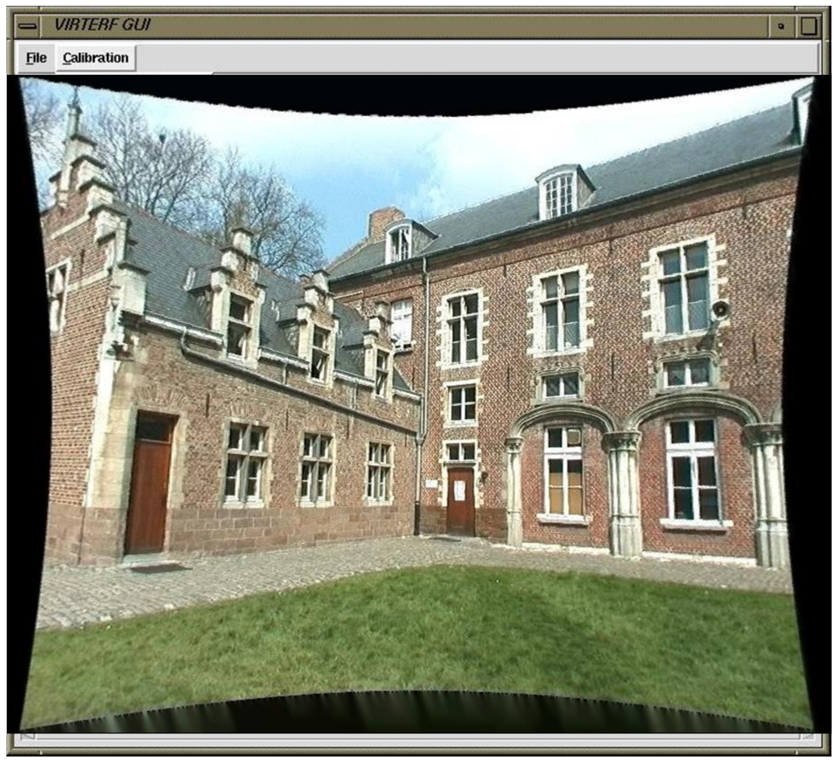
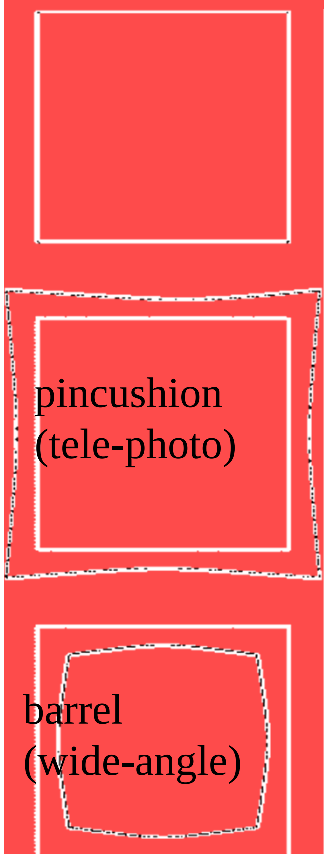
Radial Distortion

magnification/focal length different
for different angles of inclination





Can be corrected! (if parameters are know)

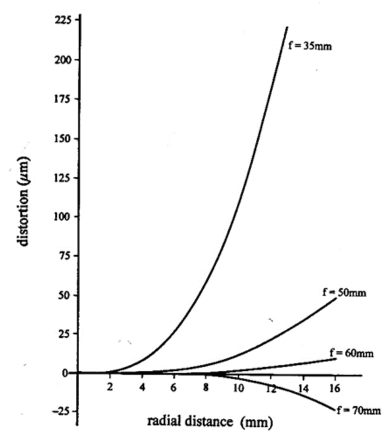
Radial distortion

- Due to spherical lenses (cheap)
- Model:

