Department of Mathematics, University of Utah

Introduction to Optimization

MATH 5770/6640, ME EN 6025 – Section 001 – Fall 2021

Homework 4

Newton's method and convex sets

Due November 2, 2021

Submit your homework assignment as a scanned copy **ON CANVAS**, to the `Homework 4` assignment.

Some of the exercises below are computational. The book problems are explained in Matlab. You need not use Matlab to complete the assignment; numerical simulation with any programming language is acceptable.

Text: *Introduction to Nonlinear Optimization*, Amir Beck,

Exercises: # 5.2,
6.2,
6.7,
6.20

Additional problems:

P1. For each of the following statements, either prove that it is true, or give a counterexample showing that it is false in general.

a. If C_1 and C_2 are convex, then $C_1 \cup C_2$ is convex.
b. If C_1 and C_2 are (not necessarily convex) cones, then $C_1 \cup C_2$ is a (not necessarily convex) cone.
c. Consider the set of points $\boldsymbol{x}$ defined by finitely many linear inequalities, i.e., the set of points $\boldsymbol{x}$ defined by $\boldsymbol{A}\boldsymbol{x} \leq \boldsymbol{b}$, where $\boldsymbol{A}$ and $\boldsymbol{b}$ are an arbitrary matrix and vector, respectively, of conforming size, and the inequality is true elementwise. Then this set is convex.
d. A convex set C is bounded.
e. A convex set C is closed.
f. (**6000-level students only**) If C_1 and C_2 are convex sets, then $\text{conv}(C_1) \cup \text{conv}(C_2) = \text{conv}(C_1 \cup C_2)$.

P2. (**6000-level students only**) Consider the set of matrices in $\mathbb{R}^{n\times n}$ given by,

$$S_+(n) \coloneqq \left\{ \boldsymbol{A} \in \mathbb{R}^{n\times n} \;\middle|\; \boldsymbol{A} = \boldsymbol{A}^T \text{ and } \boldsymbol{A} \succ \boldsymbol{0} \right\}.$$

Prove that $S_+(n)$ is a convex cone.