

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF UTAH  
**Introduction to Optimization**  
MATH 5770/6640, ME EN 6025 – Section 001 – Fall 2021  
**Homework 5**  
**Convex functions**

**Due November 16, 2021**

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Submit your homework assignment as a scanned copy **ON CANVAS**, to the Homework 5 assignment.

Text: *Introduction to Nonlinear Optimization*, Amir Beck,

Exercises: # 7.1,  
7.3,  
7.7,  
7.10 (i), (ii), (iv), (v)  
7.25

**Additional problems:**

- P1.** A function  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  is *strongly convex* if there exists some  $\alpha > 0$  such that  $f(x) - \alpha\|x\|_2^2$  is convex. Show that strong convexity  $\Rightarrow$  strict convexity  $\Rightarrow$  convexity. In addition, give counterexamples to show that the reverse implications are not true.
- P2. (6000-level students only)** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be strictly monotone (either increasing or decreasing) such that  $f$  has a well-defined functional inverse  $f^{-1} : \mathbb{R} \rightarrow \mathbb{R}$ . (I.e., the range of  $f$  is  $\mathbb{R}$  and  $f$  is bijective.) Assume  $f$  is convex. If  $f$  is increasing, what can you say about  $f^{-1}$ ? What about if  $f$  is decreasing?