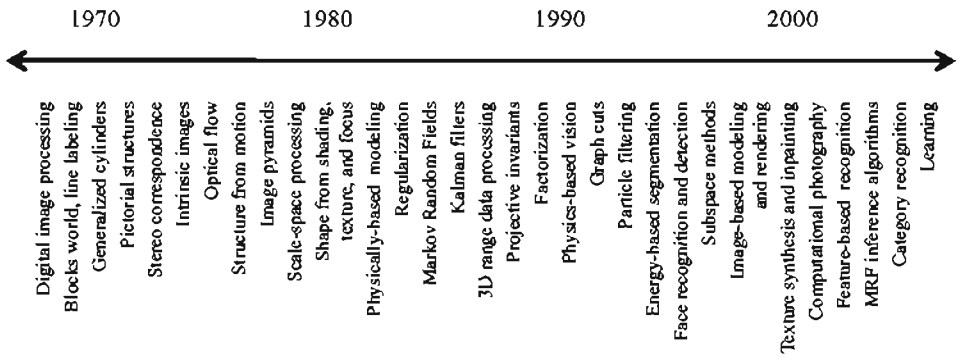


Figure 1.6 A rough timeline of some of the most active topics of research in computer vision.

1970s. When computer vision first started out in the early 1970s, it was viewed as the visual perception component of an ambitious agenda to mimic human intelligence and to endow robots with intelligent behavior. At the time, it was believed by some of the early pioneers of artificial intelligence and robotics (at places such as MIT, Stanford, and CMU) that solving the "visual input" problem would be an easy step along the path to solving more difficult problems such as higher-level reasoning and planning. According to one well-known story, in 1966, Marvin Minsky at MIT asked his undergraduate student Gerald Jay Sussman to "spend the summer linking a camera to a computer and getting the computer to describe what it saw" (Boden 2006, p. 781).[5] We now know that the problem is slightly more difficult than that.[6]

What distinguished computer vision from the already existing field of digital image processing (Rosenfeld and Pfaltz 1966; Rosenfeld and Kak 1976) was a desire to recover the three-dimensional structure of the world from images and to use this as a stepping stone towards full scene understanding. Winston (1975) and Hanson and Riseman (1978) provide two nice collections of classic papers from this early period.

Early attempts at scene understanding involved extracting edges and then inferring the 3D structure of an object or a "blocks world" from the topological structure of the 2D lines (Roberts 1965). Several *line labeling* algorithms (Figure 1.7a) were developed at that time (Huffman 1971; Clowes 1971; Waltz 1975; Rosenfeld, Hummel, and Zucker 1976; Kanade 1980). Nalwa (1993) gives a nice review of this area. The topic of edge detection was also

[5] Boden (2006) cites (Crevier 1993) as the original source. The actual Vision Memo was authored by Seymour Papert (1966) and involved a whole cohort of students.

[6] To see how far robotic vision has come in the last four decades, have a look at the towel-folding robot at http://rll.eecs.berkeley.edu/pr/icra10/ (Maitin-Shepard, Cusumano-Towner, Lei *et al.* 2010).

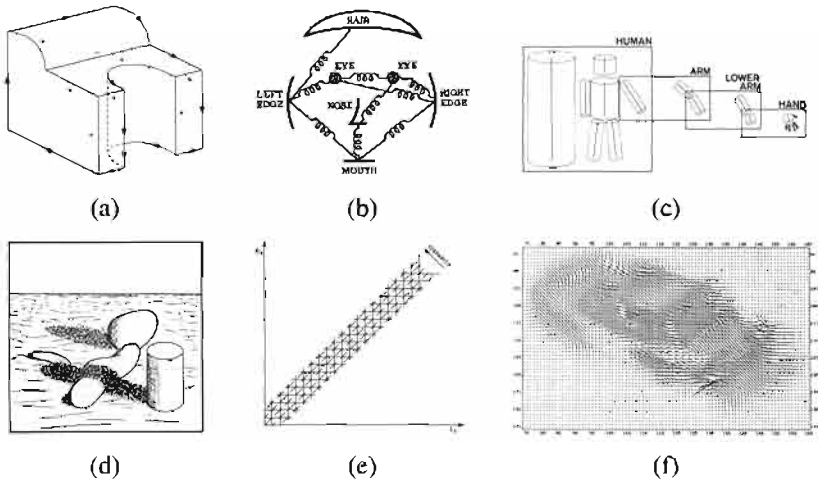


Figure 1.7 Some early (1970s) examples of computer vision algorithms: (a) line labeling (Nalwa 1993) © 1993 Addison-Wesley, (b) pictorial structures (Fischler and Elschlager 1973) © 1973 IEEE, (c) articulated body model (Marr 1982) © 1982 David Marr, (d) intrinsic images (Barrow and Tenenbaum 1981) © 1973 IEEE, (e) stereo correspondence (Marr 1982) © 1982 David Marr, (f) optical flow (Nagel and Enkelmann 1986) © 1986 IEEE.

an active area of research; a nice survey of contemporaneous work can be found in (Davis 1975).

Three-dimensional modeling of non-polyhedral objects was also being studied (Baumgart 1974; Baker 1977). One popular approach used *generalized cylinders*, i.e., solids of revolution and swept closed curves (Agin and Binford 1976; Nevatia and Binford 1977), often arranged into parts relationships[7] (Hinton 1977; Marr 1982) (Figure 1.7c). Fischler and Elschlager (1973) called such *elastic* arrangements of parts *pictorial structures* (Figure 1.7b). This is currently one of the favored approaches being used in object recognition (see Section 14.4 and Felzenszwalb and Huttenlocher 2005).

A qualitative approach to understanding intensities and shading variations and explaining them by the effects of image formation phenomena, such as surface orientation and shadows, was championed by Barrow and Tenenbaum (1981) in their paper on *intrinsic images* (Figure 1.7d), along with the related $2\frac{1}{2}$*-D sketch* ideas of Marr (1982). This approach is again seeing a bit of a revival in the work of Tappen, Freeman, and Adelson (2005).

More quantitative approaches to computer vision were also developed at the time, including the first of many feature-based stereo correspondence algorithms (Figure 1.7e) (Dev

[7] In robotics and computer animation, these linked-part graphs are often called *kinematic chains*.

