# Reduced row-echelon form

MATH 2250 Lecture 12
Book section 3.3

September 17, 2019

# Echelon matrices



We've seen that **Echelon** matrices are those of the form

$$\begin{pmatrix} * & * \\ 0 & * \end{pmatrix}, \quad \begin{pmatrix} * & * & * \\ 0 & 0 & * \end{pmatrix}, \quad \begin{pmatrix} * & * & * \\ 0 & 0 & 0 \end{pmatrix},$$

These are matrices such that

- All zero-rows are located below any non-zero row
- A *leading* nonzero entry in row $j$ lies to the *right* of all *leading* nonzero entries in previous rows.

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We add two additional properties to arrive at **reduced Echelon** matrices

- A *leading* nonzero entry in a row equals $1$.
- A *leading* nonzero entry in a row is the only nonzero entry in its column.

# Reduced Echelon matrices

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & * & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & * & * \\ 0 & 0 & 0 \end{pmatrix},$$

*Reduced* Echelon matrices are very helpful in computing solutions to linear systems.

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To obtain a *reduced* Echelon matrix from an Echelon one:

- Divide each row by the leading entry.
- Use each leading entry to eliminate entries in the rows above it.

# Reduced Echelon matrices



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We recall that **all** linear systems have either $1$, $0$, or infinitely many solutions.

Reduced Echelon matrices help us determine which of these possibilities occur.

# Examples

## Example (Problem 3.3.7)

Compute the reduced Echelon form for

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

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Compute the reduced Echelon form for

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

## Example (Problem 3.3.8)

Compute the reduced Echelon form for

$$\begin{pmatrix} 1 & -4 & -5 \\ 3 & -9 & 3 \\ 1 & -2 & 3 \end{pmatrix}$$

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## Example (Problem 3.3.13)

Compute the reduced Echelon form for

$$\begin{pmatrix} 2 & 7 & 4 & 0 \\ 1 & 3 & 2 & 1 \\ 2 & 6 & 5 & 4 \end{pmatrix}$$

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## Example (Problem 3.3.14)

Compute the reduced Echelon form for

$$\begin{pmatrix} 1 & 3 & 2 & 5 \\ 2 & 5 & 2 & 3 \\ 2 & 7 & 7 & 22 \end{pmatrix}$$