Linear algebra preliminaries

MATH 6610 Lecture 00

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Notation

We'll use some standard math notation

- $\mathbb{C}$, $\mathbb{R}$, $\mathbb{N}$
- $\in$, $\forall$, $\exists$, !
- $\left\{x \in \mathbb{C} \;\middle|\; \mathrm{Im}\{x\} \in \mathbb{N}\right\}$
- $z = x + iy$ for $x, y \in \mathbb{R} \Longrightarrow \bar{z} = z^* := x - iy$

Vectors, matrices, etc:

- $\boldsymbol{u} \in \mathbb{C}^n$
- $\boldsymbol{A} \in \mathbb{C}^{m \times n}$
- linear independence
- rank
- transpose
- determinant
- matrix inverse

Hilbertian structure

$\mathbb{C}$ endowed with the standard inner product is a Hilbert space. If $\boldsymbol{u}, \boldsymbol{v} \in \mathbb{C}^n$,

- $\langle \boldsymbol{u}, \boldsymbol{v} \rangle$, $\|\boldsymbol{u}\|$
- $\angle(\boldsymbol{u}, \boldsymbol{v})$
- $\boldsymbol{u} \perp \boldsymbol{v}$
- $\mathrm{Proj}_{\boldsymbol{v}} \boldsymbol{u}$
- orthogonal and orthonormal sets

Unitary and orthogonal matrices

We'll frequently let $\boldsymbol{U} \in \mathbb{C}^{n \times n}$ denote a unitary/orthogonal matrix.

Vector norms

A map $\|\cdot\| : \mathbb{C}^n \to \mathbb{R}$ is a *norm* if it satisfies all the following properties:

- $\|\boldsymbol{x}\| \geqslant 0 \;\; \forall \boldsymbol{x} \in \mathbb{C}^n$
- $\|\boldsymbol{x}\| = 0$ iff $\boldsymbol{x} = \boldsymbol{0}$
- $\|\boldsymbol{x} + \boldsymbol{y}\| \leqslant \|\boldsymbol{x}\| + \|\boldsymbol{y}\| \;\; \forall \boldsymbol{x}, \boldsymbol{y} \in \mathbb{C}^n$
- $\|c\boldsymbol{x}\| = |c|\|\boldsymbol{x}\| \;\; \forall \boldsymbol{x} \in \mathbb{C}^n, \; c \in \mathbb{C}$.

For example there are vector p-norms.

Matrix norms

Norms on matrices can be *induced* by vector norms, but there are also non-induced norms.