Math 5760/6890: Introduction to Mathematical Finance

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August 22, 2023

Mathematical Finance

This course concerns the *science* of the financial world:

- financial instruments and securities: loans, bonds, stocks, options, derivatives, etc.
- determination of the past/present/future value of securities
- models that predict the future value of securities

This science is exercised worldwide by banks, traders, governments, companies, researchers, etc.

Skills you'll learn/exercise

- Language
  Mathematical finance itself involves learning a nontrivial vocabulary.
- Models
  Financial instruments are, or can be valued, through numerous types of models.
- Mathematics
  Mathematics is a set of tools that are used to examine/probe/understand models.
- Simulations
  Many models are more easily explorable using computer simulations, involving coding.

In this course, we will

- become familiar with the modern financial system and markets
- understand foundational concepts in investment and risk
- internalize the time value of money, and its implications on markets and investments
- model investments and quantitatively determine returns and risks
- gain capabilities to model financial portfolios
- understand and exercise the binomial options model
- explore the Black-Scholes-Merton model for options

What is out-of-bounds

You will NOT learn the following:

- Easy ways to make money
- How to accurately predict trajectories of stocks (or any other risky security)
- How to become a sophisticated securities investor
- Identification of real-world arbitrage involving risky securities
- Advanced financial models and mathematics

Where to get information about the course?

There are two online sources of information for this course:

- Canvas
  - Syllabus
  - Homework+project assignments
  - Homework+project submission (Gradescope)
  - Grades
- Public-facing webpage
  `http://www.sci.utah.edu/~akil/math5760`
  - Syllabus
  - Slides
  - Homework+project assignments
  - Any other handouts

Prerequisites + expectations

You should be familiar with the following to feel comfortable in this class:

- probability basics (random variables, fundamental operations)
- linear algebra (linear systems, eigenvalues)
- differential equations (solutions to linear differential equations)

It will be (very) helpful if you have some background/interest in at least some real-world concepts surrounding finance:

- loans / mortgages
- stocks / bonds
- filing taxes in the US

Class logistics

This class meets Tues + Thurs from 9:10am-10:30am here (WEB L126).

This will mostly involve lectures (loosely speaking: I hope for discussion!)

For convenience, I will record (over Zoom) lectures and post them on Canvas.
(I.e., your audio is being recorded!)

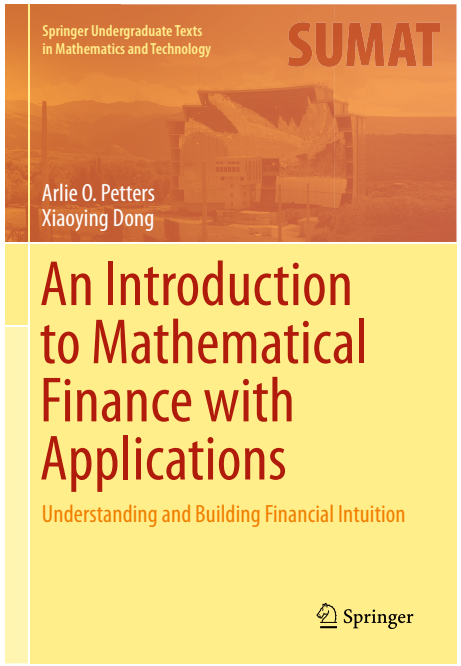
The Zoom recordings are a *convenience*; I consider in-person attendance when possible as mandatory.

But attendance is not a factor in your grade.

During class, I will supplement slides with handwritten notes/computations.

All slides will be made available on the publicly facing website.

Textbook



Access to this book is mandatory.

This book is offered through the Inclusive Access program.

See the syllabus for other useful supplements.

Homework

Homework will work in the following way for this course:

- Assignments will be collected approximately weekly (starting week 3 of class)
- Assignments will be due by 11:59pm on Tuesdays
- Submissions will be electronic through Gradescope (accessible via Canvas)
- Solutions will be posted after assignments are collected
- Late assignments will be accepted with a 25% penalty levied per day late
  Exceptions will be made only with prior approval or with documentation of extenuating circumstances.
- Your lowest homework score over the semester will be dropped

See the syllabus for how homework factors into the final grade.

You are welcome (and encouraged) to work in groups for homework assignments, but each person much turn in their own individual, original work.

Projects

There will be two projects over the course of the semester.

The projects aim to be more holistic projects around finance (interpretation, simulation, explanation).

You will have approximately two weeks to complete each project.

- The first project has a (tenative) due date of Thursday, October 19.
- The second project has a (tenative) due date of Thursday, December 7.

Projects will again be submitted through Gradescope.

See the syllabus for how projects factor into the final grade.

Exam

There is a single, final exam for this course. Per university schedule, this exam will take place on

Friday, December 15, 8:00am - 10:00am
Location: WEB L126 (the normal classroom)

The final exam will be a comprehensive, open-book, open-notes exam.

See the syllabus for how the exam factors into the final grade.

The tentative plan

Day	Date	Text Section(s)	Topic
Tuesday	August 22, 2023	1.1	Hello + basics
Thursday	August 24, 2023	1.2, 1.3	Securities and markets
Tuesday	August 29, 2023	2.1-2.4	Interest
Thursday	August 31, 2023	2.5-2.7	Net present value
Tuesday	September 5, 2023	2.8-2.10	Stocks and bonds
Thursday	September 7, 2023	—	Review: Probability, linear algebra, and differential equations
Tuesday	September 12, 2023	—	Review: Probability, linear algebra, and differential equations
Thursday	September 14, 2023	3.1	Markowitz portfolios
Tuesday	September 19, 2023	3.1-3.4	Markowitz portfolios
Thursday	September 21, 2023	3.4-3.7	N-security portfolios
Tuesday	September 26, 2023	3.4-3.7	N-security portfolios
Thursday	September 28, 2023	3.4-3.7	N-security portfolios
Tuesday	October 3, 2023	4.1	Capital market theory
Thursday	October 5, 2023	4.2	Risk measures
Tuesday	October 10, 2023	3.4-3.7	No class: Fall Break
Thursday	October 12, 2023	3.4-3.7	No class: Fall Break
Tuesday	October 17, 2023	4.3	Linear factor models
Thursday	October 19, 2023	—	No class
Tuesday	October 24, 2023	5.1	Binomial tree models
Thursday	October 26, 2023	5.1-5.2	Binomial tree models
Tuesday	October 31, 2023	5.2-5.3	Continuous-time models
Thursday	November 2, 2023	5.3-5.4	Continuous-time models
Tuesday	November 7, 2023	5.4	Continuous-time models
Thursday	November 9, 2023	6.7-6.8	The Itô integral
Tuesday	November 14, 2023	6.7-6.8	Itô's formula
Thursday	November 16, 2023	6.9	Geometric Brownian motion
Tuesday	November 21, 2023	7.2-7.3	Forward and futures
Thursday	November 23, 2023	3.4-3.7	No class: Thanksgiving break
Tuesday	November 28, 2023	7.5	Options
Thursday	November 30, 2023	8.1	The Black-Scholes-Merton model
Tuesday	December 5, 2023	8.2	Options pricing
Thursday	December 7, 2023	8.4	Risk-neutral pricing
Friday	December 15, 2023	8:00am-10:00am	**FINAL EXAM**

References I

 Petters, Arlie O. and Xiaoying Dong (2016). *An Introduction to Mathematical Finance with Applications: Understanding and Building Financial Intuition*. Springer. ISBN: 978-1-4939-3783-7.