### **Uncertainty Visualization**





### **Decision Making Under Uncertainty**

Surfaces imply certainty





### **Uncertainty Quotes**

•Richard Feynman: What is not surrounded by uncertainty cannot be the truth.

•Richard Feynman: If you thought that science was certain, well, that is just an error on your part.

•George Box: All models are wrong. Some models are useful.

•John W. Tukey: Far better an approximate answer to the right question, which is often vague, than an exact answer to the wrong question, which can always be made precise.

•Francis Bacon - If we begin with certainties, we shall end in doubts; but if we begin with doubts, and are patient in them, we shall end in certainties

•Winston Churchill: *True genius resides in the capacity for evaluation of uncertain, hazardous, and conflicting information.* 



Scientific Computing and Imaging Institute, University of Utah

## **Uncertainty Visualization**







When is the last time you've seen an error bar in a visualization of complex data ?







G.P. Bonneau, H.C. Hege, C.R. Johnson, M.M. Oliveira, K. Potter, P. Rheingans, T. Schultz. "Overview and State-ofthe-Art of Uncertainty Visualization," In Scientific Visualization: Uncertainty, Multifield, Biomedical, and Scalable Visualization, Edited by M. Chen and H. Hagen and C.D. Hansen and C.R. Johnson and A. Kauffman, Springer-Verlag, pp. 3-27. 2014.

M.G. Genton, C.R. Johnson, K. Potter, G. Stenchikov, Y. Sun. "Surface boxplots," In *Stat Journal*, Vol. 3, No. 1, pp. 1-11. 2014.

K. Potter, P. Rosen, C.R. Johnson. "From Quantification to Visualization: A Taxonomy of Uncertainty Visualization Approaches," In Uncertainty Quantification in Scientific Computing, IFIP Series, Vol. 377, Springer, pp. 226-249. 2012.

K. Potter, A. Wilson, P.-T. Bremer, D. Williams, C. Doutriaux, V. Pascucci, C.R. Johnson. "Ensemble-Vis: A Framework for the Statistical Visualization of Ensemble Data," In *Proceedings of the 2009 IEEE International Conference on Data Mining Workshops*, pp. 233-240. 2009.

C.R. Johnson, A.R. Sanderson. "A Next Step: Visualizing Errors and Uncertainty," In *IEEE Computer Graphics and Applications*, Vol. 23, No. 5, pp. 6-10. September/October,





## **Sources of Uncertainty**

- Uncertainty observed in sampled data.
- Uncertainty measures generated by models or simulations.
- Uncertainty introduced by the data processing or visualization processes.



## **Sources of Uncertainty**

- Experimental (observational, equipment limits, multiple trials)
- Numerical (approximation, interpolation, extrapolation)
- Mathematical Model (approximation to true physics/biology)
- Geometric Model (accuracy compared to true geometry)



## Categories

EPISTEMIC UNCERTAINTY

- Systematic uncertainty
- Things we could in principle know but don't in practice
- Insufficient measurement or modeling, missing data

REDUCIBLE: can be alleviated by better models, more accurate measurement





## Categories

 $A_{\text{LEATORIC}} U_{\text{NCERTAINTY}}$ 

- Statistical uncertainty
- Unknowns that differ on each run
- i.e. throwing dice

Irreducible: *cannot be eliminated through improvements in models or measurements* 





# Statistical Uncertainties - Common in Visualization

- Probability Distribution Functions (PDFs) approximate outcome through a probability function
- Probability Density continuous random variables, frequency of outcome values
- Statistics on PDFs mean, median, standard deviation



### **Traditional Display of Uncertainty**

### Boxplots (Tukey, 1977)

- Quartile range including median
- Outliers
- Assume Gaussian





## **Boxplot Modifications**

- Visual Modifications
  - Refinement for aesthetic purposes
- Density indications
  - Use the box sides to encode
- Data Characteristics
  - Sample size, confidence levels
- Additional Statistics
  - Skew, modality











