Nonhomogeneous equations: Underdetermined Coefficients

MATH 2250 Lecture 24
Book section 5.5

October 22, 2019

Nonhomogeneous equations

We've previously focused on computing solutions to constant coefficient linear *homogeneous* equations:

$$y^{(n)} + \sum_{j=0}^{n-1} a_j y^{(j)} = 0,$$

for constants $a_0, \ldots, a_{n-1}$. Our focus here is on the associated *nonhomogeneous* equation:

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Recall there are three steps to solving this DE:

1. Compute the general solution to the associated *homogeneous* equation.
2. Compute *any* particular solution.
3. Linearly combine the particular and homogeneous solutions.

Step 2 is the focus of this section.

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Find a particular solution of $y'' - 4y = 2e^{3x}$.

Example (Example 7.5.3)

Find a particular solution of $3y'' + y' - 2y = 2\cos x$.

A possible complication

If the nonhomoegeneous function $f(x)$ coincides with the homogeneous solution, the previous strategies cannot be directly applied.

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The method of undetermined coefficients forms an ansatz for the particular solution by:

- including the terms in f including the terms in all derivatives of f
- eliminating *duplication* by multiplying by x^s, where s is the order of the characteristic equation root that causes duplication.

Examples

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Example (Example 5.5.6)

Solve the initial value problem:

$$\begin{aligned} y'' - 3y' + 2y = 3e^{-x} - 10\cos 3x, \\ y(0) = 1, \quad y'(0) = 2 \end{aligned}$$