1974; Marr and Poggio 1976; Moravec 1977; Marr and Poggio 1979; Mayhew and Frisby 1981; Baker 1982; Barnard and Fischler 1982; Ohta and Kanade 1985; Grimson 1985; Pollard, Mayhew, and Frisby 1985; Prazdny 1985) and intensity-based optical flow algorithms (Figure 1.7f) (Horn and Schunck 1981; Huang 1981; Lucas and Kanade 1981; Nagel 1986). The early work in simultaneously recovering 3D structure and camera motion (see Chapter 7) also began around this time (Ullman 1979; Longuet-Higgins 1981).

A lot of the philosophy of how vision was believed to work at the time is summarized in David Marr's (1982) book.[8] In particular, Marr introduced his notion of the three levels of description of a (visual) information processing system. These three levels, very loosely paraphrased according to my own interpretation, are:

- **Computational theory:** What is the goal of the computation (task) and what are the constraints that are known or can be brought to bear on the problem?
- **Representations and algorithms:** How are the input, output, and intermediate information represented and which algorithms are used to calculate the desired result?
- **Hardware implementation:** How are the representations and algorithms mapped onto actual hardware, e.g., a biological vision system or a specialized piece of silicon? Conversely, how can hardware constraints be used to guide the choice of representation and algorithm? With the increasing use of graphics chips (GPUs) and many-core architectures for computer vision (see Section C.2), this question is again becoming quite relevant.

As I mentioned earlier in this introduction, it is my conviction that a careful analysis of the problem specification and known constraints from image formation and priors (the scientific and statistical approaches) must be married with efficient and robust algorithms (the engineering approach) to design successful vision algorithms. Thus, it seems that Marr's philosophy is as good a guide to framing and solving problems in our field today as it was 25 years ago.

1980s. In the 1980s, a lot of attention was focused on more sophisticated mathematical techniques for performing quantitative image and scene analysis.

Image pyramids (see Section 3.5) started being widely used to perform tasks such as image blending (Figure 1.8a) and coarse-to-fine correspondence search (Rosenfeld 1980; Burt and Adelson 1983a,b; Rosenfeld 1984; Quam 1984; Anandan 1989). Continuous versions of pyramids using the concept of *scale-space* processing were also developed (Witkin 1983; Witkin, Terzopoulos, and Kass 1986; Lindeberg 1990). In the late 1980s, wavelets (see Section 3.5.4) started displacing or augmenting regular image pyramids in some applications

[8] More recent developments in visual perception theory are covered in (Palmer 1999; Livingstone 2008).

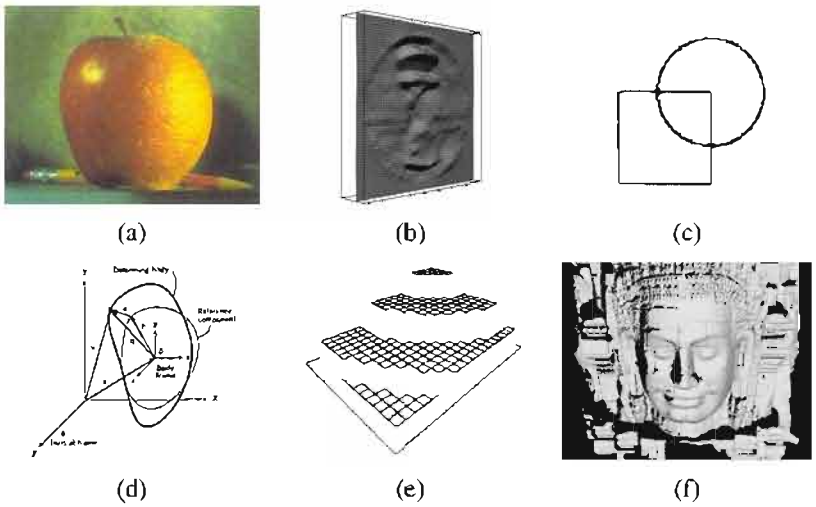


Figure 1.8 Examples of computer vision algorithms from the 1980s: (a) pyramid blending (Burt and Adelson 1983b) © 1983 ACM, (b) shape from shading (Freeman and Adelson 1991) © 1991 IEEE, (c) edge detection (Freeman and Adelson 1991) © 1991 IEEE, (d) physically based models (Terzopoulos and Witkin 1988) © 1988 IEEE, (e) regularization-based surface reconstruction (Terzopoulos 1988) © 1988 IEEE, (f) range data acquisition and merging (Banno, Masuda, Oishi *et al.* 2008) © 2008 Springer.

(Adelson, Simoncelli, and Hingorani 1987; Mallat 1989; Simoncelli and Adelson 1990a,b; Simoncelli, Freeman, Adelson *et al.* 1992).

The use of stereo as a quantitative shape cue was extended by a wide variety of *shape-from-X* techniques, including shape from shading (Figure 1.8b) (see Section 12.1.1 and Horn 1975; Pentland 1984; Blake, Zimmerman, and Knowles 1985; Horn and Brooks 1986, 1989), photometric stereo (see Section 12.1.1 and Woodham 1981), shape from texture (see Section 12.1.2 and Witkin 1981; Pentland 1984; Malik and Rosenholtz 1997), and shape from focus (see Section 12.1.3 and Nayar, Watanabe, and Noguchi 1995). Horn (1986) has a nice discussion of most of these techniques.

Research into better edge and contour detection (Figure 1.8c) (see Section 4.2) was also active during this period (Canny 1986; Nalwa and Binford 1986), including the introduction of dynamically evolving contour trackers (Section 5.1.1) such as *snakes* (Kass, Witkin, and Terzopoulos 1988), as well as three-dimensional *physically based models* (Figure 1.8d) (Terzopoulos, Witkin, and Kass 1987; Kass, Witkin, and Terzopoulos 1988; Terzopoulos and Fleischer 1988; Terzopoulos, Witkin, and Kass 1988).

Researchers noticed that a lot of the stereo, flow, shape-from-X, and edge detection al-

gorithms could be unified, or at least described, using the same mathematical framework if they were posed as variational optimization problems (see Section 3.7) and made more robust (well-posed) using regularization (Figure 1.8e) (see Section 3.7.1 and Terzopoulos 1983; Poggio, Torre, and Koch 1985; Terzopoulos 1986b; Blake and Zisserman 1987; Bertero, Poggio, and Torre 1988; Terzopoulos 1988). Around the same time, Geman and Geman (1984) pointed out that such problems could equally well be formulated using discrete *Markov Random Field* (MRF) models (see Section 3.7.2), which enabled the use of better (global) search and optimization algorithms, such as simulated annealing.

