Department of Mathematics, University of Utah

Introduction to Mathematical Finance
MATH 5760/6890 – Section 001 – Fall 2023
Homework 5
Capital Market Theory

Due: Thursday, Oct 5, 2023

Submit your homework assignment on Canvas via Gradescope.

1.) (Petters & Dong, Problem 4.1) State the definition of the *market portfolio*.

2.) (Petters & Dong, Problem 4.15, **parts a, b, d only**) Assume a risk-free rate of 1.5%. Answer the questions below using the information in the following table:

Portfolio	A	B	C	D	E	F
Expected Return	3.2%	8.1%	9.8%	5.1%	10.7%	4.8%
Standard Deviation	2.7%	9.9%	13.7%	6.2%	17%	6.1%

(a) Among the portfolios in the table, which one is closest to the market portfolio? Justify your answer.

(b) Plot the (best guess to the) capital market line (CML) based on your answer in part (a).

(d) Suppose we are willing to make an investment only with $\sigma = 6.2\%$. Is a return of 6.5% a realistic expectation for us?

3.) With a risk-free rate of $r > 0$, consider a capital asset line formed by the risk-free asset and a(ny) Markowitz efficient portfolio (σ_P, μ_P). (I.e., the Markowitz portfolio contains only risky securities.)

(a) Write the slope of the capital asset line as a function of μ_P, and maximize this expression to show that the maximum slope of the capital asset line corresponds to an efficient portfolio expected return of

$$\mu_P = -\frac{\alpha_{00} + r\alpha_{01}}{\alpha_{01} + r\alpha_{11}}, \qquad \alpha_{ij} = \boldsymbol{v}_i^T \boldsymbol{A} \boldsymbol{v}_j, \qquad i, j = 0, 1.$$

where $\boldsymbol{v}_0$ and $\boldsymbol{v}_1$ are vectors corresponding to the solution of the risk-optimal Markowitz portfolio (see slide L09-S05). Hence, this value of μ_P correponds to the market portfolio. (You may assume that critical points correspond to local maxima without verifying it for this problem.)

(b) Use the expression for the expected return μ_G of the global variance-minimizing portfolio on lecture slide L09-S05 to show that the above expression for the market portfolio mean μ_P is equivalent to,

$$\mu_P = \frac{1}{\mu_G - r}\left(\frac{\alpha_{00}}{\alpha_{11}} - r\mu_G\right),$$

(c) (**Math 6890 students only**) If $\mu_G > r$, show that the market portfolio expected return μ_P given by the formula above satisifes $\mu_P > \mu_G$.