## **The Summary Plot**

- Augment boxplot with numerous display techniques
- Emphasize characteristics other than mean/variance
- Indicate quantity and location of uncertainty









### **Summary Plot in Higher Dimensions**

- Statistics similar to summary plot
- Highlight correlations







### **Visual Encodings of Temporal Uncertainty**



Fig. 1: Six different visual encodings of start/end uncertainty of temporal intervals used in the user study: (a) gradient plot, (b) violin plot, (c) accumulated probability plot, (d) error bars, (e) centered error bars, and (f) ambiguation. We designed encodings (a)–(c) to encode statistical uncertainty and encodings (d)–(f) to encode bounded uncertainty. All encodings were used to estimate earliest start, latest start, earliest end, and latest end, as well as minimum, maximum, and average interval duration. Moreover, encodings (a)–(c) were used to estimate the probability that the interval has already started/ended at a marked position in time.



Gschwandtnei, T., Bögl, M., Federico, P., & Miksch, S. (2015). Visual encodings of temporal uncertainty: A comparative user study. IEEE transactions on visualization and computer graphics, 22(1), 539-548.





Feng, D., Kwock, L., Lee, Y., & Taylor, R. M., 2nd (2010). Matching visual saliency to confidence in plots of uncertain data. *IEEE transactions on visualization and computer graphics*, *16*(6), 980–989. doi:10.1109/TVCG.2010.176

## **2D Annotation**

- Modulate annotation lines or glyphs with uncertainty
- Minimal interference
- Uncertainty not emphasized

A. Cedilnik, P. Rheingans.Procedural Annotation of Uncertain Information.In Proc IEEE Vis, 2000.







### **Visual Entropy**

Holliman, N. S., Coltekin, A., Fernstad, S. J., Simpson, M. D., Wilson, K. J., & Woods, A. J. (2019). Visual entropy and the visualization of uncertainty. arXiv preprint arXiv:1907.12879.





Fig. 10. The urban temperature data visualization showing both hourly mean temperature values using the MetOffice color scale and the variance of those values using our new visual entropy scale, this image is an example of the high uncertainty target-present stimulus used in the experiment described below.

## Contouring

- Contours follow the line of a specific data value (ex. terrain map)
- Standard Deviation
- Fuzzy contours
- Graduated contours





. Sanyal, S. Zhang, J. Dyer, A. Mercer, P. Amburn. Noodles: A Tool for Visualization of Numerical Weather Model Ensemble Uncertainty In Proc IEEE Vis, 2010.



R.S. Allendes Osorio, K.W. Brodlie.Contouring with Uncertainty.In Theory and Practice of Computer Graphics Conf, 2008

R.S. Allendes Osorio, K.W. Brodlie.



Mean = colormap, Standard Deviation = contours

K. Potter, et al. Ensemble-Vis: A Framework for the Statistical Visualization of Ensemble Data. In IEEE ICDM Workshop on Knowledge Discovery from Climate Data: Prediction, 2009.

### **Ensembles / 2D Distributions**

- Multi-run/model simulations
- Distribution of data at every point
- Mean/std dev may not be appropriate



D. Kao, A. Luo, J. Dungan, A. Pang. Visualizing Spatially Varying Distribution Data.

In Proc Information Visualisation, 2002.



D. Kao, M. Kramer, A. Luo, J. Dungan, A. Pang.

Visualizing Distributions from Multi-Return Lidar Data to Understand Forest Structure.



Ensemble-Vis: A Framework for the Statistical Visualization of Ensemble Data.

In IEEE ICDM Workshop on Knowledge Discovery from Climate Data: Prediction, 2009.







. Sanyal, S. Zhang, J. Dyer, A. Mercer, P. Amburn. Noodles: A Tool for Visualization of Numerical Weather Model Ensemble Uncertainty In Proc IEEE Vis, 2010.

#### What is ensemble data?

Collection of data sets generated by computational simulations.