Online variants of MRF algorithms that modeled and updated uncertainties using the Kalman filter were introduced a little later (Dickmanns and Graefe 1988; Matthies, Kanade, and Szeliski 1989; Szeliski 1989). Attempts were also made to map both regularized and MRF algorithms onto parallel hardware (Poggio and Koch 1985; Poggio, Little, Gamble *et al.* 1988; Fischler, Firschein, Barnard *et al.* 1989). The book by Fischler and Firschein (1987) contains a nice collection of articles focusing on all of these topics (stereo, flow, regularization, MRFs, and even higher-level vision).

Three-dimensional range data processing (acquisition, merging, modeling, and recognition; see Figure 1.8f) continued being actively explored during this decade (Agin and Binford 1976; Besl and Jain 1985; Faugeras and Hebert 1987; Curless and Levoy 1996). The compilation by Kanade (1987) contains a lot of the interesting papers in this area.

1990s. While a lot of the previously mentioned topics continued to be explored, a few of them became significantly more active.

A burst of activity in using projective invariants for recognition (Mundy and Zisserman 1992) evolved into a concerted effort to solve the structure from motion problem (see Chapter 7). A lot of the initial activity was directed at *projective reconstructions*, which did not require knowledge of camera calibration (Faugeras 1992; Hartley, Gupta, and Chang 1992; Hartley 1994a; Faugeras and Luong 2001; Hartley and Zisserman 2004). Simultaneously, *factorization* techniques (Section 7.3) were developed to solve efficiently problems for which orthographic camera approximations were applicable (Figure 1.9a) (Tomasi and Kanade 1992; Poelman and Kanade 1997; Anandan and Irani 2002) and then later extended to the perspective case (Christy and Horaud 1996; Triggs 1996). Eventually, the field started using full global optimization (see Section 7.4 and Taylor, Kriegman, and Anandan 1991; Szeliski and Kang 1994; Azarbayejani and Pentland 1995), which was later recognized as being the same as the *bundle adjustment* techniques traditionally used in photogrammetry (Triggs, McLauchlan, Hartley *et al.* 1999). Fully automated (sparse) 3D modeling systems were built using such techniques (Beardsley, Torr, and Zisserman 1996; Schaffalitzky and Zisserman 2002; Brown and Lowe 2003; Snavely, Seitz, and Szeliski 2006).

Work begun in the 1980s on using detailed measurements of color and intensity combined

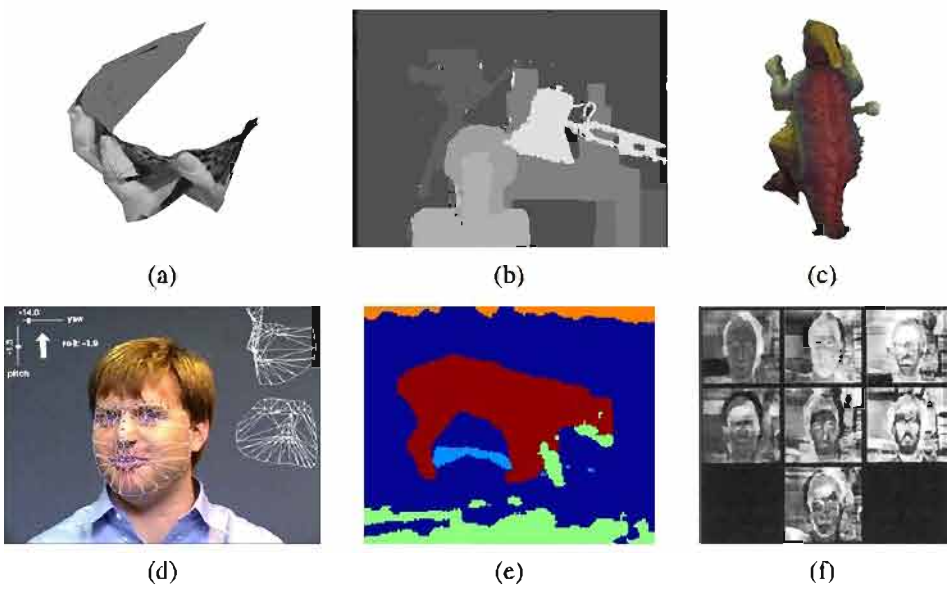


Figure 1.9 Examples of computer vision algorithms from the 1990s: (a) factorization-based structure from motion (Tomasi and Kanade 1992) © 1992 Springer, (b) dense stereo matching (Boykov, Veksler, and Zabih 2001), (c) multi-view reconstruction (Seitz and Dyer 1999) © 1999 Springer, (d) face tracking (Matthews, Xiao, and Baker 2007), (e) image segmentation (Belongie, Fowlkes, Chung *et al.* 2002) © 2002 Springer, (f) face recognition (Turk and Pentland 1991a).

with accurate physical models of radiance transport and color image formation created its own subfield known as *physics-based vision*. A good survey of the field can be found in the three-volume collection on this topic (Wolff, Shafer, and Healey 1992a; Healey and Shafer 1992; Shafer, Healey, and Wolff 1992).

Optical flow methods (see Chapter 8) continued to be improved (Nagel and Enkelmann 1986; Bolles, Baker, and Marimont 1987; Horn and Weldon Jr. 1988; Anandan 1989; Bergen, Anandan, Hanna *et al.* 1992; Black and Anandan 1996; Bruhn, Weickert, and Schnörr 2005; Papenberg, Bruhn, Brox *et al.* 2006), with (Nagel 1986; Barron, Fleet, and Beauchemin 1994; Baker, Black, Lewis *et al.* 2007) being good surveys. Similarly, a lot of progress was made on dense stereo correspondence algorithms (see Chapter 11, Okutomi and Kanade (1993, 1994); Boykov, Veksler, and Zabih (1998); Birchfield and Tomasi (1999); Boykov, Veksler, and Zabih (2001), and the survey and comparison in Scharstein and Szeliski (2002)), with the biggest breakthrough being perhaps global optimization using *graph cut* techniques (Figure 1.9b) (Boykov, Veksler, and Zabih 2001).

