Department of Mathematics, University of Utah

Introduction to Optimization

MATH 5770/6640, ME EN 6025 – Section 001 – Fall 2021

Homework 6

Convex optimization

Due December 7, 2021

Submit your homework assignment as a scanned copy **ON CANVAS**, to the `Homework 6` assignment.

Text: *Introduction to Nonlinear Optimization*, Amir Beck,

Exercises: # 8.2,
8.3,
8.4 (i), (ii), (iii), and only part (a) for each
11.2,
11.6 (iii) only,
P1 (**6000-level students only**)

Additional problems:

P1. (**6000-level students only**) Given $n \in \mathbb{N}$, let C be the closed, convex set of $n \times n$ positive semi-definite matrices,

$$C = \left\{ \boldsymbol{A} \in \mathbb{R}^{n \times n} \;\middle|\; \boldsymbol{A} \text{ is symmetric, } \boldsymbol{A} \succeq \boldsymbol{0} \right\}.$$

Given any $n \times n$ symmetric matrix $\boldsymbol{A}$, let $P_C(\boldsymbol{A})$ denote the $\| \cdot \|$-projection of $\boldsymbol{A}$ onto C. Show that both of the following statements are true,

$$P_C(\boldsymbol{A}) = \boldsymbol{U}\boldsymbol{\Lambda}_+\boldsymbol{U}^T, \qquad \| \cdot \| = \| \cdot \|_2 \text{ (the spectral or induced matrix 2-norm)}$$
$$P_C(\boldsymbol{A}) = \boldsymbol{U}\boldsymbol{\Lambda}_+\boldsymbol{U}^T, \qquad \| \cdot \| = \| \cdot \|_F \text{ (the Frobenius, or entrywise norm)}$$

where $\boldsymbol{A} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^T$ is the eigenvalue decomposition of $\boldsymbol{A}$, and $\boldsymbol{\Lambda}_+ = \max\{\boldsymbol{\Lambda}, 0\}$ with the max function applied componentwise.