Department of Mathematics, University of Utah

**Applied Complex Variables and Asymptotic Methods**
**MATH 6720 – Section 001 – Spring 2024**
**Homework 4 Solutions**
**Laurent Series**

**Due: Friday, Feb 23, 2024**

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Below, problem C in section A.B is referred to as exercise A.B.C.
Text: *Complex Variables: Introduction and Applications*, Ablowitz & Fokas,

Exercises: 3.3.3
3.3.4
3.5.1, parts a) - f)
3.5.2, parts a) - c)

Submit your homework assignment on Canvas via Gradescope.

**3.3.3.** Given the function

$$f(z) = \frac{z}{(z-2)(z+i)},$$

expand $f(z)$ in a Laurent series in powers of $z$ in the regions,
(a) $|z| < 1$
(b) $1 < |z| < 2$
(c) $|z| > 2$

**3.3.4.** Evaluate the integral $\oint_C f(z)\,\mathrm{d}z$ where $C$ is the unit circle centered at the origin and $f(z)$ is given as follows,
(a) $\frac{e^z}{z^3}$
(b) $\frac{1}{z^2 \sin z}$
(c) $\tanh z$
(d) $\frac{1}{\cos 2z}$
(e) $e^{1/z}$

**3.5.1.** Discuss the type of singularity (removable, pole and order, essential, branch, cluster, natural barrier, etc.); if the type is a pole give the strength of the pole, and give the nature (isolated or not) of all singular points associated with the following functions. Include the point at infinity.
(a) $\frac{e^{z^2}-1}{z^2}$
(b) $\frac{e^{2z}-1}{z^2}$
(c) $e^{\tan z}$
(d) $\frac{z^3}{z^2+z+1}$

(e) $\frac{z^{1/3}-1}{z-1}$

(f) $\log(1+z^{1/2})$

**3.5.2.** Evaluate the integral $\oint_C f(z)\,\mathrm{d}z$, where $C$ is a unit circle centered at the origin, and where $f(z)$ is given below.

(a) $\frac{g(z)}{z-w}$, $g(z)$ entire

(b) $\frac{z}{z^2-w^2}$

(c) $ze^{1/z^2}$