Multi-view stereo algorithms (Figure 1.9c) that produce complete 3D surfaces (see Section 11.6) were also an active topic of research (Seitz and Dyer 1999; Kutulakos and Seitz 2000) that continues to be active today (Seitz, Curless, Diebel *et al.* 2006). Techniques for producing 3D volumetric descriptions from binary silhouettes (see Section 11.6.2) continued to be developed (Potmesil 1987; Srivasan, Liang, and Hackwood 1990; Szeliski 1993; Laurentini 1994), along with techniques based on tracking and reconstructing smooth occluding contours (see Section 11.2.1 and Cipolla and Blake 1992; Vaillant and Faugeras 1992; Zheng 1994; Boyer and Berger 1997; Szeliski and Weiss 1998; Cipolla and Giblin 2000).

Tracking algorithms also improved a lot, including contour tracking using *active contours* (see Section 5.1), such as *snakes* (Kass, Witkin, and Terzopoulos 1988), *particle filters* (Blake and Isard 1998), and *level sets* (Malladi, Sethian, and Vemuri 1995), as well as intensity-based (*direct*) techniques (Lucas and Kanade 1981; Shi and Tomasi 1994; Rehg and Kanade 1994), often applied to tracking faces (Figure 1.9d) (Lanitis, Taylor, and Cootes 1997; Matthews and Baker 2004; Matthews, Xiao, and Baker 2007) and whole bodies (Sidenbladh, Black, and Fleet 2000; Hilton, Fua, and Ronfard 2006; Moeslund, Hilton, and Krüger 2006).

Image segmentation (see Chapter 5) (Figure 1.9e), a topic which has been active since the earliest days of computer vision (Brice and Fennema 1970; Horowitz and Pavlidis 1976; Riseman and Arbib 1977; Rosenfeld and Davis 1979; Haralick and Shapiro 1985; Pavlidis and Liow 1990), was also an active topic of research, producing techniques based on minimum energy (Mumford and Shah 1989) and minimum description length (Leclerc 1989), *normalized cuts* (Shi and Malik 2000), and *mean shift* (Comaniciu and Meer 2002).

Statistical learning techniques started appearing, first in the application of principal component *eigenface* analysis to face recognition (Figure 1.9f) (see Section 14.2.1 and Turk and Pentland 1991a) and linear dynamical systems for curve tracking (see Section 5.1.1 and Blake and Isard 1998).

Perhaps the most notable development in computer vision during this decade was the increased interaction with computer graphics (Seitz and Szeliski 1999), especially in the cross-disciplinary area of *image-based modeling and rendering* (see Chapter 13). The idea of manipulating real-world imagery directly to create new animations first came to prominence with *image morphing* techniques (Figure1.5c) (see Section 3.6.3 and Beier and Neely 1992) and was later applied to *view interpolation* (Chen and Williams 1993; Seitz and Dyer 1996), panoramic image stitching (Figure1.5a) (see Chapter 9 and Mann and Picard 1994; Chen 1995; Szeliski 1996; Szeliski and Shum 1997; Szeliski 2006a), and full light-field rendering (Figure 1.10a) (see Section 13.3 and Gortler, Grzeszczuk, Szeliski *et al.* 1996; Levoy and Hanrahan 1996; Shade, Gortler, He *et al.* 1998). At the same time, image-based modeling techniques (Figure 1.10b) for automatically creating realistic 3D models from collections of images were also being introduced (Beardsley, Torr, and Zisserman 1996; Debevec, Taylor, and Malik 1996; Taylor, Debevec, and Malik 1996).

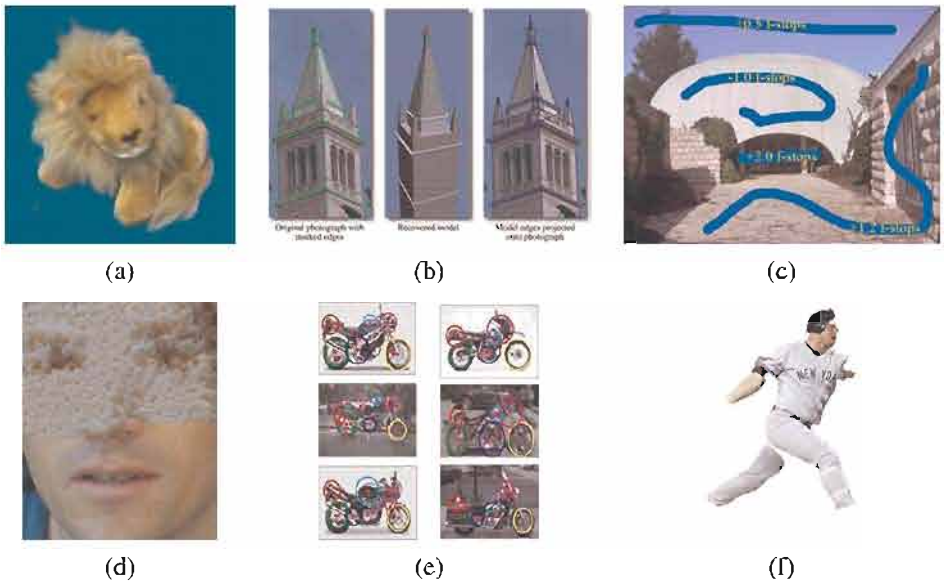


Figure 1.10 Recent examples of computer vision algorithms: (a) image-based rendering (Gortler, Grzeszczuk, Szeliski *et al.* 1996), (b) image-based modeling (Debevec, Taylor, and Malik 1996) © 1996 ACM, (c) interactive tone mapping (Lischinski, Farbman, Uyttendaele *et al.* 2006a) (d) texture synthesis (Efros and Freeman 2001), (e) feature-based recognition (Fergus, Perona, and Zisserman 2007), (f) region-based recognition (Mori, Ren, Efros *et al.* 2004) © 2004 IEEE.

