Department of Mathematics, University of Utah

Advanced Optimization

MATH6880 – 004/7875-017 – Spring 2022

Project guidelines and potential papers

A required component of this course is the in-class presentation of either a paper, or a summary of a project that you have completed. Potential types of presentations are:

- Overview of a research paper (see below for suggestions). This includes a high-level summary of the problem the paper is attempting to solve, the main ideas of the approach, a description of the numerical results (if appropriate), and a discussion of the advantages/disadvantages and open problems or future directions from this work. You need not be an expert in the content of the paper, and you need not fully understand all technical details, but you absolutely should be well-versed enough to present the paper and answer some basic questions. You are encouraged to read related references as well to gain a fuller picture of the landscape of the paper's focus.
- Presentation of a project you have pursued during this semester. "Project" is loosely defined – it can be a numerical implementation of an algorithm to solve a problem, an empirical investigation of several algorithms, a presentation of theory that you have learned, etc. The major goal is to have a well-crafted message and presentation.

In terms of topics, I am quite flexible – anything optimization "related" is fine. See below for some paper suggestions. (They are only suggestions, you may choose other options.)
In general I am happy to meet to discuss anything related to these projects, including deciding on a topic, settling on what to present, how much detail to present, etc. Please do reach out if you'd like to discuss at any point.

Timeline

Here is a proposed timeline for these presentations:

- Now until March 11 – Decide on a topic/paper and finalize a presentation date.
- By March 11 – Communicate with me your decision on topic and your preferred presentation date. We can discuss over email, virtually over Zoom, or in-person.
- March 13 – I will announce the presentation lineup and topics
- March 14 - April 21 – Presentations. I plan on one per class meeting.

You can interpret the above to mean that **March 11 is the deadline for deciding on a project topic + presentation date**. I am aiming to have one presentation per class meeting, which means that I will fill people into slots on a first-come first-served basis.

Presentation "Guidelines" and Preparation

I am leery of providing rules on presentations since I don't want to impose too many constraints. But here is a general outline of what I expect:

- Presentation length: At least 20 minutes, as this is probably the minimum amount of time required to adequately give background on the project. But I'd be very happy if your presentation turned out to be somewhat longer. As a general rule of thumb, most folks generally underestimate the amount of time they need to speak.
- Presentation format: you are welcome to use electronic slides, or present on the whiteboard. (We do not have a chalkboard in our classroom.) I would recommend slides if you're going to show numerical results/pictures. The class has an HDMI input, and I will always have a USB-C adapter + HDMI cable. If you need other adapters, please let me know as soon as you can so I can arrange to have them available. If you prefer to present virtually, please let me know well ahead of time so we can iron out logistics for that.
- Depending on the topic, I may decide to begin class with a short discussion that sets up your presentation. I will try to come prepared as well to have things to discuss after your presentation, but none of this should impact your plans for presentation.
- I would encourage you to share with me either a draft of your slides or an outline of your presentation a few days before your talk to give me notice about what you'll talk about.

Potential papers/topics

The ultimate goal of this is for you to present on a topic that you are excited about. Below I list some topics that either I am familiar with, or would be interesting for me, but you should prioritize your own interest in choosing your topics. In particular, do not consider the list below as a comprehensive collection of materials: There exist appropriate references that I have not listed in relevant sub-topics below, and there exist entire sub-topics that I have omitted.