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \sim \begin{bmatrix} f_x & s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \mathsf{R} \left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{R}^\top & -\mathbf{R}^\top \mathbf{t} \\ 0_3^\top & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \right)$$

$$\mathsf{R}(x, y) = (1 + K_1(x^2 + y^2) + K_2(x^4 + y^4) + \ldots) \begin{bmatrix} x \\ y \end{bmatrix}$$

straight lines are not straight anymore



barrel dist.

pincushion dist.

http://foto.hut.fi/opetus/260/luennot/11/atkinson_6-11_radial_distortion_zoom_lenses.jpg

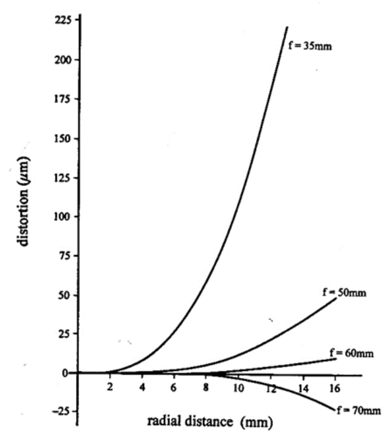
Radial distortion

- Due to spherical lenses (cheap)
- Model (following Tsai 1987 et al.):

$$p = \begin{pmatrix} 1/\lambda & 0 & 0 \\ 0 & 1/\lambda & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathcal{M} \boldsymbol{P}$$

λ is a polynomial function of $\hat{r}^2 \stackrel{\text{def}}{=} \hat{u}^2 + \hat{v}^2$, i.e., $\lambda = 1 + \kappa_1 \hat{r}^2 + \kappa_2 \hat{r}^4 + \ldots$.

straight lines are not straight anymore



barrel dist.

pincushion dist.

http://foto.hut.fi/opetus/260/luennot/11/atkinson_6-11_radial_distortion_zoom_lenses.jpg

Radial distortion example

3.3.1 Estimation of Projection Matrix

Geometrically, radial distortion changes the distance between the image center and the image point $\boldsymbol{p}$ but it does not affect the direction of the vector joining these two points. This is called the *radial alignment constraint* by Tsai, and it can be expressed algebraically by writing

$$\lambda \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \dfrac{\boldsymbol{m}_1 \cdot \boldsymbol{P}}{\boldsymbol{m}_3 \cdot \boldsymbol{P}} \\ \dfrac{\boldsymbol{m}_2 \cdot \boldsymbol{P}}{\boldsymbol{m}_3 \cdot \boldsymbol{P}} \end{pmatrix} \Longrightarrow v(\boldsymbol{m}_1 \cdot \boldsymbol{P}) - u(\boldsymbol{m}_2 \cdot \boldsymbol{P}) = 0.$$

This is a linear constraint on the vectors $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$. Given n fiducial points we obtain n equations in the eight coefficients of the vectors $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$, namely

$$\mathcal{Q}\boldsymbol{n} = 0, \quad \text{where} \quad \mathcal{Q} \stackrel{\text{def}}{=} \begin{pmatrix} v_1 \boldsymbol{P}_1^T & -u_1 \boldsymbol{P}_1^T \\ \ldots & \ldots \\ v_n \boldsymbol{P}_n^T & -u_n \boldsymbol{P}_n^T \end{pmatrix} \quad \text{and} \quad \boldsymbol{n} = \begin{pmatrix} \boldsymbol{m}_1 \\ \boldsymbol{m}_2 \end{pmatrix}. \qquad (6.3.9)$$

Note the similarity with the previous case. When $n \geq 8$, the system of equations (6.3.9) is in general overconstrained, and a solution with unit norm can be found using linear least squares.

F&P page 48 ff

Useful Links

Demo calibration (some links broken):

- http://mitpress.mit.edu/e-journals/Videre/001/articles/Zhang/CalibEnv/CalibEnv.html

Bouget camera calibration SW:

- http://www.vision.caltech.edu/bouguetj/calib_doc/

CVonline: Monocular Camera calibration:

- http://homepages.inf.ed.ac.uk/cgi/rbf/CVONLINE/entries.pl?TAG250