Department of Mathematics, University of Utah

PDEs for Engineering Students

MTH3150 – Section 002 – Spring 2021

Final exam formula sheet

Note: The final exam is cumulative!

The following are some standard thermal properties of materials. Units follow in [brackets].

- $u(x,t)$ — Temperature as a function of space and time [temperature]
- $e(x,t)$ — Thermal energy density [energy/volume]
- $\phi(x,t)$ — Thermal heat flux [energy/(time × area)]
- $\rho(x)$ — Mass density [mass/volume]
- $c(x)$ — Specific heat [energy/(mass × temperature)]
- K_0 — Thermal conductivity [energy/(time × temperature × length)]

You may find the following integrals helpful. In all the following, n and m are non-negative integers, and L is any positive number.

$$\int_0^L \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right)\,\mathrm{d}x = \begin{cases} 0, & n \neq m \\ L/2, & n = m \end{cases}$$

$$\int_0^L \cos\left(\frac{n\pi x}{L}\right)\cos\left(\frac{m\pi x}{L}\right)\,\mathrm{d}x = \begin{cases} 0, & n \neq m \\ L/2, & n = m \neq 0 \\ L, & n = m = 0 \end{cases}$$

$$\int_0^L \cos\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right)\,\mathrm{d}x = 0 \qquad (n > 0)$$

$$\int_0^L \sin\left(\frac{(2n+1)\pi x}{2L}\right)\sin\left(\frac{(2m+1)\pi x}{2L}\right)\,\mathrm{d}x = \begin{cases} 0, & n \neq m \\ L/2, & n = m \end{cases}$$

$$\int_0^L \cos\left(\frac{(2n+1)\pi x}{2L}\right)\cos\left(\frac{(2m+1)\pi x}{2L}\right)\,\mathrm{d}x = \begin{cases} 0, & n \neq m \\ L/2, & n = m \end{cases}$$

A Fourier Series on the interval $[-L, L]$ takes the form

$$\sum_{n=0}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right).$$

The coefficients of the Fourier Series associated to a function $f(x)$ are

$$\begin{aligned}
a_0 &= \frac{1}{2L}\int_{-L}^{L} f(x)\,\mathrm{d}x \\
a_n &= \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)\,\mathrm{d}x, && n \geq 1 \\
b_n &= \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)\,\mathrm{d}x, && n \geq 1
\end{aligned}$$

A Fourier Cosine series on the interval $[0, L]$ takes the form

$$\sum_{n=0}^{\infty} A_n \cos\left(\frac{n\pi x}{L}\right),$$

The coefficients of the Fourier Cosine Series associated to a function $f(x)$ are

$$\begin{aligned}
A_0 &= \frac{1}{L}\int_{0}^{L} f(x)\,\mathrm{d}x \\
A_n &= \frac{2}{L}\int_{0}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)\,\mathrm{d}x, && n \geq 1
\end{aligned}$$

A Fourier Sine series on the interval $[0, L]$ takes the form

$$\sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{L}\right),$$

The coefficients of the Fourier Sine Series associated to a function $f(x)$ are

$$B_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)\,\mathrm{d}x, \qquad n \geq 1$$

Let $f(x)$ and $F(\omega)$ be functions defined over the real number line. If f and F are Fourier transform pairs, then

$$F(\omega) = \mathcal{F}[f] = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)e^{i\omega x}\,\mathrm{d}x$$
$$f(x) = \mathcal{F}^{-1}[F] = \int_{-\infty}^{\infty} F(\omega)e^{-i\omega x}\,\mathrm{d}\omega,$$

where x is the physical variable and ω is the frequency variable. If f and F are Fourier Transform pairs, and h and H are Fourier Transform pairs, then the following table describes the relationship between g and its Fourier Transform G:

$g(x)$	$G(\omega)$
e^{-ax^2}	$\frac{1}{\sqrt{4\pi a}}e^{-\omega^2/4a}$
$\sqrt{\frac{\pi}{a}}e^{-x^2/4a}$	$e^{-a\omega^2}$
$\delta(x)$	$\frac{1}{2\pi}$
$f(x-a)$	$e^{ia\omega}F(\omega)$
$e^{-iax}f(x)$	$F(\omega-a)$
$\frac{\mathrm{d}f}{\mathrm{d}x}$	$-i\omega F(\omega)$
$ixf(x)$	$\frac{\mathrm{d}F}{\mathrm{d}\omega}$
$(f*h)(x)$	$F(\omega)H(\omega)$
$\frac{1}{2\pi}f(x)h(x)$	$(F*H)(\omega)$

Above, the convolution $*$ between two functions f and h is defined as

$$(f*h)(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(s)h(x-s)\,\mathrm{d}s.$$