Department of Mathematics, University of Utah

PDEs for Engineering Students

MTH3150 – Section 002 – Spring 2021

Midterm 2 formula sheet

The following are some standard thermal properties of materials. Units follow in [brackets].

- $u(x,t)$ — Temperature as a function of space and time [temperature]
- $e(x,t)$ — Thermal energy density [energy/volume]
- $\phi(x,t)$ — Thermal heat flux [energy/(time $\times$ area)]
- $\rho(x)$ — Mass density [mass/volume]
- $c(x)$ — Specific heat [energy/(mass $\times$ temperature)]
- K_0 — Thermal conductivity [energy/(time $\times$ temperature $\times$ length)]

You may find the following integrals helpful. In all the following, n and m are non-negative integers, and L is any positive number.

$$\int_0^L \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right)\,\mathrm{d}x = \begin{cases} 0, & n \neq m \\ L/2, & n = m \end{cases}$$

$$\int_0^L \cos\left(\frac{n\pi x}{L}\right)\cos\left(\frac{m\pi x}{L}\right)\,\mathrm{d}x = \begin{cases} 0, & n \neq m \\ L/2, & n = m \neq 0 \\ L, & n = m = 0 \end{cases}$$

$$\int_0^L \cos\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right)\,\mathrm{d}x = 0 \quad (n > 0)$$

$$\int_0^L \sin\left(\frac{(2n+1)\pi x}{2L}\right)\sin\left(\frac{(2m+1)\pi x}{2L}\right)\,\mathrm{d}x = \begin{cases} 0, & n \neq m \\ L/2, & n = m \end{cases}$$

$$\int_0^L \cos\left(\frac{(2n+1)\pi x}{2L}\right)\cos\left(\frac{(2m+1)\pi x}{2L}\right)\,\mathrm{d}x = \begin{cases} 0, & n \neq m \\ L/2, & n = m \end{cases}$$

A Fourier Series on the interval $[-L, L]$ takes the form

$$\sum_{n=0}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right).$$

The coefficients of the Fourier Series associated to a function $f(x)$ are

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,\mathrm{d}x$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)\,\mathrm{d}x, \qquad n \geq 1$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)\,\mathrm{d}x, \qquad n \geq 1$$

A Fourier Cosine series on the interval $[0, L]$ takes the form

$$\sum_{n=0}^{\infty} A_n \cos\left(\frac{n\pi x}{L}\right),$$

The coefficients of the Fourier Cosine Series associated to a function $f(x)$ are

$$A_0 = \frac{1}{L}\int_0^L f(x)\,\mathrm{d}x$$
$$A_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)\,\mathrm{d}x, \qquad n \geq 1$$

A Fourier Sine series on the interval $[0, L]$ takes the form

$$\sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{L}\right),$$

The coefficients of the Fourier Sine Series associated to a function $f(x)$ are

$$B_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)\,\mathrm{d}x, \qquad n \geq 1$$