Department of Mathematics, University of Utah

Applied Complex Variables and Asymptotic Methods

MATH 6720 – Section 001 – Spring 2024

Homework 2 Solutions

Analytic functions and integration, I

Due: Friday, Feb 2, 2024

Below, problem C in section A.B is referred to as exercise A.B.C.
Text: *Complex Variables: Introduction and Applications*, Ablowitz & Fokas,

Exercises: 2.1.1
2.1.5
2.2.1
2.2.2
2.2.3
2.4.1
2.4.4

Submit your homework assignment on Canvas via Gradescope.

2.1.1. Which of the following satisfy the Cauchy-Riemann (C-R) equations? If they satisfy the C-R equations, give the analytic function of z.

(a) $f(x,y) = x - iy + 1$
(b) $f(x,y) = y^3 - 3x^2y + i(x^3 - 3xy^2 + 2)$
(c) $f(x,y) = e^y(\cos x + i \sin y)$

2.1.5. Let $f(z)$ be analytic in some domain. Show that $f(z)$ is necessarily a constant if either the function $\overline{f(z)}$ is analytic or $f(z)$ assumes only pure imaginary values in the domain.

2.2.1. Find the location of the branch points and discuss possible branch cuts for the following functions:

(a) $\frac{1}{(z-1)^{1/2}}$
(b) $(z + 1 - 2i)^{1/4}$
(c) $2 \log z^2$
(d) $z^{\sqrt{2}}$

2.2.2. Determine all possible values and give the principal value of the following numbers (put in the form $x + iy$):

(a) $i^{1/2}$
(b) $\frac{1}{(1+i)^{1/2}}$
(c) $\log(1 + \sqrt{3}i)$
(d) $\log i^3$
(e) $i^{\sqrt{3}}$
(f) $\sin^{-1} \frac{1}{\sqrt{2}}$

2.2.3. Solve for z:
(a) $z^5 = 1$
(b) $3 + 2e^{z-i} = 1$
(c) $\tan z = 1$

2.4.1. From the basic definition of complex integration, evaluate the integral $\oint_C f(z)\,\mathrm{d}z$, where C is the parameterized unit circle enclosing the origin, $C:\ x(t) = \cos t,\ y(t) = \sin t$ or $z = e^{it}$, and where $f(z)$ is given by,
(a) z^2
(b) $\overline{z}^2$
(c) $\frac{z+1}{z^2}$

2.4.4. Use the principal branch of $\log z$ and $z^{1/2}$ to evaluate,
(a) $\int_{-1}^{1} \log z\,\mathrm{d}z$
(b) $\int_{-1}^{1} z^{1/2}\,\mathrm{d}z$