Used to simulate complex systems, mitigate uncertainty, unknowns in initial conditions, and parameter sensitivity.

- These data sets are:
- Multidimensional
- Multivariate
- Multivalued

#### **Ensemble-Vis Workflow**

- User-driven
- Component-based



#### **Trend Charts**



#### **Ensemble Overviews**



Spatial Overviews: Mean and standard deviation encoded through colormaps and



Temporal Overviews: Filmstrip and animation. Show evolution through time. Small multiples show every time step. User can select desired temporal location.

Quartile Charts: Show minimum and maximum. innerquartile range.



Plume Charts: Show every member and mean. Color coded based on model. Deselect members to hide. Drill-down to direct data display.

#### **Query Contours**

**Ensemble-Vis:** 

A Framework for the Statistical

Visualization of Ensemble Data



- \* User-driven query
- \* Select subset based on conditions
- \* Returns % of satisfying members
- \* Displayed as nested contours

#### Spaghetti Charts



- \* Show variation across space
  - \* User chosen contour value
  - \* Isocontour for each desired member
  - \* Highlights outliers and divergence



#### Implementation



#### SREF Weather Explorer

- VTK filters, Qt Widgets
- Relational database: •MySQL/ Netezza



#### ViSUS

- Climate Data Analysis Tools (CDAT) integration
- · C++, python, FLTK
- Out-of-core, streaming

## **2D Vector Fields - LIC**

- Line Interval Convolution
- 2D steady flow
- PDF describes the magnitude & direction of each vector in the field
- LIC representation of the gradient field, color encodes magnitude of uncertainty





R. S. Allendes Osorio, K. W. Brodlie.Uncertain Flow Visualization using LIC.In Theory and Practice of Computer Graphics, 2009.

## **2D Vector Fields**

- Texture-based
- Particle positions along streamlines
- Measuring errors and their influence on position





R. Botchen, D. Weiskop, T. Ertl. Texture-based visualization of uncertainty in flow fields. In IEEE Vis, 2005.



R. Botchen, D. Weiskopf, T. Ertl. Interactive visualisation of uncertainty in flow fields using texture-based techniques. In International Symposium on Flow Visualisation, 2006.

### **Reaction Diffusion Vector Field Visualization**









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A.R. Sanderson, C.R. Johnson, R.M. Kirby. "Display of Vector Fields Using a Reaction Diffusion Model," In Proceeding of IEEE Visualization 2004, pp. 115--122. 2004

A.R. Sanderson, R.M. Kirby, C.R. Johnson, L. Yang. "Advanced Reaction-Diffusion Models for Texture Synthesis," In Journal of Graphics Tools, Vol. 11, No. 3, pp. 47--71. 2006.



### **Streamlines**

- Uncertainty from numerical algs for particle tracing in fluid flow
- Highlight sensitivity of algorithm choice particularly near critical pts

S. Lodha, A. Pang, R. Sheehan, C. Wittenbrink. UFLOW: visualizing uncertainty in fluid flow. In Proc IEEE Vis, 1996.

S. Lodha, C. Wilson, R. Sheehan. "LISTEN: sounding uncertainty visualization". In Proceedings Visualization '96, pp. 189--195, 1996



Modulate pitch based on uncertainty

#### **Differences between 2 streamlines**



### Streamline Variability Plots for Characterizing the Uncertainty in Vector Field Ensembles



Fig. 1. From a set of streamlines in an ensemble of vector fields (left), our method generates an abstract visualization of the major trends in this set (middle). For each trend, a region of high confidence and a representative streamline-median is extracted. The relative strength of a trend is indicated by the thickness of its median line and by the bar plot on the right. Our method works in 2D and 3D (right), as well as for particle trajectories in time-dependent fields.



Ferstl, F., Bürger, K., & Westermann, R. (2015). Streamline variability plots for characterizing the uncertainty in vector field ensembles. IEEE Transactions on Visualization and Computer Graphics, 22(1), 767-776.