2000s. This past decade has continued to see a deepening interplay between the vision and graphics fields. In particular, many of the topics introduced under the rubric of image-based rendering, such as image stitching (see Chapter 9), light-field capture and rendering (see Section 13.3), and *high dynamic range* (HDR) image capture through exposure bracketing (Figure1.5b) (see Section 10.2 and Mann and Picard 1995; Debevec and Malik 1997), were re-christened as *computational photography* (see Chapter 10) to acknowledge the increased use of such techniques in everyday digital photography. For example, the rapid adoption of exposure bracketing to create high dynamic range images necessitated the development of *tone mapping* algorithms (Figure 1.10c) (see Section 10.2.1) to convert such images back to displayable results (Fattal, Lischinski, and Werman 2002; Durand and Dorsey 2002; Reinhard, Stark, Shirley *et al.* 2002; Lischinski, Farbman, Uyttendaele *et al.* 2006a). In addition to merging multiple exposures, techniques were developed to merge flash images with non-flash counterparts (Eisemann and Durand 2004; Petschnigg, Agrawala, Hoppe *et al.* 2004) and to interactively or automatically select different regions from overlapping images (Agarwala,

Dontcheva, Agrawala *et al.* 2004).

Texture synthesis (Figure 1.10d) (see Section 10.5), quilting (Efros and Leung 1999; Efros and Freeman 2001; Kwatra, Schödl, Essa *et al.* 2003) and inpainting (Bertalmio, Sapiro, Caselles *et al.* 2000; Bertalmio, Vese, Sapiro *et al.* 2003; Criminisi, Pérez, and Toyama 2004) are additional topics that can be classified as computational photography techniques, since they re-combine input image samples to produce new photographs.

A second notable trend during this past decade has been the emergence of feature-based techniques (combined with learning) for object recognition (see Section 14.3 and Ponce, Hebert, Schmid *et al.* 2006). Some of the notable papers in this area include the *constellation model* of Fergus, Perona, and Zisserman (2007) (Figure 1.10e) and the *pictorial structures* of Felzenszwalb and Huttenlocher (2005). Feature-based techniques also dominate other recognition tasks, such as scene recognition (Zhang, Marszalek, Lazebnik *et al.* 2007) and panorama and location recognition (Brown and Lowe 2007; Schindler, Brown, and Szeliski 2007). And while *interest point* (patch-based) features tend to dominate current research, some groups are pursuing recognition based on contours (Belongie, Malik, and Puzicha 2002) and region segmentation (Figure 1.10f) (Mori, Ren, Efros *et al.* 2004).

Another significant trend from this past decade has been the development of more efficient algorithms for complex global optimization problems (see Sections 3.7 and B.5 and Szeliski, Zabih, Scharstein *et al.* 2008; Blake, Kohli, and Rother 2010). While this trend began with work on graph cuts (Boykov, Veksler, and Zabih 2001; Kohli and Torr 2007), a lot of progress has also been made in message passing algorithms, such as *loopy belief propagation* (LBP) (Yedidia, Freeman, and Weiss 2001; Kumar and Torr 2006).

The final trend, which now dominates a lot of the visual recognition research in our community, is the application of sophisticated machine learning techniques to computer vision problems (see Section 14.5.1 and Freeman, Perona, and Schölkopf 2008). This trend coincides with the increased availability of immense quantities of partially labelled data on the Internet, which makes it more feasible to learn object categories without the use of careful human supervision.

1.3 Book overview

In the final part of this introduction, I give a brief tour of the material in this book, as well as a few notes on notation and some additional general references. Since computer vision is such a broad field, it is possible to study certain aspects of it, e.g., geometric image formation and 3D structure recovery, without engaging other parts, e.g., the modeling of reflectance and shading. Some of the chapters in this book are only loosely coupled with others, and it is not strictly necessary to read all of the material in sequence.

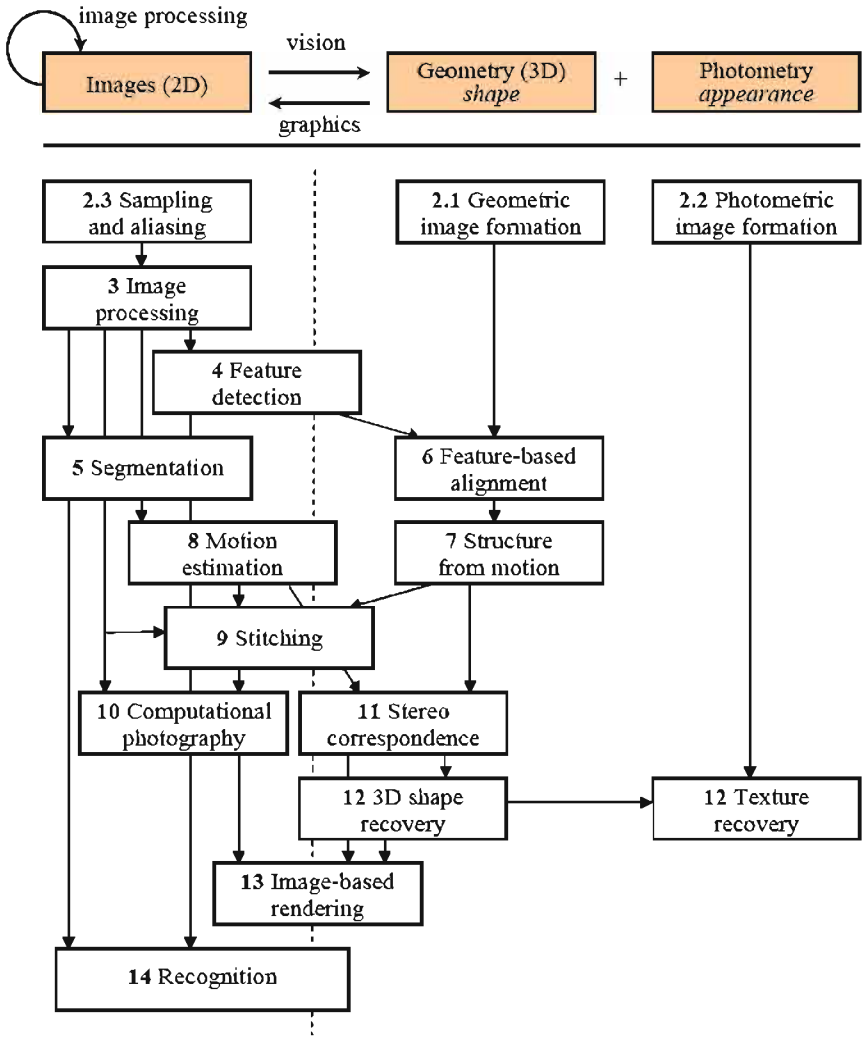


Figure 1.11 Relationship between images, geometry, and photometry, as well as a taxonomy of the topics covered in this book. Topics are roughly positioned along the left–right axis depending on whether they are more closely related to image-based (left), geometry-based (middle) or appearance-based (right) representations, and on the vertical axis by increasing level of abstraction. The whole figure should be taken with a large grain of salt, as there are many additional subtle connections between topics not illustrated here.

