# DE solutions through integration

MATH 2250 Lecture 02
Book section 1.2

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# "Easy" differential equations

The easiest differential equations in this class are those that don't (really) involve the dependent variable, e.g.,

$$y' = f(x),$$

for a given function $f(x)$.

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## Example

Compute the family of solution $y(x)$ to the DE

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## Example

Compute the solution to the initial value problem,

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \exp(t) + \log(t), \qquad\qquad x(1) = 0$$

For the first example, the family of solutions covers *all possible* solutions to the DE, and hence this is a **general solution**.

For the second example, the initial data restricts to a *single solution*, hence this is a **particular solution**.

# Higher-order equations

The same technique can be used to solve higher-order equations.

## Example

Solve the initial value problem

$$y'' = 1 + \sin t, \qquad y'(0) = 0, \qquad y(0) = 1$$

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Solve the initial value problem

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...and the same game can be repeated for even higher-order differential equations.

# Acceleration



Consider a variable $x$ denoting a position that depends on time $t$, $x(t)$.

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We have special names for certain derivatives of such a position function:

- $v = x'(t)$: "velocity"
- $a = x''(t)$: "acceleration"
- $j = x^{(3)}(t) = x'''(t)$: "jerk"
- $s = x^{(4)}(t)$: "snap" or "jounce"

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DE models positing constant acceleration are such commonplace that they deserve special note.

A rigid object is in free fall close to the surface of the Earth. If $y(t)$ denotes the object's position, derive a DE model for $y(t)$.

How many initial conditions are needed? What is the solution for the object's trajectory given these initial values?