Department of Mathematics, University of Utah

Applied Complex Variables and Asymptotic Methods

MTH6720 – Section 01 – Spring 2024

Midterm formula sheet

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1. Given any integer k, a point $a \in \mathbb{C}$, and a simple closed loop C not intersecting a, then,

$$\oint_C (z-a)^k \, \mathrm{d}z = \begin{cases} 2\pi i, & k = -1 \text{ and } C \text{ encloses } a \\ 0, & \text{otherwise} \end{cases}$$

2. Suppose f is analytic on an open simply connected domain containing a simple closed loop C. Then for all integers $n \geq 0$ and all z enclosed by C,

$$f^{(n)}(z) = \frac{n!}{2\pi i} \oint_C \frac{f(w)}{(w-z)^{n+1}} \, \mathrm{d}w,$$

3. The coefficients for a z-centered Laurent series of the function f are given by,

$$c_n = \frac{1}{2\pi i} \oint_C \frac{f(w)}{(w-z)^{n+1}} \, \mathrm{d}w$$

4. If a continuous f is bounded over a contour C of finite length, i.e., $|f(z)| \leq M < \infty$ for all $z \in C$ and $\int_C |\, \mathrm{d}z| = L < \infty$, then

$$\left| \int_C f(z) \, \mathrm{d}z \right| \leq ML$$

Midterm **Name:** _____

MATH6720, Section 01 March 1, 2024

This test is:

- closed-book
- closed-notes (but you may refer to the formula sheet provided)
- no-calculator
- 50 minutes

Indicate your answers clearly, and show your work. Partial credit will be awarded based on work shown. Full credit will not be awarded without some work shown.

There are 4 questions, some with multiple parts. You are **required** to complete question 1, and in addition you must complete **two out of three** problems of your choice from questions 2, 3, and 4. Each question you complete is worth a total of 20 points. (60 points total)

The anatomy of this midterm is as follows:

- Cover page (this page)
- A single page containing the statements for all 4 questions (one page, no space for work)
- You are given a separate blue book in which to complete the questions.