Figure 1.11 shows a rough layout of the contents of this book. Since computer vision involves going from images to a structural description of the scene (and computer graphics the converse), I have positioned the chapters horizontally in terms of which major component they address, in addition to vertically according to their dependence.

Going from left to right, we see the major column headings as Images (which are 2D in nature), Geometry (which encompasses 3D descriptions), and Photometry (which encompasses object appearance). (An alternative labeling for these latter two could also be *shape* and *appearance*—see, e.g., Chapter 13 and Kang, Szeliski, and Anandan (2000).) Going from top to bottom, we see increasing levels of modeling and abstraction, as well as techniques that build on previously developed algorithms. Of course, this taxonomy should be taken with a large grain of salt, as the processing and dependencies in this diagram are not strictly sequential and subtle additional dependencies and relationships also exist (e.g., some recognition techniques make use of 3D information). The placement of topics along the horizontal axis should also be taken lightly, as most vision algorithms involve mapping between at least two different representations.[9]

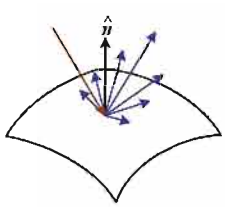
Interspersed throughout the book are sample **applications**, which relate the algorithms and mathematical material being presented in various chapters to useful, real-world applications. Many of these applications are also presented in the exercises sections, so that students can write their own.

At the end of each section, I provide a set of **exercises** that the students can use to implement, test, and refine the algorithms and techniques presented in each section. Some of the exercises are suitable as written homework assignments, others as shorter one-week projects, and still others as open-ended research problems that make for challenging final projects. Motivated students who implement a reasonable subset of these exercises will, by the end of the book, have a computer vision software library that can be used for a variety of interesting tasks and projects.

As a reference book, I try wherever possible to discuss which techniques and algorithms work well in practice, as well as providing up-to-date pointers to the latest research results in the areas that I cover. The exercises can be used to build up your own personal library of self-tested and validated vision algorithms, which is more worthwhile in the long term (assuming you have the time) than simply pulling algorithms out of a library whose performance you do not really understand.

The book begins in Chapter 2 with a review of the image formation processes that create the images that we see and capture. Understanding this process is fundamental if you want to take a scientific (model-based) approach to computer vision. Students who are eager to just start implementing algorithms (or courses that have limited time) can skip ahead to the

[9] For an interesting comparison with what is known about the human visual system, e.g., the largely parallel *what* and *where* pathways, see some textbooks on human perception (Palmer 1999; Livingstone 2008).



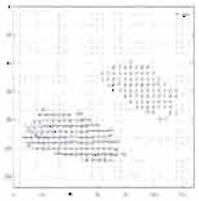
2. Image Formation

3. Image Processing

4. Features

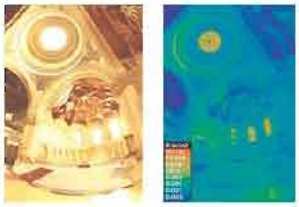
5. Segmentation

6-7. Structure from Motion



8. Motion

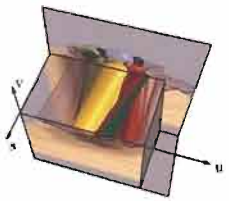
9. Stitching



10. Computational Photography

11. Stereo

12. 3D Shape



13. Image-based Rendering

14. Recognition

Figure 1.12 A pictorial summary of the chapter contents. Sources: Brown, Szeliski, and Winder (2005); Comaniciu and Meer (2002); Snavely, Seitz, and Szeliski (2006); Nagel and Enkelmann (1986); Szeliski and Shum (1997); Debevec and Malik (1997); Gortler, Grzeszczuk, Szeliski *et al.* (1996); Viola and Jones (2004)—see the figures in the respective chapters for copyright information.

next chapter and dip into this material later. In Chapter 2, we break down image formation into three major components. Geometric image formation (Section 2.1) deals with points, lines, and planes, and how these are mapped onto images using *projective geometry* and other models (including radial lens distortion). Photometric image formation (Section 2.2) covers *radiometry*, which describes how light interacts with surfaces in the world, and *optics*, which projects light onto the sensor plane. Finally, Section 2.3 covers how sensors work, including topics such as sampling and aliasing, color sensing, and in-camera compression.

Chapter 3 covers image processing, which is needed in almost all computer vision applications. This includes topics such as linear and non-linear filtering (Section 3.3), the Fourier transform (Section 3.4), image pyramids and wavelets (Section 3.5), geometric transformations such as image warping (Section 3.6), and global optimization techniques such as *regularization* and *Markov Random Fields* (MRFs) (Section 3.7). While most of this material is covered in courses and textbooks on image processing, the use of optimization techniques is more typically associated with computer vision (although MRFs are now being widely used in image processing as well). The section on MRFs is also the first introduction to the use of Bayesian inference techniques, which are covered at a more abstract level in Appendix B. Chapter 3 also presents applications such as seamless image blending and image restoration.

In Chapter 4, we cover feature detection and matching. A lot of current 3D reconstruction and recognition techniques are built on extracting and matching *feature points* (Section 4.1), so this is a fundamental technique required by many subsequent chapters (Chapters 6, 7, 9 and 14). We also cover edge and straight line detection in Sections 4.2 and 4.3.

Chapter 5 covers region segmentation techniques, including active contour detection and tracking (Section 5.1). Segmentation techniques include top-down (split) and bottom-up (merge) techniques, mean shift techniques that find modes of clusters, and various graph-based segmentation approaches. All of these techniques are essential building blocks that are widely used in a variety of applications, including performance-driven animation, interactive image editing, and recognition.

In Chapter 6, we cover geometric alignment and camera calibration. We introduce the basic techniques of feature-based alignment in Section 6.1 and show how this problem can be solved using either linear or non-linear least squares, depending on the motion involved. We also introduce additional concepts, such as uncertainty weighting and robust regression, which are essential to making real-world systems work. Feature-based alignment is then used as a building block for 3D pose estimation (*extrinsic calibration*) in Section 6.2 and camera (*intrinsic*) calibration in Section 6.3. Chapter 6 also describes applications of these techniques to photo alignment for flip-book animations, 3D pose estimation from a hand-held camera, and single-view reconstruction of building models.

