Department of Mathematics, University of Utah

Applied Complex Variables and Asymptotic Methods

MATH 6720 – Section 001 – Spring 2024

Homework 1

Basics of complex numbers

Due Friday, Jan 19, 2024

Submit your solutions online through Gradescope. Below, problem C in section A.B is referred to as exercise A.B.C.
Text: *Complex Variables: Introduction and Applications*, Ablowitz & Fokas,

Exercises: 1.1.2 (b,d)
1.1.3
1.2.1
1.2.5
Supplement 1.1
Supplement 1.2

1.1.2. Express each of the following in the form $a + bi$, where a and b are real:
(b) $\frac{1}{1+i}$
(d) $|3 + 4i|$

1.1.3. Solve for the roots of the following equations:
(a) $z^3 = 4$
(b) $z^4 = -1$
(c) $(az + b)^3 = c$, where $a, b, c > 0$
(d) $z^4 + 2z^2 + 2 = 0$

1.2.1. Sketch the regions associated with the following inequalities. Determine if the region is open, closed, bounded, or compact.
(a) $|z| \leq 1$
(b) $|2z + 1 + i| < 4$
(c) $\Re z \geq 4$
(d) $|z| \leq |z + 1|$
(e) $0 < |2z - 1| \leq 2$

1.2.5. Use any method to determine series expansions for the following functions:
(a) $\frac{\sin z}{z}$
(b) $\frac{\cosh z - 1}{z^2}$
(c) $\frac{e^z - 1 - z}{z}$

Supplement 1.1. Use de Moivre's Theorem to show that the two expressions,

$$\cos(n\theta), \qquad \frac{\sin((n+1)\theta)}{\sin\theta},$$

for arbitrary $n \in \mathbb{N}$, are both degree-n polynomials of $\cos\theta$.

Supplement 1.2. Prove the triangle inequality: Given $z_1, z_2 \in \mathbb{C}$, then

$$||z_1| - |z_2|| \le |z_1 + z_2| \le |z_1| + |z_2|.$$