## **3D Meteorological Trajectory**

- •Estimate uncertainty due to interpolation
- User seeded trajectories
- Prune trajectories with high uncertainty





R. Boller, S. Braun, J. Miles, D. Laidlaw. Application of Uncertainty Visualization Methods to Meteorological Trajectories. In Earth Science Informatics, 3(1-2), 2010.

### **DTI Tensor Uncertainty Visualization**



(a) Uncertainity cones [50]



(c) Decomposed ensemble representation [110]





(b) HiFiVE Glyphs [90]



× ×

(d) ODF glyphs [96]



(a) Spaghetti plot [51]



(c) Illustrative visualization [15]



(b) Wrapped streamlines [19]



(d) Connectivity mapping [55]



Siddiqui, F., Höllt, T., & Vilanova, A. (2021). Uncertainty in the DTI Visualization Pipeline. Anisotropy Across Fields and Scales, 125.

### **Topological Uncertainty**





M. Otto, T. Germer, H.C. Hege, H. Theisel. Uncertain 2D Vector Field Topology. In CGF, 29(2), 2010.

### Visualizing uncertainty in topological structures





Merge Tree: a topological summary of scalar fields





Merge trees that arise from an ensemble of scalar fields

- 1. Compute an average merge tree from an ensemble
- 2. Uncertainty visualization of the average tree captures structural variations among the ensembles

Lin Yan, Yusu Wang, Elizabeth Munch, Ellen Gasparovic, Bei Wang. A Structural Average of Labeled Merge Trees for Uncertainty Visualization, IEEE VIS, 2019. arXiv: 1908.00113.

### **Volumetric Data - Isosurfacing**

- Isosurfaces show where a volumetric data value lies in space
- Map uncertainties to:
  - -hue, saturation, brightness
  - -texture mapping
- Isovalue eases display

P. Rhodes, R. Laramee, R.D. Bergeron, T. Sparr. Uncertainty Visualization Methods in Isosurface Rendering.

In EUROGRAPHICS 2003 Short Papers, 2003.





64^3



16^3



### Visualization Uncertainties - Isosurfaces

- Uncertainty from differences in isosurface creation
- Compare
  - marching cubes & marching cubes with ambiguous cell correction
  - interpolation schemes

A. Jospeh, S. Lodha, J. Renteria, A. Pang.UISURF: Visualizing Uncertainty in Isosurfaces.In Proc Computer Graphics and Imaging, 1999.





### **Possibilistic Marching Cubes**

Possibility theory is mathematically the simplest uncertainty theory for dealing with incomplete information. It is a natural means for quantifying epistemic uncertainty coming from lack of knowledge.



**FIG. 7:** Uncertain isocontours of the pressure field: (a) Ensemble of isocontours of the pressure field extracted from fluid simulation. (b) Possibilities (from PossMC) visualization. (c) Pignistic probabilities (from PossMC) visualization. (d) Necessities (from PossMC) visualization (the contour is faded looking due to the chosen colorbar: small necessity values are represented by white and light blue colors). The visualization has been overlaid on top of a LIC [49] visualization of one of the ensemble members.



He, Y., Mirzargar, M., Hudson, S., Kirby, R. M., & Whitaker, R. "An uncertainty visualization technique using possibility theory: Possibilistic marching cubes." International Journal for Uncertainty Quantification 5.5 (2015).

### Isosurface uncertainty

VIS 2003

### Two kinds of boundaries



Flow-line curvature (ter Haar Romeny et al., 1991) for uncertainty visualization:

- Material boundaries are intrinsic •
- If small  $\Delta$  isovalue  $\Rightarrow$  big  $\Delta$  isosurface orientation, isosurface probably not a good material boundary
- Qualitative indicator of surface model uncertainty Scientific Computing and Imaging Institute, University of Utah SCI VIS 2003



SCI

### Flow-line curvature results

VIS 2003

### Thumb from Visible Human Female, fresh CT:


### **Volumetric Data-Volume Rendering**

- Show data with high or low uncertainty
- Map data to color & uncertainty to opacity
- Add discontinuities to regions of high uncertainty (speckles, noise)

S. Djurcilov, K. Kim, P. Lermusiaux, A. Pang. Visualizing Scalar Volumetric Data with Uncertainty. In Computers and Graphics, vol. 26, 2002.



sualizing Scalar Volumetric Data with Uncertainty Computers and Graphics, vol. 26, 2002.





