Acceleration and Velocity Models

MATH 2250 Lecture 08
Book section 2.3

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Models for drag

We have seen that in free fall near the Earth's surface, the vertical position $x(t)$ of an object obeys the DE

$$x''(t) = -g,$$

where g is the acceleration of gravity.

We seek to make this model more realistic.

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The atmosphere (the air) imparts a force that counteracts motion through air resistance or "drag".

Basic principle: the faster an object moves, the more air resistance encountered.

A simple model: the force of resistance is given by

$$F_R = -kv^p, \quad \Longrightarrow \quad x''(t) = -g - kv^p,$$

where $v(t) = x'(t)$, and k and p are positive constants.

- Usually $1 \leqslant p \leqslant 2$, the value of which depends on whether something moves very quickly or very slowly.
- The value of k is determined by shape and size of the object.

Examples

Example

Compute the solution to the IVP

$$x''(t) = -g - kv, \qquad x(0) = x_0, \qquad x'(0) = v_0.$$

Show that v appoaches a constant, the *terminal velocity*, for large t.

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Example

Suppose that a body moves through a resisting medium with resistance proportional to its velocity v, so that $v'(t) = -kv$. (a) Show that its velocity and position at time t are given by $v(t) = v_0 e^{-kt}$ and,

$$x(t) = x_0 + \left(\frac{v_0}{k}\right)\left(1 - e^{-kt}\right).$$

(b) Conclude that the body travels only a finite distance, and find that distance.