Department of Mathematics, University of Utah

Applied Complex Variables and Asymptotic Methods

MATH 6720 – Section 001 – Spring 2024

Homework 5 Solutions

Residue Calculus, I

Due: Friday, March 15, 2024

Below, problem C in section A.B is referred to as exercise A.B.C.

Text: *Complex Variables: Introduction and Applications*, Ablowitz & Fokas,

Exercises: 4.1.1, parts (b), (d), and (e)
4.1.2, parts (b) and (c)
4.1.8

Submit your homework assignment on Canvas via Gradescope.

4.1.1. Evaluate the integrals $\frac{1}{2\pi i}\oint_C f(z)\,\mathrm{d}z$, where C is the unit circle centered at the origin and $f(z)$ is given below.

(b) $\frac{\cosh(1/z)}{z}$

(d) $\frac{\log(z+2)}{2z+1}$, principal branch

(e) $\frac{z+1/z}{z(2z-1/2z)}$

4.1.2. Evaluate the integrals $\frac{1}{2\pi i}\oint_C f(z)\,\mathrm{d}z$, where C is the unit circle centered at the origin with $f(z)$ given below. Do these problems by both (i) enclosing the singular points inside C and (ii) enclosing the singular points outside C (by including the point at infinity). Show that you obtain the same result in both cases.

(b) $\frac{z^2+1}{z^3}$

(c) $z^2 e^{-1/z}$

4.1.8. Suppose $f(z)$ is a meromorphic function (i.e., $f(z)$ is analytic everywhere in the finite z plane except at isolated points where it has poles) with N simple zeros (i.e., $f(z_0) = 0$, $f'(z_0) \neq 0$) and M simple poles inside a circle C. Show that,

$$\frac{1}{2\pi i}\oint_C \frac{f'(z)}{f(z)}\,\mathrm{d}z = N - M.$$

Akil Narayan: akil (at) sci.utah.edu