## **muView Visualization System**

Visualizing uncertainty in cardiac ischemia simulations





P. Rosen, B. Burton, K. Potter, C.R. Johnson. "muView: A Visual Analysis System for Exploring Uncertainty in Myocardial Ischemia Simulations," In Visualization in Medicine and Life Sciences III, Springer Nature, pp. 49--69. 2016.

## **Uncertainty Visualization**







Images Courtesy of Claes Lundström, Patric Ljung, Anders Persson, Anders Ynnerman

## **Uncertainty Visualization**





Claes Lundström, Patric Ljung, Anders Persson, Anders Ynnerman. Uncertainty Visualization in Medical Volume Rendering Using Probabilistic Animation, IEEE Transactions on Visualization and Computer Graphics, 13(2007): no. 5

### High Dimensional Transfer Functions Overview

- Create Transfer Functions (TFs) from user selected samples in spatial domain and error/uncertainty.
- Multiple linked views.





## Functional Box Plot

Boxplot statistics on 2D functions

Defined on the function, rather than point-wise

*Functional Boxplots.* <u>Ying Sun</u>, <u>Marc G. Genton</u>. J. of Comp. and Graphical Statistics 20:2, 2011, 316-334.



## Functional Box Plot

#### **Band Depth**

# The amount of time a function lies within the set of functions

*Functional Boxplots.* <u>Ying Sun</u>, <u>Marc G. Genton</u>. J. of Comp. and Graphical Statistics 20:2, 2011, 316-334. (a) Functional Boxplot



## **Contour Box Plots**

$$S \in \mathrm{sB}(S_1, \ldots S_j) \iff \bigcap_{k=1}^j S_k \subset S \subset \bigcup_{k=1}^j S_k$$



Whitaker, Mirzargar, Kirby, *IEEE Transactions on Visualization and Computer Graphics*, Vol. 19, No. 12, pp. 2713--2722, 2013.



M.G. Genton, C.R. Johnson, K. Potter, G. Stenchikov, Y. Sun. "Surface boxplots," In *Stat Journal*, Vol. 3, No. 1, pp. 1-11. 2014.

### **Ensemble Curved Boxplot**



M. Mirzargar, R. Whitaker, R. M. Kirby. "Curve Boxplot: Generalization of Boxplot for Ensembles of Curves," IEEE Transactions on Visualization and Computer Graphics, Vol. 20, No. 12, IEEE, pp. 2654-63. December, 2014.



## Surface Box Plots

Extension of band depth to 3D
Images rather than curves
Volume-based band-depth









#### Surface Boxplots.

Marc G. Genton, Christopher Johnson, Kristin Potter, Georgiy Stenchikov, and Ying Sun. Stat. 3:1, 2014, 1–11.

### Data-driven space-filling curves

- Data-driven space-filling curves
  - better coherency preservation (data value + position) than existing methods
  - 2D and 3D data
  - regular grids and multiscale
- A flexible Hamiltonian path method





L. Zhou, C. R. Johnson, D. Weiskopf. "Data-Driven Space-Filling Curves," In IEEE Transactions on Visualization and Computer Graphics, Vol. 27, No. 2, IEEE, pp. 1591-1600. 2021.

