Math 6620: Analysis of Numerical Methods, II

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Math 6620: Analysis of Numerical Methods, II

- In-person (unless otherwise specified)
- Grading based on ∼biweekly homework assignments and 2 exams
- Core topics covered are finite-difference methods for PDEs, time-differencing for IVPs, global/spectral methods

Most non-technical information in what follows is on the syllabus.

Theme of this class

Analysis of Numerical Methods, II, or "Basics of numerical solutions to O/PDEs"

Major goals (and warnings) for this class:

- Provide an overview/survey of basic numerical methods
- Not to provide substantial in-depth and detailed knowledge of solving particular PDEs

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- Present you with resources where you could find more detailed information
- Some (important!) mathematical and algorithmic minutiae are glossed over
- Furnish experience for implementing numerical schemes

Course meetings:

- MWF @ 12:55 - 1:45pm, HEB 2002
- Can be virtual (Zoom) on request
- Attendance not required
- I'll use course slides exclusively – these will be uploaded to the course webpage after class

High-level logistics

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Grading for the course:

- Grade exclusively based on exams + homework assignments
- Two exams: one midterm (Feb 28), and one final exam (April 30)
- Exams are standard written exams, no extra material allowed

A rough outline of what we'll discuss in the "core" topics:

- Background: Basic theory of linear partial differential equations
- Examples, and elliptic, parabolic, hyperbolic problems
- Desiderata for numerical schemes – stability, convergence, efficiency
- Finite-difference methods
- Numerical methods for initial value problems
- Global Fourier and spectral methods

Throughout, we'll discuss theory, algorithms, and software considerations.

Course references

There is no formal textbook for this course.

I'll provide slides (like these) that are a fairly detailed outline of topics covered. I apologize in advance for the highly probable typos and mistakes

There are numerous textbooks that I'll draw from for material, and I'll identify appropriate texts and chapter/section numbers during class meetings:

Randall J. LeVeque (2007). *Finite Difference Methods for Ordinary and Partial Differential Equations: Steady-State and Time-Dependent Problems*. SIAM. ISBN: 978-0-89871-783-9

Hans Petter Langtangen and Svein Linge (2017). *Finite Difference Computing with PDEs: A Modern Software Approach*. Springer. ISBN: 978-3-319-55456-3

Heinz-Otto Kreiss, Joseph Oliger, and Bertil Gustafsson (2013). *Time-Dependent Problems and Difference Methods*. John Wiley & Sons. ISBN: 978-1-118-54852-3

Uri M. Ascher and Linda R. Petzold (1998). *Computer Methods for Ordinary Differential Equations and Differential-Algebraic Equations*. SIAM. ISBN: 978-1-61197-139-2

Claudio Canuto et al. (2011). *Spectral Methods: Fundamentals in Single Domains*. 1st ed. 2006. Corr. 4th printing 2010 edition. Berlin ; New York: Springer. ISBN: 978-3-540-30725-9

Jie Shen, Tao Tang, and Li-Lian Wang (2011). *Spectral Methods: Algorithms, Analysis and Applications*. Springer Science & Business Media. ISBN: 978-3-540-71041-7

Jan S. Hesthaven, Sigal Gottlieb, and David Gottlieb (2007). *Spectral Methods for Time-Dependent Problems*. Cambridge University Press. ISBN: 0-521-79211-8

Grading: Homeworks and exams

Homeworks:

- Due approximately every 2 weeks
- Exercises with theory/written and coding components
- Provide direct templates for exam problems
- You are encouraged to work in groups, but each person must craft and submit individual work

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Exams: 2 in-class exams

- Midterm exam: Wednesday, Feb 28
- Final exam: Tuesday, April 30

This is a qualifying exam course for the Mathematics PhD track.

A grade of A will earn you a "High pass" for the corresponding qualifying exam.

A grade of A- or B+ will earn you a "Pass" for the corresponding qualifying exam.

The grad bulletin is the authority on this: `http://www.math.utah.edu/dept/gradbull.pdf`

General guidelines

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I will send any general announcements through email.

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See the syllabus for detailed information and policies on student integrity, inclusivity, discrimination/harrassment, social equity, the English language medium, undocumented student support, student veterans, ADA, student wellness, UCC, sexual misconduct, campus safety, and the Office of the Dean of Students.

Semester calendar
(Subject to change!)

Day	Date	Topic
Monday	January 8, 2024	Hello
Wednesday	January 10, 2024	Basic PDE concepts
Friday	January 12, 2024	Finite differences for 1D Poisson problems
Monday	January 15, 2024	No class: Margin Luther King Jr. Day
Wednesday	January 17, 2024	Finite differences for 1D Poisson problems
Friday	January 19, 2024	Finite differences for multidimensional Poisson problems
Monday	January 22, 2024	Finite differences for multidimensional Poisson problems
Wednesday	January 24, 2024	Initial value problems, I
Friday	January 26, 2024	Initial value problems, II
Monday	January 29, 2024	Initial value problems, III
Wednesday	January 31, 2024	Multi-stage methods, I
Friday	February 2, 2024	Multi-stage methods, II
Monday	February 5, 2024	Multi-step methods, I
Wednesday	February 7, 2024	Multi-step methods, II
Friday	February 9, 2024	Multi-step methods, III
Monday	February 12, 2024	Time-stepping miscellany
Wednesday	February 14, 2024	Stability + well-posedness for linear PDEs
Friday	February 16, 2024	Parabolic equations, I
Monday	February 19, 2024	No class: Presidents Day
Wednesday	February 21, 2024	Parabolic equations, II
Friday	February 23, 2024	Parabolic equations, III
Monday	February 26, 2024	Von Neumann stability
Wednesday	February 28, 2024	**Midterm Exam**
Friday	March 1, 2024	No class
Monday	March 4, 2024	No class: Spring break
Wednesday	March 6, 2024	No class: Spring break
Friday	March 8, 2024	No class: Spring break
Monday	March 11, 2024	Fourier series, I
Wednesday	March 13, 2024	Fourier series, II
Friday	March 15, 2024	Fourier interpolation
Monday	March 18, 2024	Non-smooth problems
Wednesday	March 20, 2024	Weak solutions
Friday	March 22, 2024	Weighted residual methods
Monday	March 25, 2024	Fourier Galerkin methods
Wednesday	March 27, 2024	Fourier collocation methods
Friday	March 29, 2024	Time-dependent Fourier spectral methods, I
Monday	April 1, 2024	Time-dependent Fourier spectral methods, II
Wednesday	April 3, 2024	Orthogonal polynomials, I
Friday	April 5, 2024	Orthogonal polynomials, II
Monday	April 8, 2024	Polynomial spectral methods, I
Wednesday	April 10, 2024	Polynomial spectral methods, II
Friday	April 12, 2024	Polynomial spectral methods, III
Monday	April 15, 2024	Buffer day
Wednesday	April 17, 2024	Buffer day
Friday	April 19, 2024	Buffer day
Monday	April 22, 2024	Review
Wednesday	April 24, 2024	Reading day
~~Thursday~~	~~April 25, 2024~~ 8:00am-10:00am	**FINAL EXAM**

Tues. Apr 30, 2024 1pm-3pm

 Ascher, Uri M. and Linda R. Petzold (1998). *Computer Methods for Ordinary Differential Equations and Differential-Algebraic Equations*. SIAM. ISBN: 978-1-61197-139-2.

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