Midterm 2 practice
MATH2250, Section 04

Name: ______________________________

October 4, 2019

This test is:

- closed-book
- closed-notes
- no-calculator
- 50 minutes

Indicate your answers clearly, and show your work. Partial credit will be awarded based on work shown. Full credit will not be awarded without some work shown.

For questions with multiple parts, **clearly indicate** your solution to each portion of the question.

Fun fact of life: if your work is not legible, I will not be able to read it. The ramifications of this outcome should be clear.

There are 4 questions with multiple parts; each question is worth a total of 10 points.

All pages are one-sided. If on any problem you require more space, use the back of the page.

Do not turn this page until directed to begin

1. (10 pts total) Given the following matrix and vectors:

$$\boldsymbol{A} = \begin{pmatrix} 2 & 2 & -2 \\ 1 & 3 & 0 \\ 1 & 1 & 0 \end{pmatrix}, \qquad \boldsymbol{v} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}, \qquad \boldsymbol{w} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$

a) (3 pts) Compute $\boldsymbol{A}\boldsymbol{v}$.

b) (3 pts) Compute $\boldsymbol{A}^{-1}$.

$$\boldsymbol{A} = \begin{pmatrix} 2 & 2 & -2 \\ 1 & 3 & 0 \\ 1 & 1 & 0 \end{pmatrix}, \qquad \boldsymbol{v} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}, \qquad \boldsymbol{w} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$

c) (4 pts) Compute the solution $\boldsymbol{x}$ to $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{w}$.

2.
(10 pts total) Define $\boldsymbol{A}$, $\boldsymbol{b}_1$, and $\boldsymbol{b}_2$ as:

$$\boldsymbol{A} = \begin{pmatrix} 1 & 4 & -1 \\ 2 & 3 & 0 \\ 1 & 1 & 1 \end{pmatrix}, \qquad \boldsymbol{b}_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \qquad \boldsymbol{b}_2 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

Compute $\boldsymbol{A}^{-1}$ <u>and</u> use it to compute the vectors $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ that satisfy $\boldsymbol{A}\boldsymbol{x}_1 = \boldsymbol{b}_1$ and $\boldsymbol{A}\boldsymbol{x}_2 = \boldsymbol{b}_2$.

3. (10 pts total) This question concerns vector spaces and subspaces.
a) (5 pts) Let V be the set of points $\boldsymbol{x} = (x_1, x_2, x_3, x_4)$ in $\mathbb{R}^4$ satisfying

$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ -1 & 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Is V a subspace? If so, prove it <u>and</u> provide a basis for V. If not, demonstrate why it is not a subspace.

b) (5 pts) Let W be the set of points $\boldsymbol{x} = (x_1, x_2, x_3, x_4)$ in $\mathbb{R}^4$ satisfying

$$|x_1 + x_2 + x_3 + x_4| = 1$$

Is W a subspace? If so, prove it <u>and</u> provide a basis for W. If not, demonstrate why it is not a subspace.

4. (10 pts) Consider a body that moves horizontally through a medium whose resistance is proportional to the velocity v. Assume that at time $t = 0$ the velocity is 5 m/s, and that at time $t = 1$ the velocity is 2 m/s. What is the total distance traveled by the object after $t = 0$?