### Example - Brain Atlas

- Brain atlas of 2D MRI scans (176\*208 pixels); curve generation time: 3m49s
- Surface Boxplot; linearized based on the median





## **3D Transfer Function**



### **Visualizing Uncertainty Using Volume Rendering**





## **High Dimensional Transfer Functions**

- Create Transfer Functions (TFs) from user selected samples in • spatial domain and error/uncertainty.
- Multiple linked views.







### **Uncertainty-Aware Volume Visualization**



Visualization pipeline [Brodlie et al., 2012]



## Volume Rendering With Nonparametric Statistics





T. Athawale, B. Ma, E. Sakhaee, C.R. Johnson, and A. Entezari. Nonparametric Statistical Framework for Direct Volume Rendering of Uncertain Data. IEEE Visualization 2020, Oct. 2020.

### Models of Uncertainty

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### Tangle Function (Qualitative Comparisons)





Gaussian mixtures (four Gaussians) (Monte Carlo)

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Mean





Quantile interpolation

(four quantiles)



Quantile interpolation (eight quantiles)



Quantile interpolation (two quantiles)

### Tangle Function (Quantitative Comparisons)



### Visualizing Uncertain Multivariate Data Using Feature Confidence Level-Sets

- We explore whether extending the method by Zehner et al. to compute "Feature Confidence Level-Sets" is useful.
  - We would effectively be replacing "additional" feature level-sets with feature confidence level-sets.
- Assume each grid point has a distribution of values represented using a mean and standard deviation.
- For each grid point we compute an upper and lower confidence value using mean, standard deviation and confidence interval %.
- We perform a range intersection to determine if a trait exists at a grid point.



S. Sane, T. Athawale, and C.R. Johnson. Visualization of Uncertain Multivariate Data via Feature Confidence Level-Sets. EuroVis 2021.



Probabilistic Asymptotic Decider for Topological Ambiguity Resolution in Level-Set Extraction for Uncertain 2D Data



### Uncertainty Visualization of the Marching Squares and Marching Cubes Topology Cases - VIS 2021





Figure 5: Uncertainty visualizations for the stag beetle [21] hixel dataset at k = 900. The noise in the data results in breaking of the beetle leg in image (b). In probabilistic marching cubes, it is difficult to distinguish between the regions of high and topological uncertainty, which is easier using our visualizations in images (d-f). The relatively high sensitivity of the beetle leg topology to noise is detected in images (d-f) by the red regions. In image (f), the most probable level-set (gray) is overlaid with the entropy volume level-set (red) for entropy isovalue 5.

## **Perceptual Uncertainty**







Scientific Computing and Imaging Institute, University of Utah

## **Perceptual Uncertainty**







Scientific Computing and Imaging Institute, University of Utah



Path of water through a karst limestone structure of a ground sample analysis visualizing stone porosity and the spatial arrangement of the flow traces.



### **NIH Visible Male**





### **Visible Human - High Resolution**











#### The Need for High Resolution Visualization

"...the data show for the first time how detailed transport and chemistry effects can influence the mixing of reactive scalars. It may be advantageous to incorporate these effects within molecular mixing models. It is worth noting that at present it is impossible to obtain this type of information any other way than by using the type of highly resolved simulation performed here."



Lower Resolution

**High Resolution** 



Fig. 2. 372 evaluation paths that we observed across a sample of 86 publications with uncertainty visualization evaluations. The number of inlinks and outlinks differ for some nodes due to the same evaluation path representing multiple codes at a single level (e.g., Analysis).



#### In Pursuit of Error: A Survey of Uncertainty Visualization Evaluation

Jessica Hullman, Xiaoli Qiao, Michael Correll, Alex Kale, Matthew Kay IEEE Trans. Visualization & Comp. Graphics (Proc. InfoVis), 2019

## Summary

- Decision making, exploration, and understanding with uncertainty
- Currently, the study of uncertainty is usually performed in along disciplinary lines.
- We need more unified, interdisciplinary treatments of uncertainty:
- Representation, Quantification, Propagation, and Visualization of Uncertainty
- Need to also concentrate on *Certainty*



## **More Information**

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