Partial Differential Equations

3150

MATH ~~66~~10 Lecture 00

Section/chapter 1

January 21, 2021

Partial Differential Equations (PDEs)

PDEs are, unsurprisingly, equations involving partial derivatives.

Typically the function being differentiated is unknown, and the goal is to solve (compute an explicit expression for) the function.

Partial Differential Equations (PDEs)

PDEs are, unsurprisingly, equations involving partial derivatives.

Typically the function being differentiated is unknown, and the goal is to solve (compute an explicit expression for) the function.

Types of equations:

- *Algebraic* equations: Solve for x:

$$x^2 - 4 = 0 \qquad x = \pm 2$$

- (Ordinary) *differential* equations: Solve for $y(x)$:

$$y(x) \qquad \frac{\mathrm{d}y}{\mathrm{d}x} = 3y \qquad y(x) = \exp(3x),\ \pi \exp(3x),\ 25\exp(3x), \ldots$$

- *Partial differential* equations: Solve for $u(x,t)$:

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}.$$

ODE: $\frac{dy}{dx} = 3y \longrightarrow$ infinitely many solutions

(contrast w/ algebraic equations)

$y(x) = C \cdot \exp(3x)$ for any constant C.

A unique solution can be prescribed by furnishing an <u>initial condition</u>.

PDE: $u = u(x, t)$

"space" "time"

$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}$, $u(x,t) = \sin(x+t)$.

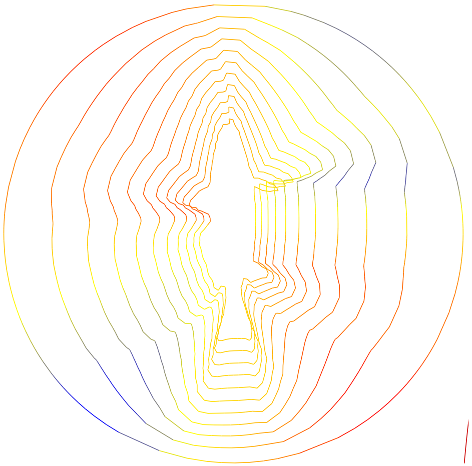
Like ODE's, infinitely many solutions exist, unless additional conditions are given ("initial", "boundary")

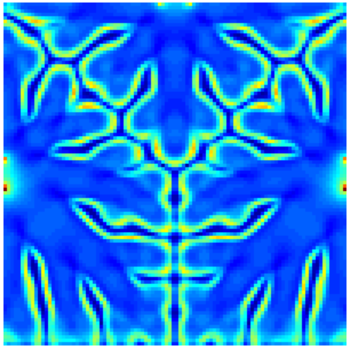
Partial Differential Equations (PDEs)

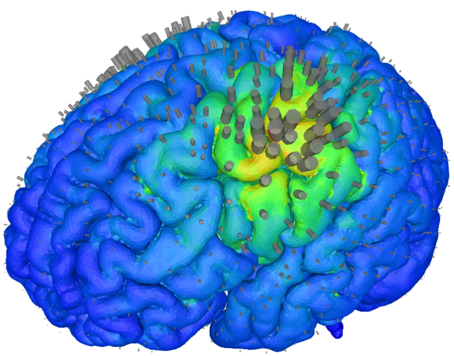
PDEs are, essentially, *mathematical models*.

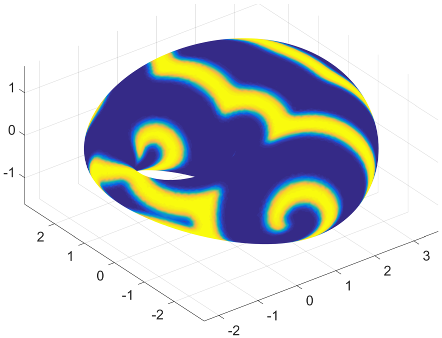
- Astronomical/cosmological models
- Biophysical models
- Chemical flows and reactions
- Data analysis and clustering
- Fluid dynamics
- Imaging
- Neurological models
- Optimization and design
- Population dynamics, swarm behavior
- Structural mechanics/dynamics

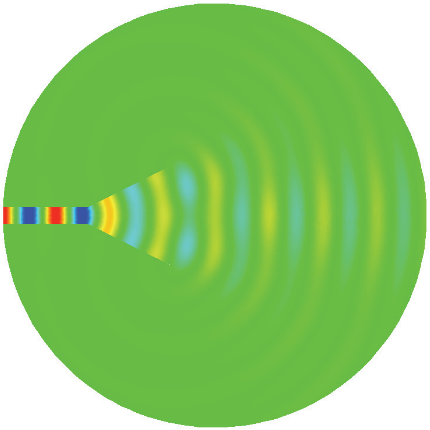
PDE applications











Scope of this class

$u_t = \frac{\partial u}{\partial t}$, $u_{xx} = \frac{\partial^2 u}{\partial x^2}$

This class is a first look into PDEs.

Specifically, we consider the following linear PDEs:

- The heat equation: $u_t = u_{xx}$ $u = u(x, t)$, u: temperature.
- Laplace's equation: $u_{xx} + u_{yy} = 0$ $u = u(x, y)$
- The wave equation: $u_{tt} = u_{xx}$ $u = u(x, t)$

We are interested in (a) solving these PDEs, (b) understanding what kind of behavior these PDEs model.

Scope of this class

This class is a first look into PDEs.

Specifically, we consider the following linear PDEs:

- The heat equation: $u_t = u_{xx}$
- Laplace's equation: $u_{xx} + u_{yy} = 0$
- The wave equation: $u_{tt} = u_{xx}$ $+ u_{yy}$ $\quad u = u(x, y, t)$

We are interested in (a) solving these PDEs, (b) understanding what kind of behavior these PDEs model.

There are many more PDEs that we don't cover in this class.

$u = u(x,t)$

$u_t + u \cdot u_x = 0$ (Burgers' eqn.)

$u_t = u_x$

Prerequisites

We assume some background:

- Fluency in calculus (derivatives+integrals of common functions, u-substitution, integration by parts, ...)
- Familiarity with ordinary differential equations (simple harmonic oscillators)

This class will be difficult without knowledge of the above.

OH: Monday 11am-12pm (via Zoom)
Thursday 3pm-4pm ← not today (starting Jan 25.)
or by appointment