Chapter 7 covers the topic of *structure from motion*, which involves the simultaneous recovery of 3D camera motion and 3D scene structure from a collection of tracked 2D fea-

tures. This chapter begins with the easier problem of 3D point *triangulation* (Section 7.1), which is the 3D reconstruction of points from matched features when the camera positions are known. It then describes two-frame structure from motion (Section 7.2), for which algebraic techniques exist, as well as robust sampling techniques such as RANSAC that can discount erroneous feature matches. The second half of Chapter 7 describes techniques for multi-frame structure from motion, including factorization (Section 7.3), bundle adjustment (Section 7.4), and constrained motion and structure models (Section 7.5). It also presents applications in view morphing, sparse 3D model construction, and match move.

In Chapter 8, we go back to a topic that deals directly with image intensities (as opposed to feature tracks), namely dense intensity-based motion estimation (*optical flow*). We start with the simplest possible motion models, translational motion (Section 8.1), and cover topics such as hierarchical (coarse-to-fine) motion estimation, Fourier-based techniques, and iterative refinement. We then present parametric motion models, which can be used to compensate for camera rotation and zooming, as well as affine or planar perspective motion (Section 8.2). This is then generalized to spline-based motion models (Section 8.3) and finally to general per-pixel optical flow (Section 8.4), including layered and learned motion models (Section 8.5). Applications of these techniques include automated morphing, frame interpolation (slow motion), and motion-based user interfaces.

Chapter 9 is devoted to *image stitching*, i.e., the construction of large panoramas and composites. While stitching is just one example of *computation photography* (see Chapter 10), there is enough depth here to warrant a separate chapter. We start by discussing various possible motion models (Section 9.1), including planar motion and pure camera rotation. We then discuss global alignment (Section 9.2), which is a special (simplified) case of general bundle adjustment, and then present *panorama recognition*, i.e., techniques for automatically discovering which images actually form overlapping panoramas. Finally, we cover the topics of *image compositing* and *blending* (Section 9.3), which involve both selecting which pixels from which images to use and blending them together so as to disguise exposure differences.

Image stitching is a wonderful application that ties together most of the material covered in earlier parts of this book. It also makes for a good mid-term course project that can build on previously developed techniques such as image warping and feature detection and matching. Chapter 9 also presents more specialized variants of stitching such as whiteboard and document scanning, video summarization, *panography*, full 360° spherical panoramas, and interactive photomontage for blending repeated action shots together.

Chapter 10 presents additional examples of *computational photography*, which is the process of creating new images from one or more input photographs, often based on the careful modeling and calibration of the image formation process (Section 10.1). Computational photography techniques include merging multiple exposures to create *high dynamic range* images (Section 10.2), increasing image resolution through blur removal and *super-resolution* (Sec-

tion 10.3), and image editing and compositing operations (Section 10.4). We also cover the topics of texture analysis, synthesis and *inpainting* (hole filling) in Section 10.5, as well as non-photorealistic rendering (Section 10.5.2).

In Chapter 11, we turn to the issue of stereo correspondence, which can be thought of as a special case of motion estimation where the camera positions are already known (Section 11.1). This additional knowledge enables stereo algorithms to search over a much smaller space of correspondences and, in many cases, to produce dense depth estimates that can be converted into visible surface models (Section 11.3). We also cover multi-view stereo algorithms that build a true 3D surface representation instead of just a single depth map (Section 11.6). Applications of stereo matching include head and gaze tracking, as well as depth-based background replacement (*Z-keying*).

Chapter 12 covers additional 3D shape and appearance modeling techniques. These include classic *shape-from-X* techniques such as shape from shading, shape from texture, and shape from focus (Section 12.1), as well as shape from smooth occluding contours (Section 11.2.1) and silhouettes (Section 12.5). An alternative to all of these *passive* computer vision techniques is to use *active rangefinding* (Section 12.2), i.e., to project patterned light onto scenes and recover the 3D geometry through triangulation. Processing all of these 3D representations often involves interpolating or simplifying the geometry (Section 12.3), or using alternative representations such as surface point sets (Section 12.4).

The collection of techniques for going from one or more images to partial or full 3D models is often called *image-based modeling* or *3D photography*. Section 12.6 examines three more specialized application areas (architecture, faces, and human bodies), which can use *model-based reconstruction* to fit parameterized models to the sensed data. Section 12.7 examines the topic of *appearance modeling*, i.e., techniques for estimating the texture maps, albedos, or even sometimes complete *bi-directional reflectance distribution functions* (BRDFs) that describe the appearance of 3D surfaces.

In Chapter 13, we discuss the large number of image-based rendering techniques that have been developed in the last two decades, including simpler techniques such as view interpolation (Section 13.1), layered depth images (Section 13.2), and sprites and layers (Section 13.2.1), as well as the more general framework of light fields and Lumigraphs (Section 13.3) and higher-order fields such as environment mattes (Section 13.4). Applications of these techniques include navigating 3D collections of photographs using *photo tourism* and viewing 3D models as *object movies*.

In Chapter 13, we also discuss video-based rendering, which is the temporal extension of image-based rendering. The topics we cover include video-based animation (Section 13.5.1), periodic video turned into *video textures* (Section 13.5.2), and 3D video constructed from multiple video streams (Section 13.5.4). Applications of these techniques include video denoising, morphing, and tours based on 360° video.

Week	Material	Project
(1.)	Chapter 2 Image formation	
2.	Chapter 3 Image processing	
3.	Chapter 4 Feature detection and matching	P1
4.	Chapter 6 Feature-based alignment	
5.	Chapter 9 Image stitching	P2
6.	Chapter 8 Dense motion estimation	
7.	Chapter 7 Structure from motion	PP
8.	Chapter 14 Recognition	
(9.)	Chapter 10 Computational photography	
10.	Chapter 11 Stereo correspondence	
(11.)	Chapter 12 3D reconstruction	
12.	Chapter 13 Image-based rendering	
13.	Final project presentations	FP

Table 1.1 Sample syllabi for 10-week and 13-week courses. The weeks in parentheses are not used in the shorter version. P1 and P2 are two early-term mini-projects, PP is when the (student-selected) final project proposals are due, and FP is the final project presentations.