General convex optimization

- [5], *Living on the edge: A geometric theory of phase transitions in convex optimization*
- [54], *Scalable Semidefinite Programming*
- [18], *Convex Relaxation of Discrete Vector-Valued Optimization Problems*
- [41], *Characterizing Bad Semidefinite Programs: Normal Forms and Short Proofs*
- [23], *On the Simplicity and Conditioning of Low Rank Semidefinite Programs*
- [11], *On Representer Theorems and Convex Regularization*

General (not necessarily convex) optimization

- [30], *Semi-Infinite Programming: Theory, Methods, and Applications*
- [53], *High-Dimensional Gaussian Sampling: A Review and a Unifying Approach Based on a Stochastic Proximal Point Algorithm*
- [33], *Newton's Method in Mixed Precision*
- [16], *First-Order Methods for Nonconvex Quadratic Minimization*
- [9], *Solving Large-Scale Sparse PCA to Certifiable (Near) Optimality*
- [43], *The generalized orthogonal Procrustes problem in the high noise regime*
- [21], *Identifying and Attacking the Saddle Point Problem in High-dimensional Non-convex Optimization*

- [10], *Optimization Methods for Large-Scale Machine Learning*
- [45], *EM algorithm and variants: an informal tutorial*
- [13], *An Interior Point Algorithm for Large-Scale Nonlinear Programming*

Neural networks

- [3], *Fast Convex Pruning of Deep Neural Networks*
- [1], *Learning regularization parameters of inverse problems via deep neural networks*
- [19], *Global Minima of Overparameterized Neural Networks*
- [28], *Generative Adversarial Nets*
- [46], *Improving GANs Using Optimal Transport*

Optimal control and PDE-constrained optimization

- [7], *Optimal control of the cylinder wake in the laminar regime by trust-region methods and POD reduced-order models*
- [20], *Surrogate-Based Optimization Using Multifidelity Models with Variable Parameterization and Corrected Space Mapping*
- [44], *Optimal Solvers for PDE-Constrained Optimization*
- [55], *Second-Order Necessary Conditions for Stochastic Optimal Control Problems*
- [2], *A trust-region framework for constrained optimization using reduced order modeling*

Optimal transport

- [40], *Optimal Transport with Proximal Splitting*
- [26], *Stochastic Optimization for Large-scale Optimal Transport*
- [42], *Computational Optimal Transport*

Inference and inverse problems

- [17], *Designing Optimal Spectral Filters for Inverse Problems*
- [25], *Bayesian inference with optimal maps*
- [32], *Sensor Selection via Convex Optimization*
- [37], *Stein Variational Gradient Descent: A General Purpose Bayesian Inference Algorithm*
- [29], *Analysis of Discrete Ill-Posed Problems by Means of the L-Curve*
- [50], *Inverse Problems: A Bayesian Perspective*

Nonnegative matrix factorization

- [52], *On the Complexity of Nonnegative Matrix Factorization*
- [8], *Algorithms and applications for approximate nonnegative matrix factorization*
- [27], *On the geometric interpretation of the nonnegative rank*
- [24], *When Does Non-Negative Matrix Factorization Give a Correct Decomposition into Parts?*
- [36], *Projected Gradient Methods for Nonnegative Matrix Factorization*

Matrix completion and subset selection

- [15], *Exact Matrix Completion via Convex Optimization*
- [14], *The Power of Convex Relaxation: Near-Optimal Matrix Completion*
- [4], *Greedy column subset selection: new bounds and distributed algorithms*
- [51], *Column Subset Selection, Matrix Factorization, and Eigenvalue Optimization*
- [35], *Non-convex low-rank matrix recovery with arbitrary outliers via median-truncated gradient descent*
- [31], *A Lipschitz Matrix for Parameter Reduction in Computational Science*

Feasibility problems and least squares

- [49], *A Randomized Kaczmarz Algorithm with Exponential Convergence*
- [22], *A Sampling Kaczmarz–Motzkin Algorithm for Linear Feasibility*
- [39], *Stochastic gradient descent, weighted sampling, and the randomized Kaczmarz algorithm*
- [6], *On Projection Algorithms for Solving Convex Feasibility Problems*

Graph partitioning, clustering

- [47], *Graph clustering*
- [48], *Normalized cuts and image segmentation*
- [38], *A tutorial on spectral clustering*
- [34], *Semi-supervised graph clustering: a kernel approach*
- [12], *Recent Advances in Graph Partitioning*

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