CS 6170: Computational Topology, Spring 2019
Lecture 07

Topological Data Analysis for Data Scientists

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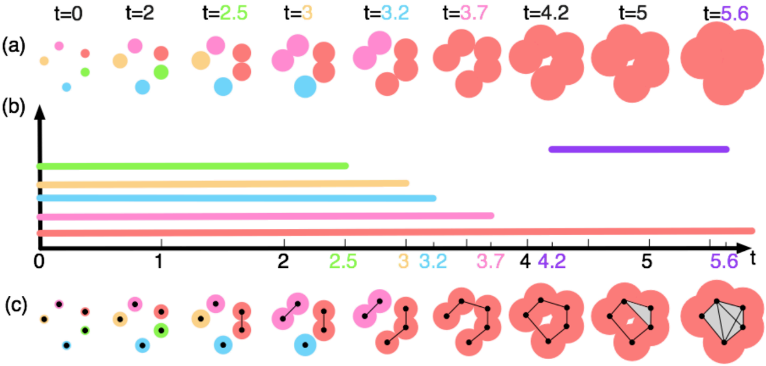
Persistent Homology in a Nutshell

Book Chapter C.VII, C.VIII

Persistent homology: simplicial representations



Wong et al. (2016)

Ripser Demo

https://github.com/Ripser/ripser

Ripser setup

```
git clone https://github.com/Ripser/ripser.git
cd ripser
make
./ripser examples/sphere_3_192.lower_distance_matrix
```

Ripser setup

```
Beis-MacBook-Pro-2018:ripser beiphillips$ ./ripser examples/sphere_3_192.lower_distance_matrix
value range: [0.00367531,2]
sparse distance matrix with 192 points and 17857/18431 entries
persistence intervals in dim 0:
 [0,0.00367531)
 [0,0.00391946)
 [0,0.0120098)
 [0,0.0173134)
 [0,0.0217315)
 [0,0.0311057)
 [0,0.037954)
 [0,0.0387451)
 [0,0.0433426)
 [0,0.0471186)
 [0,0.0487061)
 [0,0.048826)
 [0,0.0496187)
 [0,0.0515157)
 [0,0.0522132)
 [0,0.0559886)
 [0,0.0560443)
 [0,0.057488)
 [0,0.0593071)
 [0,0.0623736)
  ......
 [0,0.332695)
 [0, )
persistence intervals in dim 1:
 [0.542696,0.558863)
 [0.531636,0.578093)
 [0.530723,0.576869)
 [0.463389,0.505345)
 [0.445398,0.448892)
 [0.443911,0.54761)
 [0.431628,0.477277)
 [0.413789,0.487379)
 [0.412572,0.46308)
  ......
```

Ripser: example 1

square.txt

```
0 0
0 1
1 1
1 0
```

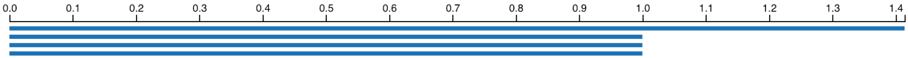
```
[beis-mbp-2018:ripser beiphillips$ ./ripser ../data/square.txt --format point-cloud
point cloud with 4 points in dimension 2
value range: [1,1.41421]
distance matrix with 4 points
persistence intervals in dim 0:
 [0,1)
 [0,1)
 [0,1)
 [0, )
persistence intervals in dim 1:
 [1,1.41421)
```

Ripser

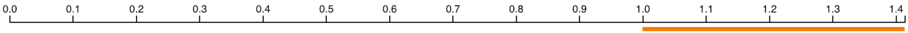
Load a point cloud to compute Vietoris–Rips persistence barcodes in dimensions 0 to 1 and up to distance :

Choose File square.txt

Persistence intervals in dimension 0:



Persistence intervals in dimension 1:



Elapsed time: 0.012 seconds

Ripser: example 2

triangle.txt

```
0 0
2 0
0 1
```

```
beis-mbp-2018:ripser beiphillips$ ./ripser ../data/triangle.txt --format point-cloud
point cloud with 3 points in dimension 2
value range: [1,2.23607]
sparse distance matrix with 3 points and 2/4 entries
persistence intervals in dim 0:
 [0,1)
 [0,2)
 [0, )
persistence intervals in dim 1:
```

Ripser: example 2

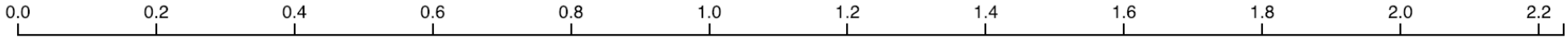
Ripser

Load a [point cloud] to compute Vietoris–Rips persistence barcodes in dimensions [0] to [1] and up to distance [ ]:

[Choose File] triangle.txt

Persistence intervals in dimension 0:

Persistence intervals in dimension 1:



Elapsed time: 0.011 seconds

Ripser: example 3

cube.txt

```
5 0 0
5 5 0
0 5 0
0 0 0
5 0 5
5 5 5
0 5 5
0 0 5
```

```
[beis-mbp-2018:ripser beiphillips$ ./ripser ../data/cube.txt --format point-cloud
point cloud with 8 points in dimension 3
value range: [5,8.66025]
distance matrix with 8 points
persistence intervals in dim 0:
 [0,5)
 [0,5)
 [0,5)
 [0,5)
 [0,5)
 [0,5)
 [0,5)
 [0, )
persistence intervals in dim 1:
 [5,7.07107)
 [5,7.07107)
 [5,7.07107)
 [5,7.07107)
 [5,7.07107)
```

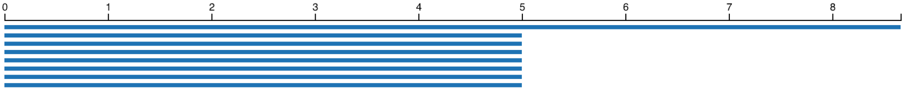
Ripser: example 3

Ripser

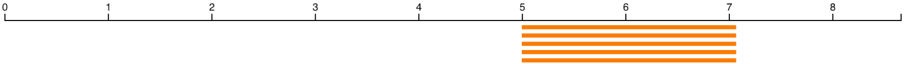
Load a point cloud to compute Vietoris–Rips persistence barcodes in dimensions 0 to 1 and up to distance :

Choose File cube.txt

Persistence intervals in dimension 0:



Persistence intervals in dimension 1:

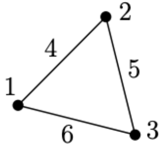


Elapsed time: 0.023 seconds

Persistent Homology Computation By Examples

Book Chapter C.VII, C.VIII

Persistent homology: Example 1



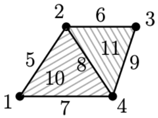
$$\partial = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline & & & 1 & & 1 \\ & & & 1 & 1 & \\ & & & & 1 & 1 \\ & & & & & \\ & & & & & \\ & & & & & \end{array}$$

$$\Rightarrow \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6(+5+4) \\ \hline & & & 1 & & \cancel{1} \\ & & & \boxed{1} & 1 & \cancel{1} \\ & & & & \boxed{1} & \cancel{1} \\ & & & & & \\ & & & & & \\ & & & & & \end{array}$$

What is the barcode?

Dim 0: [1,), [2, 4), [3, 5). Dim 1: [6,)

Persistent homology: Example 2



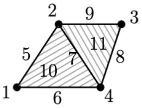
$$
\partial = \begin{array}{c}
 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11
\end{array}
\begin{array}{ccccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11
\end{array}
\left[\begin{array}{ccccccccccc}
 & & & & 1 & & 1 & & & & \\
 & & & & 1 & 1 & & 1 & & & \\
 & & & & & 1 & & & 1 & & \\
 & & & & & & 1 & 1 & 1 & & \\
 & & & & & & & & & 1 & 1 \\
 & & & & & & & & & & \\
 & & & & & & & & & 1 & \\
 & & & & & & & & & 1 & 1 \\
 & & & & & & & & & & 1 \\
 & & & & & & & & & & \\
 & & & & & & & & & &
\end{array}\right]
$$

$$
\Rightarrow
\begin{array}{c|ccccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8(+7+5) & 9 & 10 & 11 \\
\hline
1 & & & & & 1 & & 1 & \cancel{1} & \cancel{1} & & \\
2 & & & & & \boxed{1} & 1 & & \cancel{1} & \cancel{1} & & \\
3 & & & & & & \boxed{1} & & & \cancel{1} & & \\
4 & & & & & & & \boxed{1} & \cancel{1} & \cancel{1} & & \\
5 & & & & & & & & & & 1 & 1 \\
6 & & & & & & & & & & & \\
7 & & & & & & & & & & 1 & \\
8 & & & & & & & & & & \boxed{1} & 1 \\
9 & & & & & & & & & & & \boxed{1} \\
10 & & & & & & & & & & & \\
11 & & & & & & & & & & &
\end{array}
$$

Dim 0: [1,), [2,5), [3, 6), [4, 7). Dim 1: [8,10), [9,11).

Persistent homology: Example 3

Same simplicial complex, different ordering.



$\delta =$

	1	2	3	4	5	6	7	8	9	10	11
1					1	1					
2					1		1		1		
3								1	1		
4						1	1	1			
5										1	
6										1	
7										1	1
8											1
9											1
10											
11											

$\Rightarrow$

	1	2	3	4	5	6	7(+6+5)	8(+6)	9(+8+5)	10	11
1					1	1	~~1~~	**1**	~~1~~		
2					$\boxed{1}$		~~1~~		~~1~~		
3								$\boxed{1}$	~~1~~		
4						$\boxed{1}$	~~1~~	~~1~~			
5										1	
6										1	
7										$\boxed{1}$	1
8											1
9											$\boxed{1}$
10											
11											

Dim 0: [1,), [2,5), [3, 8), [4, 6). Dim 1: [7,10), [9,11).

References I

Edelsbrunner, H. and Harer, J. (2010). *Computational Topology: An Introduction*. American Mathematical Society, Providence, RI, USA.

Wong, E., Palande, S., Wang, B., Zielinski, B., Anderson, J., and Fletcher, P. T. (2016). Kernel partial least squares regression for relating functional brain network topology to clinical measures of behavior. *International Symposium on Biomedical Imaging (ISBI)*.