Chapter 14 describes different approaches to recognition. It begins with techniques for detecting and recognizing faces (Sections 14.1 and 14.2), then looks at techniques for finding and recognizing particular objects (*instance recognition*) in Section 14.3. Next, we cover the most difficult variant of recognition, namely the recognition of broad *categories*, such as cars, motorcycles, horses and other animals (Section 14.4), and the role that scene context plays in recognition (Section 14.5).

To support the book's use as a textbook, the appendices and associated Web site contain more detailed mathematical topics and additional material. Appendix A covers linear algebra and numerical techniques, including matrix algebra, least squares, and iterative techniques. Appendix B covers Bayesian estimation theory, including maximum likelihood estimation, robust statistics, Markov random fields, and uncertainty modeling. Appendix C describes the supplementary material available to complement this book, including images and data sets, pointers to software, course slides, and an on-line bibliography.

1.4 Sample syllabus

Teaching all of the material covered in this book in a single quarter or semester course is a Herculean task and likely one not worth attempting. It is better to simply pick and choose

topics related to the lecturer's preferred emphasis and tailored to the set of mini-projects envisioned for the students.

Steve Seitz and I have successfully used a 10-week syllabus similar to the one shown in Table 1.1 (omitting the parenthesized weeks) as both an undergraduate and a graduate-level course in computer vision. The undergraduate course[10] tends to go lighter on the mathematics and takes more time reviewing basics, while the graduate-level course[11] dives more deeply into techniques and assumes the students already have a decent grounding in either vision or related mathematical techniques. (See also the *Introduction to Computer Vision* course at Stanford,[12] which uses a similar curriculum.) Related courses have also been taught on the topics of 3D photography[13] and computational photography.[14]

When Steve and I teach the course, we prefer to give the students several small programming projects early in the course rather than focusing on written homework or quizzes. With a suitable choice of topics, it is possible for these projects to build on each other. For example, introducing feature matching early on can be used in a second assignment to do image alignment and stitching. Alternatively, direct (optical flow) techniques can be used to do the alignment and more focus can be put on either graph cut seam selection or multi-resolution blending techniques.

We also ask the students to propose a final project (we provide a set of suggested topics for those who need ideas) by the middle of the course and reserve the last week of the class for student presentations. With any luck, some of these final projects can actually turn into conference submissions!

No matter how you decide to structure the course or how you choose to use this book, I encourage you to try at least a few small programming tasks to get a good feel for how vision techniques work, and when they do not. Better yet, pick topics that are fun and can be used on your own photographs, and try to push your creative boundaries to come up with surprising results.

1.5 A note on notation

For better or worse, the notation found in computer vision and multi-view geometry textbooks tends to vary all over the map (Faugeras 1993; Hartley and Zisserman 2004; Girod, Greiner, and Niemann 2000; Faugeras and Luong 2001; Forsyth and Ponce 2003). In this book, I use the convention I first learned in my high school physics class (and later multi-variate

[10] http://www.cs.washington.edu/education/courses/455/
[11] http://www.cs.washington.edu/education/courses/576/
[12] http://vision.stanford.edu/teaching/cs223b/
[13] http://www.cs.washington.edu/education/courses/558/06sp/
[14] http://graphics.cs.cmu.edu/courses/15-463/

calculus and computer graphics courses), which is that vectors $\boldsymbol{v}$ are lower case bold, matrices $\boldsymbol{M}$ are upper case bold, and scalars (T, s) are mixed case italic. Unless otherwise noted, vectors operate as column vectors, i.e., they post-multiply matrices, $\boldsymbol{M}\boldsymbol{v}$, although they are sometimes written as comma-separated parenthesized lists $\boldsymbol{x} = (x, y)$ instead of bracketed column vectors $\boldsymbol{x} = [x\ y]^T$. Some commonly used matrices are $\boldsymbol{R}$ for rotations, $\boldsymbol{K}$ for calibration matrices, and $\boldsymbol{I}$ for the identity matrix. Homogeneous coordinates (Section 2.1) are denoted with a tilde over the vector, e.g., $\tilde{\boldsymbol{x}} = (\tilde{x}, \tilde{y}, \tilde{w}) = \tilde{w}(x, y, 1) = \tilde{w}\bar{\boldsymbol{x}}$ in $\mathcal{P}^2$. The cross product operator in matrix form is denoted by $[\,]_\times$.

1.6 Additional reading

This book attempts to be self-contained, so that students can implement the basic assignments and algorithms described here without the need for outside references. However, it does presuppose a general familiarity with basic concepts in linear algebra and numerical techniques, which are reviewed in Appendix A, and image processing, which is reviewed in Chapter 3.

Students who want to delve more deeply into these topics can look in (Golub and Van Loan 1996) for matrix algebra and (Strang 1988) for linear algebra. In image processing, there are a number of popular textbooks, including (Crane 1997; Gomes and Velho 1997; Jähne 1997; Pratt 2007; Russ 2007; Burger and Burge 2008; Gonzales and Woods 2008). For computer graphics, popular texts include (Foley, van Dam, Feiner *et al.* 1995; Watt 1995), with (Glassner 1995) providing a more in-depth look at image formation and rendering. For statistics and machine learning, Chris Bishop's (2006) book is a wonderful and comprehensive introduction with a wealth of exercises. Students may also want to look in other textbooks on computer vision for material that we do not cover here, as well as for additional project ideas (Ballard and Brown 1982; Faugeras 1993; Nalwa 1993; Trucco and Verri 1998; Forsyth and Ponce 2003).

There is, however, no substitute for reading the latest research literature, both for the latest ideas and techniques and for the most up-to-date references to related literature.[15] In this book, I have attempted to cite the most recent work in each field so that students can read them directly and use them as inspiration for their own work. Browsing the last few years' conference proceedings from the major vision and graphics conferences, such as CVPR, ECCV, ICCV, and SIGGRAPH, will provide a wealth of new ideas. The tutorials offered at these conferences, for which slides or notes are often available on-line, are also an invaluable resource.

[15] For a comprehensive bibliography and taxonomy of computer vision research, Keith Price's Annotated Computer Vision Bibliography http://www.visionbib.com/bibliography/contents.html is an invaluable resource.